Exercise

1. Find $\int \sec x \; dx$ three ways:

   (a) using $\frac{\sec x + \tan x}{\sec x + \tan x}$,

   (b) using $t = \tan \frac{x}{2}$.

   (c) rewriting $\sec x$ as $\frac{\cos x}{\cos^2 x}$,

   (d) Show that each of the anti-derivatives obtained equals $\ln(\sec x + \tan x)$.

2. Evaluate
$$\int \csc x \; dx.$$

3. Evaluate, in two ways,
$$\int \frac{\sqrt{9 - x^2}}{x} \, dx$$

   (a) using $x = 3 \sin \theta$,

   (b) using $t = \sqrt{9 - x^2}$.

   (c) Show the anti-derivatives obtained are equal.

4. Evaluate
$$\int \frac{dx}{1 + \sin x}.$$

5. Find
$$\int \frac{dx}{\sqrt{x} - \sqrt[3]{x}}.$$
   Try the substitution $u = \sqrt[6]{x}$. Note that 6 is the least common multiple of 2 and 3.