15-11-30-Th12

Chapter 3 Questions to Guide Your Review

1. What is the derivative of a function f? How is its domain related to the domain of f? Give examples.
2. What role does the derivative play in defining slopes, tangents, and rates of change?
3. How can you sometimes graph the derivative of a function when all you have is a table of the function's values?
4. What does it mean for a function to be differentiable on an open interval? On a closed interval?
5. How are derivatives and one-sided derivatives related?
6. Describe geometrically when a function typically does *not* have a derivative at a point.
7. How is a function's differentiability at a point related to its continuity there, if at all?
8. Could the unit step function

$$U(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases}$$

possibly be the derivative of some other function on $[-1, 1]$? Explain.

9. What rules do you know for calculating derivatives? Give some examples.
10. Explain how the three formulas

a. $\frac{d}{dx}(x^n) = nx^{n-1}$

b. $\frac{d}{dx}(cu) = c\frac{du}{dx}$

c. $\frac{d}{dx}(u_1 + u_2 + \cdots + u_n) = \frac{du_1}{dx} + \frac{du_2}{dx} + \cdots + \frac{du_n}{dx}$

enable us to differentiate any polynomial.

11. What formula do we need, in addition to the three listed in Question 10, to differentiate rational functions?
12. What is a second derivative? A third derivative? How many derivatives do the functions you know have? Give examples.
13. What is the relationship between a function's average and instantaneous rates of change? Give an example.
14. How do derivatives arise in the study of motion? What can you learn about a body's motion along a line by examining the derivatives of the body's position function? Give examples.
15. How can derivatives arise in economics?
16. Give examples of still other applications of derivatives.
17. What do the limits $\lim_{h\to 0}((\sin h)/h)$ and $\lim_{h\to 0}((\cos h - 1)/h)$ have to do with the derivatives of the sine and cosine functions? What *are* the derivatives of these functions?
18. Once you know the derivatives of $\sin x$ and $\cos x$, how can you find the derivatives of $\tan x$, $\cot x$, $\sec x$, and $\csc x$? What *are* the derivatives of these functions?
19. At what points are the six basic trigonometric functions continuous? How do you know?
20. What is the rule for calculating the derivative of a composite of two differentiable functions? How is such a derivative evaluated? Give examples.
21. What is the formula for the slope dy/dx of a parametrized curve $x = f(t), y = g(t)$? When does the formula apply? When can you expect to be able to find d^2y/dx^2 as well? Give examples.
22. If u is a differentiable function of x, how do you find $(d/dx)(u^n)$ if n is an integer? If n is a rational number? Give examples.
23. What is implicit differentiation? When do you need it? Give examples.
24. How do related rates problems arise? Give examples.
25. Outline a strategy for solving related rates problems. Illustrate with an example.
26. What is the linearization $L(x)$ of a function $f(x)$ at a point $x = a$? What is required of f at a for the linearization to exist? How are linearizations used? Give examples.
27. If x moves from a to a nearby value $a + dx$, how do you estimate the corresponding change in the value of a differentiable function $f(x)$? How do you estimate the relative change? The percentage change? Give an example.

Chapter 3 Practice Exercises

Derivatives of Functions

Find the derivatives of the functions in Exercises 1-40.

1. $y = x^5 - 0.125x^2 + 0.25x$
2. $y = 3 - 0.7x^3 + 0.3x^7$
3. $y = x^3 - 3(x^2 + \pi^2)$
4. $y = x^7 + \sqrt{7}x - \frac{1}{\pi + 1}$
5. $y = (x + 1)^2(x^2 + 2x)$
6. $y = (2x - 5)(4 - x)^{-1}$
7. $y = (\theta^2 + \sec\theta + 1)^3$
8. $y = \left(-1 - \frac{\csc\theta}{2} - \frac{\theta^2}{4}\right)^2$
9. $s = \frac{\sqrt{t}}{1 + \sqrt{t}}$
10. $s = \frac{1}{\sqrt{t} - 1}$

