4^{th} roots of 4

$4 = 4 + 0i$

$4 = 4[\cos 0 + i \sin 0]$

Let $w = \rho[\cos\phi + i\sin\phi]$

Suppose

$w^4 = 4$

$(\rho[\cos\phi + i\sin\phi])^4 = 4[\cos 0 + i\sin 0]$

$\rho^4[\cos 4\phi + i\sin 4\phi] = 4[\cos 0 + i\sin 0]$

$\Rightarrow$ ① $\rho^4 = 4 \Rightarrow \rho = \sqrt[4]{4} \Rightarrow \rho = \sqrt{2}$

$\boxed{\rho = \sqrt{2}}$

② $4\phi = 0 + 2k\pi$, $k = 0, 1, 2, 3$

$\phi = \frac{2k\pi}{4}$

$\phi = \frac{k\pi}{2}$

$k=0$ $\phi_1 = 0$ $w_1 = \sqrt{2}[\cos 0 + i\sin 0] = \sqrt{2}[1+0] = \sqrt{2}$

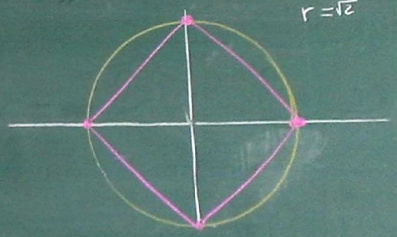
$k=1$ $\phi_2 = \pi/2$ $w_2 = \sqrt{2}[\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}] = \sqrt{2}[0+i] = 0 + i\sqrt{2}$

$k=2$ $\phi_3 = \pi$ $w_3 = \sqrt{2}[\cos\pi + i\sin\pi] = \sqrt{2}[-1+0] = -\sqrt{2}$

$k=3$ $\phi_4 = \frac{3\pi}{2}$ $w_4 = \sqrt{2}[\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}] = \sqrt{2}[0-i] = 0 - \sqrt{2}i$

Ans the 4, 4^{th} roots of 4 are

$\sqrt{2}, -\sqrt{2}, i\sqrt{2}, -i\sqrt{2}$



All 6 6th roots of 2.

Soln

$$2 = 2 + 0i$$
$$= 2[\cos 0 + i \sin 0]$$

Let $w = \rho[\cos\phi + i\sin\phi]$

Suppose

$$w^6 = 2.$$

That means

$$\rho^6[\cos 6\phi + i \sin 6\phi] = 2[\cos 0 + i \sin 0]$$

$\Rightarrow$ ① $\rho^6 = 2 \equiv \rho = \sqrt[6]{2}$.

② $6\phi = 0 + 2k\pi$, $k = 0, 1, 2, \cdots, 5$

$$\phi = \frac{k\pi}{3} \quad \left\{ \phi = k(\pi/3) \right.$$

$k=0$	$\phi_1 = 0$
$k=1$	$\phi_2 = \frac{\pi}{3}$
$k=2$	$\phi_3 = \frac{2\pi}{3}$
$k=3$	$\phi_4 = \pi$
$k=4$	$\phi_5 = \frac{4\pi}{3}$
$k=5$	$\phi_6 = \frac{5\pi}{3}$

$w_1 = \sqrt[6]{2}[\cos 0 + i \sin 0]$ $= \sqrt[6]{2}$

$w_2 = \sqrt[6]{2}[\cos \frac{\pi}{3} + i \sin \pi/3]$ $= \sqrt[6]{2}[\frac{1}{2} + i\sqrt{3}/2]$ I

$w_3 = \sqrt[6]{2}[\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}]$ $= \sqrt[6]{2}[\frac{-1}{2} + i\sqrt{3}/2]$ II

$w_4 = \sqrt[6]{2}[\cos \pi + i \sin \pi]$ $= -\sqrt[6]{2}$

$w_5 = \sqrt[6]{2}[\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}]$ $= \sqrt[6]{2}[-1/2 - i\frac{\sqrt{3}}{2}]$ III

$w_6 = \sqrt[6]{2}[\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}]$ $= \sqrt[6]{2}[\frac{1}{2} - i\frac{\sqrt{3}}{2}]$ IV

$r = \sqrt[6]{2}$