

Summary of translation

The conclusion we arrived at, stated below, is true for a any function.

Theorem

If $g(x) = f(x - h) + k$ for every number x in the domain of f , then function g is function f translated $|h|$ units horizontally and $|k|$ units vertically.

The translation is

$$\left\{ \begin{array}{ll} \text{right } h \text{ units,} & \text{if } h > 0 \\ \text{left } h \text{ units,} & \text{if } h < 0 \\ \text{up } k \text{ units,} & \text{if } k > 0 \\ \text{down } k \text{ units,} & \text{if } k < 0 \end{array} \right.$$



You will often see $y - k = f(x - h)$ or sometimes $g(x) - k = f(x - h)$ instead of $g(x) = f(x - h) + k$.

Questions

[1] Notice that $g(x) = \frac{1}{x-3}$ and $f(x) = \frac{1}{x}$ produce different values for the same argument. For example, $f(4) = \frac{1}{4}$ but $g(4) = 1$. In fact, there is no x for which $f(x) = g(x)$. In what sense is g simply f translated to an different location?

[2] Suppose $g(x) = \frac{3}{x}$ and $f(x) = \frac{1}{x}$. Is g a translation of f ?

[3] Suppose $g(x) = \frac{2}{x}$, $f(x) = \frac{1}{x}$, and $h(x) = \frac{2}{x-7} - 3$. Is g more like f or more like h ?

[4] Suppose $g(x) = \frac{-4x+17}{x-5}$. Show that g is f translated where $f: f(x) = \frac{-3}{x}$.

* The notation " $f: f(x) = \frac{1}{x}$ " means "The function f defined by $f(x) = \frac{1}{x}$."