11. $y = 2\tan^2 x - \sec^2 x$

12. $y = \frac{1}{\sin^2 x} - \frac{2}{\sin x}$

13. $s = \cos^4(1 - 2t)$

14. $s = \cot^3\left(\frac{2}{t}\right)$

15. $s = (\sec t + \tan t)^5$

16. $s = \csc^5(1 - t + 3t^2)$

17. $r = \sqrt{2\theta \sin \theta}$

18. $r = 2\theta\sqrt{\cos \theta}$

19. $r = \sin \sqrt{2\theta}$

20. $r = \sin\left(\theta + \sqrt{\theta + 1}\right)$

21. $y = \frac{1}{2}x^2 \csc \frac{2}{x}$

22. $y = 2\sqrt{x} \sin \sqrt{x}$

23. $y = x^{-1/2}\sec(2x)^2$

24. $y = \sqrt{x}\csc(x + 1)^3$

25. $y = 5\cot x^2$

26. $y = x^2 \cot 5x$

27. $y = x^2 \sin^2(2x^2)$

28. $y = x^{-2}\sin^2(x^3)$

29. $s = \left(\frac{4t}{t + 1}\right)^{-2}$

30. $s = \frac{-1}{15(15t - 1)^3}$

31. $y = \left(\frac{\sqrt{x}}{1 + x}\right)^2$

32. $y = \left(\frac{2\sqrt{x}}{2\sqrt{x} + 1}\right)^2$

33. $y = \sqrt{\frac{x^2 + x}{x^2}}$

34. $y = 4x\sqrt{x + \sqrt{x}}$

35. $r = \left(\frac{\sin \theta}{\cos \theta - 1}\right)^2$

36. $r = \left(\frac{1 + \sin \theta}{1 - \cos \theta}\right)^2$

37. $y = (2x + 1)\sqrt{2x + 1}$

38. $y = 20(3x - 4)^{1/4}(3x - 4)^{-1/5}$

39. $y = \frac{3}{(5x^2 + \sin 2x)^{3/2}}$

40. $y = (3 + \cos^3 3x)^{-1/3}$

Implicit Differentiation

In Exercises 41–48, find dy/dx.

41. $xy + 2x + 3y = 1$

42. $x^2 + xy + y^2 - 5x = 2$

43. $x^3 + 4xy - 3y^{4/3} = 2x$

44. $5x^{4/5} + 10y^{6/5} = 15$

45. $\sqrt{xy} = 1$

46. $x^2y^2 = 1$

47. $y^2 = \frac{x}{x + 1}$

48. $y^2 = \sqrt{\frac{1 + x}{1 - x}}$

In Exercises 49 and 50, find dp/dq.

49. $p^3 + 4pq - 3q^2 = 2$

50. $q = (5p^2 + 2p)^{-3/2}$

In Exercises 51 and 52, find dr/ds.

51. $r\cos 2s + \sin^2 s = \pi$

52. $2rs - r - s + s^2 = -3$

53. Find d^2y/dx^2 by implicit differentiation:

a. $x^3 + y^3 = 1$

b. $y^2 = 1 - \frac{2}{x}$

54. a. By differentiating $x^2 - y^2 = 1$ implicitly, show that $dy/dx = x/y$.

b. Then show that $d^2y/dx^2 = -1/y^3$.

Numerical Values of Derivatives

55. Suppose that functions $f(x)$ and $g(x)$ and their first derivatives have the following values at $x = 0$ and $x = 1$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	1	1	−3	1/2
1	3	5	1/2	−4

Find the first derivatives of the following combinations at the given value of x.

a. $6f(x) - g(x)$, $x = 1$

b. $f(x)g^2(x)$, $x = 0$

c. $\frac{f(x)}{g(x) + 1}$, $x = 1$

d. $f(g(x))$, $x = 0$

e. $g(f(x))$, $x = 0$

f. $(x + f(x))^{3/2}$, $x = 1$

g. $f(x + g(x))$, $x = 0$

56. Suppose that the function $f(x)$ and its first derivative have the following values at $x = 0$ and $x = 1$.

x	$f(x)$	$f'(x)$
0	9	−2
1	−3	1/5

Find the first derivatives of the following combinations at the given value of x.

