

Exam 2

Exam 2 will test proficiency using the rules for derivatives to find derivatives of functions. The exam will have enough questions so that you will have an average of approximately 2 minutes per question. You will not be given any extra time to complete the exam.

What is included in the exam

- (1) Using all the rules from textbook sections [3.3] to [3.7] to find derivatives of functions.

What is not included in the exam

- (1) Implicit differentiation.
- (2) Finding a derivative by using only the definition of derivative. This would be section [3.2].

Finding derivatives is a basic skill of mathematics (like multiplication in 4th grade) and must become *automatic*. It does not require any deep thinking. It is merely a matter of knowing the rules, recognizing patterns, and applying the rules.

The path to proficiency is practice! Do not be satisfied until you are getting all of the answers to all of the problems correct on the first try.

There are many examples and exercise problems in your textbook to use for practice. There is value to doing the same problems more than once. You have been provided with an additional problem set (a copy is included with this announcement). If you would like even more practice problems, just ask me.

My advice is to practice a couple of times each day starting today.

EXERCISES 2.5

1–4 ■ Find dy/dx and $dy/dx|_{x=1}$ in two ways: (a) using the Chain Rule and (b) without using the Chain Rule, as in Example 2.

1. $y = u^2$, $u = x^2 + 2x + 3$

2. $y = u^2 - 2u + 3$, $u = 5 - 6x$

3. $y = u^3$, $u = x + (1/x)$

4. $y = u - u^2$, $u = \sqrt{x} + \sqrt[3]{x}$

5–48 ■ Find the derivative of the function.

5. $F(x) = (x^2 + 4x + 6)^5$

6. $F(x) = (x^3 - 5x)^4$

7. $G(x) = (3x - 2)^{10}(5x^2 - x + 1)^{12}$

8. $g(t) = (6t^2 + 5)^3(t^3 - 7)^4$

9. $f(t) = (2t^2 - 6t + 1)^{-8}$

10. $f(t) = \frac{1}{(t^2 - 2t - 5)^4}$

11. $g(x) = \sqrt{x^2 - 7x}$

12. $k(x) = \sqrt[3]{1 + \sqrt{x}}$

13. $h(t) = \left(t - \frac{1}{t}\right)^{3/2}$

14. $F(s) = \sqrt{s^3 + 1}(s^2 + 1)^4$

15. $F(y) = \left(\frac{y - 6}{y + 7}\right)^3$

16. $s(t) = \sqrt[4]{\frac{t^3 + 1}{t^3 - 1}}$

17. $f(z) = \frac{1}{\sqrt[5]{2z - 1}}$

18. $f(x) = \frac{x}{\sqrt{7 - 3x}}$

19. $y = (2x - 5)^4(8x^2 - 5)^{-3}$

20. $y = (x^2 + 1)\sqrt[3]{x^2 + 2}$

21. $y = \tan 3x$

22. $y = 4 \sec 5x$

23. $y = \cos(x^3)$

24. $y = \cos^3 x$

25. $y = (1 + \cos^2 x)^6$

26. $y = \tan(x^2) + \tan^2 x$

27. $y = \cos(\tan x)$

28. $y = \sin(\sin x)$

29. $y = \sec^2 2x - \tan^2 2x$

30. $y = \sqrt{1 + 2 \tan x}$

31. $y = \csc(x/3)$

32. $y = \cot \sqrt[3]{1 + x^2}$

33. $y = \sin^3 x + \cos^3 x$

34. $y = \sin^2(\cos 4x)$

35. $y = \sin \frac{1}{x}$

36. $y = \frac{\sin^2 x}{\cos x}$

37. $y = \frac{1 + \sin 2x}{1 - \sin 2x}$

38. $y = x \sin \frac{1}{x}$

39. $y = \tan^2(x^3)$

40. $y = (\sin \sqrt{x^2 + 1})^{\sqrt{2}}$

41. $y = \cos^2(\cos x) + \sin^2(\cos x)$

42. $y = \sin(\sin(\sin x))$

43. $y = \sqrt{x + \sqrt{x}}$

44. $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$

45. $f(x) = [x^3 + (2x - 1)^3]^3$

46. $g(t) = \sqrt[4]{(1 - 3t)^4 + t^4}$

47. $y = \sin(\tan \sqrt{\sin x})$

48. $y = \sqrt{\cos(\sin^2 x)}$

49–52 ■ Find the equation of the tangent line to the curve at the given point.

49. $y = (x^3 - x^2 + x - 1)^{10}$, $(1, 0)$

50. $y = \sqrt{x + (1/x)}$, $(1, \sqrt{2})$

51. $y = \frac{8}{\sqrt{4 + 3x}}$, $(4, 2)$

52. $y = \sin x + \cos 2x$, $(\pi/6, 1)$

53. (a) Find an equation of the tangent line to the curve $y = \tan(\pi x^2/4)$ at the point $(1, 1)$.

(b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.

54. (a) The curve $y = |x|/\sqrt{2 - x^2}$ is called a **bullet-nose curve**. Find an equation of the tangent line to this curve at the point $(1, 1)$.

(b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.

