Implication

p	q	$p \implies q$
T	T	T
T	F	F
F	T	T
F	F	T

TABLE 1.1. Truth table for $p \implies q$

The third and fourth rows of the truth table for $p \implies q$ are sometimes perplexing. Our goal is to understand why those two rows are exactly what we would want them to be.

Consider the proposition

(1.1) If Socrates is human, then Socrates is mortal.

A little reflection should convince you that proposition 1.1 means

(1.2) Either Socrates is not human or Socrates is mortal.

So, Proposition 1.1 and Proposition 1.2 should be true and false together. That is, Proposition 1.1 is true if and only if Proposition 1.2 is true.

If we let p represent "Socrates is human" and q represent "Socrates is mortal", then Table 1.2 is the truth table for Proposition 1.2.

p	q	$\neg p \, V q$
T	T	T
T	F	F
F	T	T
F	F	T

TABLE 1.2. Truth table for $\neg p \vee q$

Since Proposition 1.1 and Proposition 1.2 are true and false together, the body of Table 1.1 should be identical to the body of Table 1.2.

You might be surprised that you have known all along that the last two rows of the truth table for implication (Table 1.1) are as they should be.

Ask yourself how you would argue a statement of the form $p \implies q$ is false. Constructing a counterexample would do the job. Now, only an example in which p is true and q is false will be a counterexample. That is row 2 of the truth table.

You know better than to argue against a statement of the form $p \implies q$ by making up examples that match rows 3 or 4 of Table 1.1. Matching row 3 or row 4 will not show $p \implies q$ is false. But if a proposition is not false then it is true.

So, you knew the truth table for $p \implies q$ even before we started this discussion.