a. $\sqrt{x}\, f(x)$, $x = 1$

b. $\sqrt{f(x)}$, $x = 0$

c. $f(\sqrt{x})$, $x = 1$

d. $f(1 - 5\tan x)$, $x = 0$

e. $\frac{f(x)}{2 + \cos x}$, $x = 0$

f. $10\sin\left(\frac{\pi x}{2}\right) f^2(x)$, $x = 1$

57. Find the value of dy/dt at $t = 0$ if $y = 3\sin 2x$ and $x = t^2 + \pi$.

58. Find the value of ds/du at $u = 2$ if $s = t^2 + 5t$ and $t = (u^2 + 2u)^{1/3}$.

59. Find the value of dw/ds at $s = 0$ if $w = \sin\left(\sqrt{r} - 2\right)$ and $r = 8\sin(s + \pi/6)$.

60. Find the value of dr/dt at $t = 0$ if $r = (\theta^2 + 7)^{1/3}$ and $\theta^2 t + \theta = 1$.

61. If $y^3 + y = 2\cos x$, find the value of d^2y/dx^2 at the point (0, 1).

62. If $x^{1/3} + y^{1/3} = 4$, find d^2y/dx^2 at the point (8, 8).

Derivative Definition

In Exercises 63 and 64, find the derivative using the definition.

63. $f(t) = \frac{1}{2t + 1}$

64. $g(x) = 2x^2 + 1$

65. a. Graph the function

$$f(x) = \begin{cases} x^2, & -1 \le x < 0 \\ -x^2, & 0 \le x \le 1. \end{cases}$$

b. Is f continuous at $x = 0$?

c. Is f differentiable at $x = 0$?

Give reasons for your answers.

66. a. Graph the function

$$f(x) = \begin{cases} x, & -1 \le x < 0 \\ \tan x, & 0 \le x \le \pi/4. \end{cases}$$

b. Is f continuous at $x = 0$?

c. Is f differentiable at $x = 0$?

Give reasons for your answers.

67. a. Graph the function

$$f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 2 - x, & 1 < x \le 2. \end{cases}$$

b. Is f continuous at $x = 1$?

c. Is f differentiable at $x = 1$?

Give reasons for your answers.

68. For what value or values of the constant m, if any, is

$$f(x) = \begin{cases} \sin 2x, & x \le 0 \\ mx, & x > 0 \end{cases}$$

a. continuous at $x = 0$?

b. differentiable at $x = 0$?

Give reasons for your answers.

Slopes, Tangents, and Normals

69. **Tangents with specified slope** Are there any points on the curve $y = (x/2) + 1/(2x - 4)$ where the slope is $-3/2$? If so, find them.

70. **Tangents with specified slope** Are there any points on the curve $y = x - 1/(2x)$ where the slope is 3? If so, find them.

71. **Horizontal tangents** Find the points on the curve $y = 2x^3 - 3x^2 - 12x + 20$ where the tangent is parallel to the x-axis.

72. **Tangent intercepts** Find the x- and y-intercepts of the line that is tangent to the curve $y = x^3$ at the point $(-2, -8)$.

73. **Tangents perpendicular or parallel to lines** Find the points on the curve $y = 2x^3 - 3x^2 - 12x + 20$ where the tangent is

a. perpendicular to the line $y = 1 - (x/24)$.

b. parallel to the line $y = \sqrt{2} - 12x$.

74. **Intersecting tangents** Show that the tangents to the curve $y = (\pi \sin x)/x$ at $x = \pi$ and $x = -\pi$ intersect at right angles.

75. **Normals parallel to a line** Find the points on the curve $y = \tan x$, $-\pi/2 < x < \pi/2$, where the normal is parallel to the line $y = -x/2$. Sketch the curve and normals together, labeling each with its equation.

76. **Tangent and normal lines** Find equations for the tangent and normal to the curve $y = 1 + \cos x$ at the point $(\pi/2, 1)$. Sketch the curve, tangent, and normal together, labeling each with its equation.

77. **Tangent parabola** The parabola $y = x^2 + C$ is to be tangent to the line $y = x$. Find C.

78. **Slope of tangent** Show that the tangent to the curve $y = x^3$ at any point (a, a^3) meets the curve again at a point where the slope is four times the slope at (a, a^3).

79. **Tangent curve** For what value of c is the curve $y = c/(x + 1)$ tangent to the line through the points $(0, 3)$ and $(5, -2)$?

80. **Normal to a circle** Show that the normal line at any point of the circle $x^2 + y^2 = a^2$ passes through the origin.

Tangents and Normals to Implicitly Defined Curves

In Exercises 81–86, find equations for the lines that are tangent and normal to the curve at the given point.

