

## Derivative Calculations

In Exercises 1–8, given  $y = f(u)$  and  $u = g(x)$ , find  $dy/dx = f'(g(x))g'(x)$ .

1.  $y = 6u - 9$ ,  $u = (1/2)x^4$
2.  $y = 2u^3$ ,  $u = 8x - 1$
3.  $y = \sin u$ ,  $u = 3x + 1$
4.  $y = \cos u$ ,  $u = -x/3$
5.  $y = \cos u$ ,  $u = \sin x$
6.  $y = \sin u$ ,  $u = x - \cos x$
7.  $y = \tan u$ ,  $u = 10x - 5$
8.  $y = -\sec u$ ,  $u = x^2 + 7x$

In Exercises 9–18, write the function in the form  $y = f(u)$  and  $u = g(x)$ . Then find  $dy/dx$  as a function of  $x$ .

9.  $y = (2x + 1)^5$
10.  $y = (4 - 3x)^9$
11.  $y = \left(1 - \frac{x}{7}\right)^{-7}$
12.  $y = \left(\frac{x}{2} - 1\right)^{-10}$
13.  $y = \left(\frac{x^2}{8} + x - \frac{1}{x}\right)^4$
14.  $y = \left(\frac{x}{5} + \frac{1}{5x}\right)^5$
15.  $y = \sec(\tan x)$
16.  $y = \cot\left(\pi - \frac{1}{x}\right)$
17.  $y = \sin^3 x$
18.  $y = 5 \cos^{-4} x$

Find the derivatives of the functions in Exercises 19–38.

19.  $p = \sqrt{3 - t}$
20.  $q = \sqrt{2r - r^2}$
21.  $s = \frac{4}{3\pi} \sin 3t + \frac{4}{5\pi} \cos 5t$

$$22. s = \sin\left(\frac{3\pi t}{2}\right) + \cos\left(\frac{3\pi t}{2}\right) \quad [16-11-07Th]$$

$$23. r = (\csc \theta + \cot \theta)^{-1} \quad 24. r = -(\sec \theta + \tan \theta)^{-1}$$

$$25. y = x^2 \sin^4 x + x \cos^{-2} x \quad 26. y = \frac{1}{x} \sin^{-5} x - \frac{x}{3} \cos^3 x$$

$$27. y = \frac{1}{21}(3x - 2)^7 + \left(4 - \frac{1}{2x^2}\right)^{-1}$$

$$28. y = (5 - 2x)^{-3} + \frac{1}{8}\left(\frac{2}{x} + 1\right)^4$$

$$29. y = (4x + 3)^4(x + 1)^{-3} \quad 30. y = (2x - 5)^{-1}(x^2 - 5x)^6$$

$$31. h(x) = x \tan(2\sqrt{x}) + 7 \quad 32. k(x) = x^2 \sec\left(\frac{1}{x}\right)$$

$$33. f(\theta) = \left(\frac{\sin \theta}{1 + \cos \theta}\right)^2 \quad 34. g(t) = \left(\frac{1 + \cos t}{\sin t}\right)^{-1}$$

$$35. r = \sin(\theta^2) \cos(2\theta) \quad 36. r = \sec \sqrt{\theta} \tan\left(\frac{1}{\theta}\right)$$

$$37. q = \sin\left(\frac{t}{\sqrt{t+1}}\right) \quad 38. q = \cot\left(\frac{\sin t}{t}\right)$$

In Exercises 39–48, find  $dy/dt$ .

39.  $y = \sin^2(\pi t - 2)$
40.  $y = \sec^2 \pi t$
41.  $y = (1 + \cos 2t)^{-4}$
42.  $y = (1 + \cot(t/2))^{-2}$

Do #1-57  
odd

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43.  $y = \sin(\cos(2t - 5))$
44.  $y = \cos\left(5 \sin\left(\frac{t}{3}\right)\right)$
45.  $y = \left(1 + \tan^4\left(\frac{t}{12}\right)\right)^3$
46.  $y = \frac{1}{6}(1 + \cos^2(7t))^3$
47.  $y = \sqrt{1 + \cos(t^2)}$
48.  $y = 4 \sin(\sqrt{1 + \sqrt{t}})$

## Second Derivatives

Find  $y''$  in Exercises 49–52.

49.  $y = \left(1 + \frac{1}{x}\right)^3$
50.  $y = (1 - \sqrt{x})^{-1}$
51.  $y = \frac{1}{9} \cot(3x - 1)$
52.  $y = 9 \tan\left(\frac{x}{3}\right)$

## Finding Numerical Values of Derivatives

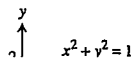
In Exercises 53–58, find the value of  $(f \circ g)'$  at the given value of  $x$ .

