Roots of complex numbers

in the complex numbers $\mathbb{C}$

<u>FALSE</u> Every polynomial eqn has a soln in $\mathbb{R}$.

<u>TRUE</u> Every polynomial eqn of degree n has n solutions (including multiplicities) in $\mathbb{C}$,

Fundamental Thm Algebra.

<u>Complex roots</u>

Find all n roots of z, z is complex number.

<u>Soln</u>

Let $z = r[\cos\theta + i\sin\theta]$

$w = \rho[\cos\phi + i\sin\phi]$

Suppose $w^n = z$

So,

$$\left(\rho[\cos\phi + i\sin\phi]\right)^n$$

$$= r[\cos\theta + i\sin\theta]$$

by de Moivre Thm

$$\rho^n[\cos n\phi + i\sin n\phi] = r[\cos\theta + i\sin\theta]$$

$$\Rightarrow \text{①}\ \rho^n = r \equiv \rho = \sqrt[n]{r}$$

$$\text{②}\ n\phi = \theta + 2k\pi,\ \underline{k = 0, 1, 2, \dots},\ k = 0, 1, 2, \dots, n-1$$

$$\phi = \frac{\theta + 2k\pi}{n}$$

so

when

$$k=0 \quad \phi_1 = \frac{\theta+0}{n} = \frac{\theta}{n}$$

$$k=1 \quad \phi_2 = \frac{\theta+2\pi}{n}$$

$$k=2 \quad \phi_3 = \frac{\theta+4\pi}{n}$$

$$\vdots$$

$$k=n-1 \quad \phi_n = \frac{\theta+(n-1)2\pi}{n}$$

How come you stop at $k=n-1$

$$k=1 \quad \frac{\theta}{n} + \frac{0}{n}$$

$$k=2 \quad \frac{\theta}{n} + \frac{2\pi}{n}$$

$$k=3 \quad \frac{\theta}{n} + \frac{4\pi}{n}$$

$$\vdots$$

$$k=n-1 \quad \frac{\theta}{n} + \frac{(n-1)}{n}2\pi$$

$$k=n \quad \frac{\theta}{n} + \frac{n}{n}\cdot 2\pi$$

$$= \frac{\theta}{n} + 2\pi = \frac{\theta}{n}$$

Roots

$k=0 \quad w_1 = \sqrt[n]{r}\left[\cos \frac{\theta+0}{n} + i\sin\frac{\theta+0}{n}\right]$

$k=1 \quad w_2 = \sqrt[n]{r}\left[\cos \frac{\theta+2\pi}{n} + i\sin\frac{\theta+2\pi}{n}\right]$

$\vdots$

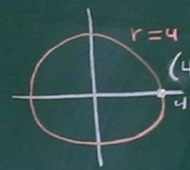
$k=n-1 \quad w_n = \sqrt[n]{r}\left[\cos \frac{\theta+(n-1)2\pi}{n} + i\sin\frac{\theta+(n-1)2\pi}{n}\right]$

EX 1

Find all 4 $4^{th}$ roots of 4.

Soln

Write 4 as complex number trig form




$(4\cos 0 + i\sin 0)$

$4 = 4[\cos 0 + i\sin 0]$

Let $w = \rho[\cos\phi + i\sin\phi]$

and suppose

$w^4 = 4$

$\Rightarrow \overbrace{\left[\rho(\cos[\phi] + i\sin[\phi])\right]^4}^{w^4} = \overbrace{4[\cos 0 + i\sin 0]}^{4}$

de Moivre's Thm

$\rho^n[\cos n\phi + i\sin n\phi] = 4[\cos 0 + i\sin 0]$

ctd . . .