81. $x^2 + 2y^2 = 9, \quad (1, 2)$

82. $x^3 + y^2 = 2, \quad (1, 1)$

83. $xy + 2x - 5y = 2, \quad (3, 2)$

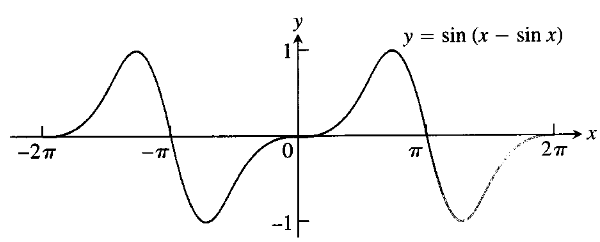
84. $(y - x)^2 = 2x + 4, \quad (6, 2)$

85. $x + \sqrt{xy} = 6, \quad (4, 1)$

86. $x^{3/2} + 2y^{3/2} = 17, \quad (1, 4)$

87. Find the slope of the curve $x^3y^3 + y^2 = x + y$ at the points $(1, 1)$ and $(1, -1)$.

88. The graph shown suggests that the curve $y = \sin(x - \sin x)$ might have horizontal tangents at the x-axis. Does it? Give reasons for your answer.



Tangents to Parametrized Curves

In Exercises 89 and 90, find an equation for the line in the xy-plane that is tangent to the curve at the point corresponding to the given value of t. Also, find the value of d^2y/dx^2 at this point.

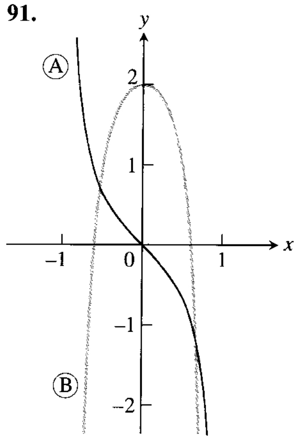
89. $x = (1/2)\tan t, \quad y = (1/2)\sec t, \quad t = \pi/3$

90. $x = 1 + 1/t^2, \quad y = 1 - 3/t, \quad t = 2$

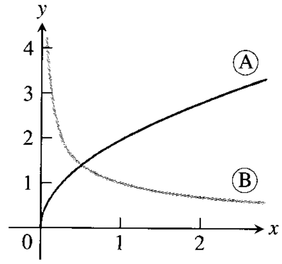
Analyzing Graphs

Each of the figures in Exercises 91 and 92 shows two graphs, the graph of a function $y = f(x)$ together with the graph of its derivative $f'(x)$. Which graph is which? How do you know?

91.



92.



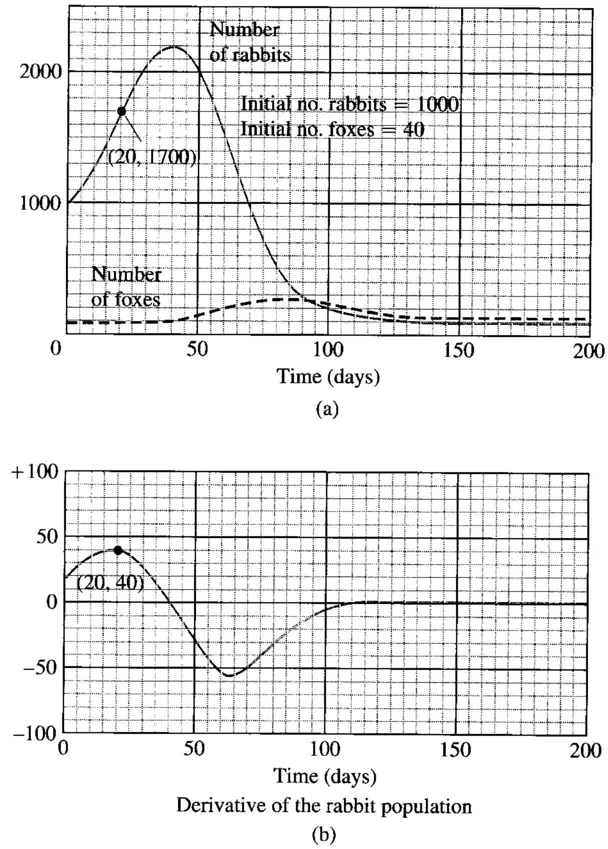


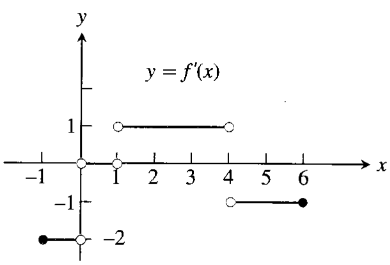
FIGURE 3.53 Rabbits and foxes in an arctic predator-prey food chain.