53.  $f(u) = u^5 + 1$ ,  $u = g(x) = \sqrt{x}$ ,  $x = 1$
54.  $f(u) = 1 - \frac{1}{u}$ ,  $u = g(x) = \frac{1}{1-x}$ ,  $x = -1$
55.  $f(u) = \cot \frac{\pi u}{10}$ ,  $u = g(x) = 5\sqrt{x}$ ,  $x = 1$
56.  $f(u) = u + \frac{1}{\cos^2 u}$ ,  $u = g(x) = \pi x$ ,  $x = 1/4$
57.  $f(u) = \frac{2u}{u^2 + 1}$ ,  $u = g(x) = 10x^2 + x + 1$ ,  $x = 0$
58.  $f(u) = \left(\frac{u-1}{u+1}\right)^2$ ,  $u = g(x) = \frac{1}{x^2} - 1$ ,  $x = -1$

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1.  $12x^3$
3.  $3 \cos(3x + 1)$
5.  $-\sin(\sin x) \cos x$
7.  $10 \sec^2(10x - 5)$
9. With  $u = (2x + 1)$ ,  $y = u^5$ :  $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = 5u^4 \cdot 2 = 10(2x + 1)^4$
11. With  $u = (1 - (x/7))$ ,  $y = u^{-7}$ :  $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = -7u^{-8} \cdot \left(-\frac{1}{7}\right) = \left(1 - \frac{x}{7}\right)^{-8}$
13. With  $u = ((x^2/8) + x - (1/x))$ ,  $y = u^4$ :  $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = 4u^3 \cdot \left(\frac{x}{4} + 1 + \frac{1}{x^2}\right) = 4\left(\frac{x^2}{8} + x - \frac{1}{x}\right)^3 \left(\frac{x}{4} + 1 + \frac{1}{x^2}\right)$
15. With  $u = \tan x$ ,  $y = \sec u$ :  $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = (\sec u \tan u)(\sec^2 x) = \sec(\tan x) \tan(\tan x) \sec^2 x$
17. With  $u = \sin x$ ,  $y = u^3$ :  $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = 3u^2 \cos x = 3 \sin^2 x (\cos x)$
19.  $-\frac{1}{2\sqrt{3-t}}$
21.  $\frac{4}{\pi}(\cos 3t - \sin 5t)$
23.  $\frac{\csc \theta}{\cot \theta + \csc \theta}$
25.  $2x \sin^4 x + 4x^2 \sin^3 x \cos x + \cos^{-2} x + 2x \cos^{-3} x \sin x$
27.  $(3x - 2)^6 - \frac{1}{x^3 \left(4 - \frac{1}{2x^2}\right)^2}$
29.  $\frac{(4x + 3)^3(4x + 7)}{(x + 1)^4}$
31.  $\sqrt{x} \sec^2(2\sqrt{x}) + \tan(2\sqrt{x})$
33.  $\frac{2 \sin \theta}{(1 + \cos \theta)^2}$
35.  $\frac{dr}{d\theta} = -2 \sin(\theta^2) \sin 2\theta + 2\theta \cos(2\theta) \cos(\theta^2)$

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37.  $\frac{dq}{dt} = \left(\frac{t+2}{2(t+1)^{3/2}}\right) \cos\left(\frac{t}{\sqrt{t+1}}\right)$
39.  $2\pi \sin(\pi t - 2) \cos(\pi t - 2)$
41.  $\frac{8 \sin(2t)}{(1 + \cos 2t)^5}$
43.  $-2 \cos(\cos(2t - 5))(\sin(2t - 5))$
45.  $\left(1 + \tan^4\left(\frac{t}{12}\right)\right)^2 \left(\tan^3\left(\frac{t}{12}\right) \sec^2\left(\frac{t}{12}\right)\right)$
47.  $-\frac{t \sin(t^2)}{\sqrt{1 + \cos(t^2)}}$
49.  $\frac{6}{x^3} \left(1 + \frac{1}{x}\right) \left(1 + \frac{2}{x}\right)$
51.  $2 \csc^2(3x - 1) \cot(3x - 1)$
53.  $5/2$
55.  $-\pi/4$
57.  $0$
59. (a)  $2/3$  (b)  $2\pi + 5$  (c)  $15 - 8\pi$  (d)  $37/6$  (e)  $-1$
- (f)  $\sqrt{2}/24$  (g)  $5/32$  (h)  $-5/(3\sqrt{17})$
61.  $5$
63. (a)  $1$  (b)  $1$
65. (a)  $y = \pi x + 2 - \pi$  (b)  $\pi/2$
67. 
69. 