55. (a) If $f(x) = \sqrt{1 - x^2}/x$, find $f'(x)$.

(b) Check to see that your answer to part (a) is reasonable by comparing the graphs of f and f' .

56. (a) If $f(x) = 1/(\cos^2 \pi x + 9 \sin^2 \pi x)$, find $f'(x)$.

(b) Check to see that your answer to part (a) is reasonable by comparing the graphs of f and f' .

57. Find all points on the graph of the function $f(x) = 2 \sin x + \sin^2 x$ at which the tangent line is horizontal.

58. Find the x -coordinates of all points on the curve $y = \sin 2x - 2 \sin x$ at which the tangent line is horizontal.

59. Suppose that $F(x) = f(g(x))$ and $g(3) = 6$, $g'(3) = 4$, $f'(3) = 2$, and $f'(6) = 7$. Find $F'(3)$.

60. Suppose that $w = u \circ v$ and $u(0) = 1$, $v(0) = 2$, $u'(0) = 3$, $u'(2) = 4$, $v'(0) = 5$, and $v'(2) = 6$. Find $w'(0)$.

61. The displacement of a particle on a vibrating string is given by the equation

$$s(t) = 10 + \frac{1}{4} \sin(10\pi t)$$

where s is measured in centimeters and t in seconds. Find the velocity of the particle after t seconds.

62. If the equation of motion of a particle is given by $s = A \cos(\omega t + \delta)$, the particle is said to undergo *simple harmonic motion*.

(a) Find the velocity of the particle at time t .
(b) When is the velocity 0?

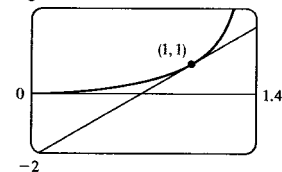
63. A Cepheid variable star is a star whose brightness alternately increases and decreases. The most easily visible such star is Delta Cephei, for which the interval between times of maximum brightness is 5.4 days. The average brightness of this star is 4.0 and its brightness changes by ± 0.35 . In view of these data, the brightness of Delta Cephei at time t , where t is measured in days, has been modeled by the function

$$B(t) = 4.0 + 0.35 \sin(2\pi t/5.4)$$

(a) Find the rate of change of the brightness after t days.
(b) Find, correct to two decimal places, the rate of increase after one day.

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1. $4u(x+1)$, 48 3. $3u^2(1-1/x^2)$, 0
 5. $F'(x) = 10(x^2 + 4x + 6)^4(x+2)$
 7. $G'(x) = 6(3x-2)^9(5x^2-x+1)^{11}(85x^2-51x+9)$
 9. $f'(t) = -16(2t^2-6t+1)^{-9}(2t-3)$
 11. $g'(x) = (2x-7)/(2\sqrt{x^2-7x})$
 13. $h'(t) = \frac{3}{2}(t-1/t)^{1/2}(1+1/t^2)$
 15. $F'(y) = 39(y-6)^2/(y+7)^4$
 17. $f'(z) = -\frac{2}{3}(2z-1)^{-6/5}$
 19. $y' = 8(2x-5)^3(8x^2-5)^{-4}(-4x^2+30x-5)$
 21. $y' = 3\sec^2 3x$ 23. $y' = -3x^2\sin(x^3)$
 25. $y' = -12\cos x \sin x (1+\cos^2 x)^5$
 27. $y' = -\sin(\tan x)\sec^2 x$ 29. $y' = 0$
 31. $y' = -(1/3)\csc(x/3)\cot(x/3)$
 33. $y' = 3\sin x \cos x (\sin x - \cos x)$
 35. $y' = -\cos(1/x)/x^2$ 37. $y' = 4(\cos 2x)/(1-\sin 2x)^2$
 39. $y' = 6x^2 \tan(x^3)\sec^2(x^3)$ 41. $y' = 0$
 43. $y' = [1 + 1/(2\sqrt{x})]/(2\sqrt{x} + \sqrt{x})$
 45. $f'(x) = 9[x^3 + (2x-1)^3]^2(9x^2 - 8x + 2)$
 47. $y' = \cos(\tan \sqrt{\sin x})(\sec^2 \sqrt{\sin x})[1/(2\sqrt{\sin x})](\cos x)$
 49. $y = 0$ 51. $3x + 16y = 44$
 53. (a) $y = \pi x - \pi + 1$ (b) $\frac{3}{3}$



55. (a) $-1/(x^2\sqrt{1-x^2})$
 57. $((\pi/2) + 2n\pi, 3)$, $((3\pi/2) + 2n\pi, -1)$, n any integer
 59. 28 61. $v(t) = (5\pi/2)\cos(10\pi t)$ cm/s
 63. (a) $dB/dt = (7\pi/54)\cos(2\pi t/5.4)$ (b) 0.16
 65. (a) On $(0, \infty)$ (b) $G'(x) = h'(\sqrt{x})/(2\sqrt{x})$
 67. (a) $F'(x) = -\sin x f'(\cos x)$ (b) $G'(x) = -\sin(f(x))f'(x)$
 69. 0 75. $f'(x) = x/|x|$
 77. $h'(x) = |2x-1| + 2x(2x-1)/|2x-1|$