93. Use the following information to graph the function $y = f(x)$ for $-1 \leq x \leq 6$.

i. The graph of f is made of line segments joined end to end.

ii. The graph starts at the point $(-1, 2)$.

iii. The derivative of f, where defined, agrees with the step function shown here.



94. Repeat Exercise 93, supposing that the graph starts at $(-1, 0)$ instead of $(-1, 2)$.

Exercises 95 and 96 are about the graphs in Figure 3.53 (right-hand column). The graphs in part (a) show the numbers of rabbits and foxes in a small arctic population. They are plotted as functions of time for 200 days. The number of rabbits increases at first, as the rabbits reproduce. But the foxes prey on rabbits and, as the number of foxes increases, the rabbit population levels off and then drops. Figure 3.53b shows the graph of the derivative of the rabbit population. We made it by plotting slopes.

95. a. What is the value of the derivative of the rabbit population in Figure 3.53 when the number of rabbits is largest? Smallest?

b. What is the size of the rabbit population in Figure 3.53 when its derivative is largest? Smallest (negative value)?

96. In what units should the slopes of the rabbit and fox population curves be measured?

Trigonometric Limits

97. $\lim_{x \to 0} \dfrac{\sin x}{2x^2 - x}$

98. $\lim_{x \to 0} \dfrac{3x - \tan 7x}{2x}$

99. $\lim_{r \to 0} \dfrac{\sin r}{\tan 2r}$

100. $\lim_{\theta \to 0} \dfrac{\sin(\sin \theta)}{\theta}$

101. $\lim_{\theta \to (\pi/2)^-} \dfrac{4\tan^2\theta + \tan\theta + 1}{\tan^2\theta + 5}$

102. $\lim_{\theta \to 0^+} \dfrac{1 - 2\cot^2\theta}{5\cot^2\theta - 7\cot\theta - 8}$

103. $\lim_{x \to 0} \dfrac{x \sin x}{2 - 2\cos x}$

104. $\lim_{\theta \to 0} \dfrac{1 - \cos\theta}{\theta^2}$

Show how to extend the functions in Exercises 105 and 106 to be continuous at the origin.

105. $g(x) = \dfrac{\tan(\tan x)}{\tan x}$

106. $f(x) = \dfrac{\tan(\tan x)}{\sin(\sin x)}$

Related Rates

107. Right circular cylinder The total surface area S of a right circular cylinder is related to the base radius r and height h by the equation $S = 2\pi r^2 + 2\pi rh$.

a. How is dS/dt related to dr/dt if h is constant?

b. How is dS/dt related to dh/dt if r is constant?

c. How is dS/dt related to dr/dt and dh/dt if neither r nor h is constant?

d. How is dr/dt related to dh/dt if S is constant?

108. Right circular cone The lateral surface area S of a right circular cone is related to the base radius r and height h by the equation $S = \pi r\sqrt{r^2 + h^2}$.

a. How is dS/dt related to dr/dt if h is constant?

b. How is dS/dt related to dh/dt if r is constant?

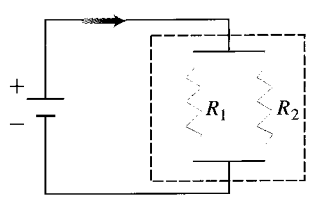
c. How is dS/dt related to dr/dt and dh/dt if neither r nor h is constant?

109. Circle's changing area The radius of a circle is changing at the rate of $-2/\pi$ m/sec. At what rate is the circle's area changing when $r = 10$ m?

110. Cube's changing edges The volume of a cube is increasing at the rate of 1200 cm^3/min at the instant its edges are 20 cm long. At what rate are the lengths of the edges changing at that instant?

111. Resistors connected in parallel If two resistors of R_1 and R_2 ohms are connected in parallel in an electric circuit to make an R-ohm resistor, the value of R can be found from the equation

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$



If R_1 is decreasing at the rate of 1 ohm/sec and R_2 is increasing at the rate of 0.5 ohm/sec, at what rate is R changing when $R_1 = 75$ ohms and $R_2 = 50$ ohms?

112. Impedance in a series circuit The impedance Z (ohms) in a series circuit is related to the resistance R (ohms) and reactance X (ohms) by the equation $Z = \sqrt{R^2 + X^2}$. If R is increasing at 3 ohms/sec and X is decreasing at 2 ohms/sec, at what rate is Z changing when $R = 10$ ohms and $X = 20$ ohms?

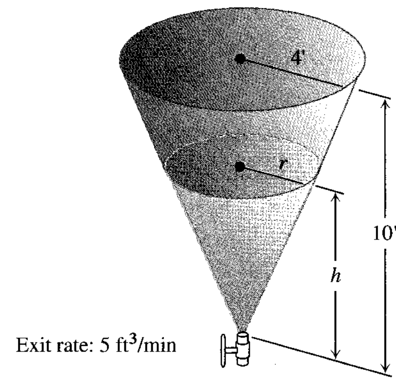
113. Speed of moving particle The coordinates of a particle moving in the metric xy-plane are differentiable functions of time t with $dx/dt = 10$ m/sec and $dy/dt = 5$ m/sec. How fast is the particle moving away from the origin as it passes through the point $(3, -4)$?

114. Motion of a particle A particle moves along the curve $y = x^{3/2}$ in the first quadrant in such a way that its distance from the origin increases at the rate of 11 units per second. Find dx/dt when $x = 3$.

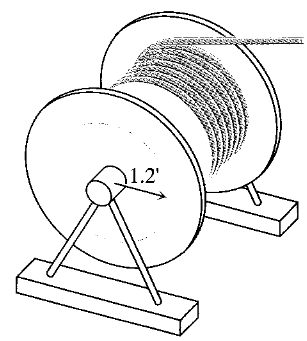
115. Draining a tank Water drains from the conical tank shown in the accompanying figure at the rate of 5 ft^3/min.

a. What is the relation between the variables h and r in the figure?

b. How fast is the water level dropping when $h = 6$ ft?



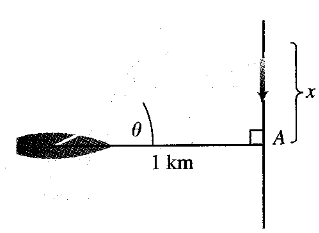
116. Rotating spool As television cable is pulled from a large spool to be strung from the telephone poles along a street, it unwinds from the spool in layers of constant radius (see accompanying figure). If the truck pulling the cable moves at a steady 6 ft/sec (a touch over 4 mph), use the equation $s = r\theta$ to find how fast (radians per second) the spool is turning when the layer of radius 1.2 ft is being unwound.



117. Moving searchlight beam The figure shows a boat 1 km offshore, sweeping the shore with a searchlight. The light turns at a constant rate, $d\theta/dt = -0.6$ rad/sec.

a. How fast is the light moving along the shore when it reaches point A?

b. How many revolutions per minute is 0.6 rad/sec?



118. Points moving on coordinate axes Points A and B move along the x- and y-axes, respectively, in such a way that the distance r (meters) along the perpendicular from the origin to the line AB remains constant. How fast is OA changing, and is it increasing, or decreasing, when $OB = 2r$ and B is moving toward O at the rate of $0.3r$ m/sec?

Linearization

119. Find the linearizations of

a. $\tan x$ at $x = -\pi/4$ **b.** $\sec x$ at $x = -\pi/4$.

Graph the curves and linearizations together.

120. We can obtain a useful linear approximation of the function $f(x) = 1/(1 + \tan x)$ at $x = 0$ by combining the approximations

$$\frac{1}{1+x} \approx 1 - x \quad \text{and} \quad \tan x \approx x$$

to get

$$\frac{1}{1 + \tan x} \approx 1 - x.$$

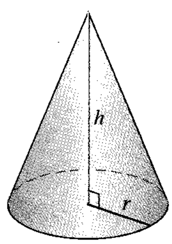
Show that this result is the standard linear approximation of $1/(1 + \tan x)$ at $x = 0$.

121. Find the linearization of $f(x) = \sqrt{1 + x} + \sin x - 0.5$ at $x = 0$.

122. Find the linearization of $f(x) = 2/(1 - x) + \sqrt{1 + x} - 3.1$ at $x = 0$.

Differential Estimates of Change

123. Surface area of a cone Write a formula that estimates the change that occurs in the lateral surface area of a right circular cone when the height changes from h_0 to $h_0 + dh$ and the radius does not change.



$V = \frac{1}{3}\pi r^2 h$

$S = \pi r\sqrt{r^2 + h^2}$

(Lateral surface area)

124. Controlling error

a. How accurately should you measure the edge of a cube to be reasonably sure of calculating the cube's surface area with an error of no more than 2%?

b. Suppose that the edge is measured with the accuracy required in part (a). About how accurately can the cube's volume be calculated from the edge measurement? To find out, estimate the percentage error in the volume calculation that might result from using the edge measurement.

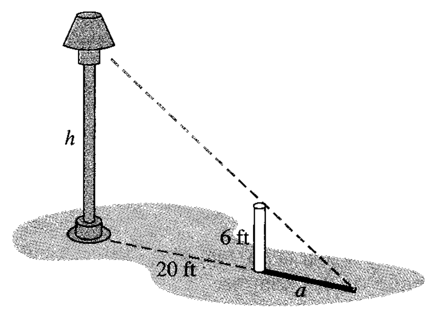
125. Compounding error The circumference of the equator of a sphere is measured as 10 cm with a possible error of 0.4 cm. This measurement is then used to calculate the radius. The radius is then used to calculate the surface area and volume of the sphere. Estimate the percentage errors in the calculated values of

a. the radius.

b. the surface area.

c. the volume.

126. Finding height To find the height of a lamppost (see accompanying figure), you stand a 6 ft pole 20 ft from the lamp and measure the length a of its shadow, finding it to be 15 ft, give or take an inch. Calculate the height of the lamppost using the value $a = 15$ and estimate the possible error in the result.



Practice Exercises, pp. 235–240

1. $5x^4 - 0.25x + 0.25$ **3.** $3x(x - 2)$

5. $2(x + 1)(2x^2 + 4x + 1)$

7. $3(\theta^2 + \sec\theta + 1)^2(2\theta + \sec\theta\tan\theta)$

9. $\dfrac{1}{2\sqrt{t}(1 + \sqrt{t})^2}$ **11.** $2\sec^2 x\tan x$

13. $8\cos^3(1 - 2t)\sin(1 - 2t)$ **15.** $5(\sec t)(\sec t + \tan t)^5$

17. $\dfrac{\theta\cos\theta + \sin\theta}{\sqrt{2\theta\sin\theta}}$ **19.** $\dfrac{\cos\sqrt{2\theta}}{\sqrt{2\theta}}$

21. $x\csc\left(\dfrac{2}{x}\right) + \csc\left(\dfrac{2}{x}\right)\cot\left(\dfrac{2}{x}\right)$

23. $\dfrac{1}{2}x^{1/2}\sec(2x)^2\left[16\tan(2x)^2 - x^{-2}\right]$

25. $-10x\csc^2(x^2)$ **27.** $8x^3\sin(2x^2)\cos(2x^2) + 2x\sin^2(2x^2)$

29. $\dfrac{-(t + 1)}{8t^3}$ **31.** $\dfrac{1 - x}{(x + 1)^3}$ **33.** $\dfrac{-1}{2x^2\left(1 + \dfrac{1}{x}\right)^{1/2}}$

35. $\dfrac{-2\sin\theta}{(\cos\theta-1)^2}$ **37.** $3\sqrt{2x+1}$ **39.** $-9\left[\dfrac{5x+\cos 2x}{(5x^2+\sin 2x)^{5/2}}\right]$

41. $-\dfrac{y+2}{x+3}$ **43.** $\dfrac{-3x^2-4y+2}{4x-4y^{1/3}}$ **45.** $-\dfrac{y}{x}$

47. $\dfrac{1}{2y(x+1)^2}$ **49.** $\dfrac{dp}{dq}=\dfrac{6q-4p}{3p^2+4q}$

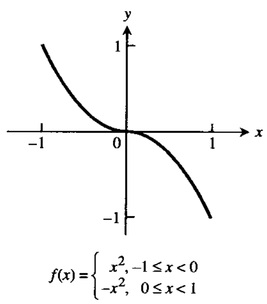
51. $\dfrac{dr}{ds}=(2r-1)(\tan 2s)$

53. (a) $\dfrac{d^2y}{dx^2}=\dfrac{-2xy^3-2x^4}{y^5}$ **(b)** $\dfrac{d^2y}{dx^2}=\dfrac{-2xy^2-1}{x^4y^3}$

55. (a) 7 **(b)** −2 **(c)** 5/12 **(d)** 1/4 **(e)** 12 **(f)** 9/2 **(g)** 3/4

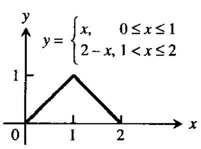
57. 0 **59.** $\sqrt{3}$ **61.** $-\dfrac{1}{2}$ **63.** $\dfrac{-2}{(2t+1)^2}$

65. (a)



(b) Yes **(c)** Yes

67. (a)

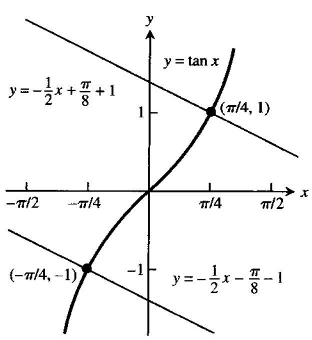


(b) Yes **(c)** No

69. $\left(\dfrac{5}{2},\dfrac{9}{4}\right)$ and $\left(\dfrac{3}{2},-\dfrac{1}{4}\right)$ **71.** $(-1, 27)$ and $(2, 0)$

73. (a) $(-2, 16), (3, 11)$ **(b)** $(0, 20), (1, 7)$

75.



77. $\dfrac{1}{4}$ **79.** 4 **81.** Tangent: $y=-\dfrac{1}{4}x+\dfrac{9}{4}$, normal: $y=4x-2$

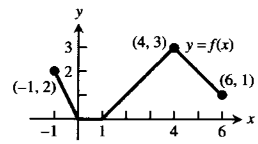
83. Tangent: $y=2x-4$, normal: $y=-\dfrac{1}{2}x+\dfrac{7}{2}$

85. Tangent: $y=-\dfrac{5}{4}x+6$, normal: $y=\dfrac{4}{5}x-\dfrac{11}{5}$

87. $(1, 1)$: $m=-\dfrac{1}{2}$; $(1, -1)$: m not defined

89. $y=\left(\dfrac{\sqrt{3}}{2}\right)x+\dfrac{1}{4}, \dfrac{1}{4}$ **91.** B = graph of f, A = graph of f'

93.



95. (a) 0, 0 **(b)** 1700 rabbits, ≈1400 rabbits

97. −1 **99.** 1/2 **101.** 4 **103.** 1

107. (a) $\dfrac{dS}{dt}=(4\pi r+2\pi h)\dfrac{dr}{dt}$ **(b)** $\dfrac{dS}{dt}=2\pi r\dfrac{dh}{dt}$

(c) $\dfrac{dS}{dt}=(4\pi r+2\pi h)\dfrac{dr}{dt}+2\pi r\dfrac{dh}{dt}$

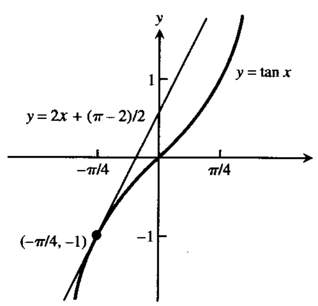
(d) $\dfrac{dr}{dt}=-\dfrac{r}{2r+h}\dfrac{dh}{dt}$

109. $-40\ \text{m}^2/\text{sec}$ **111.** 0.02 ohm/sec **113.** 22 m/sec

115. (a) $r=\dfrac{2}{5}h$ **(b)** $-\dfrac{125}{144\pi}$ ft/min

117. (a) $\dfrac{3}{5}$ km/sec or 600 m/sec **(b)** $\dfrac{18}{\pi}$ rpm

119. (a) $L(x)=2x+\dfrac{\pi-2}{2}$



(b) $L(x)=-\sqrt{2}x+\dfrac{\sqrt{2}(4-\pi)}{4}$