Math 11 - Adv. Algebra & Trig

Annotated Solutions to Selected Problems from NEM

NEM Exercise 1.2 # 11

If $\frac{r^2}{4}(3x)^r\left(\frac{2}{9x^2}\right)^{6-r}$ can be simplified to $\frac{k}{x^3}$, find the values of the constants r and k.

Solution

We are given that the complicated expression simplifies to $\frac{k}{x^3}$, this means

$$\frac{r^2}{4}(3x)^r\left(\frac{2}{9x^2}\right)^{6-r}=\frac{k}{x^3} \tag{1}$$

If we are to solve for two unknowns r and k, we ought to have two independent equations. Suppose we would have another equation like Eqn (1). Imagine we divide the one equation by the other. The result of such a division would be no letter k on the RHS. Looking ahead, we expect the fact that r appears as both an exponent and a base to cause trouble. But, the division that would eliminate k on the RHS will eliminate $\frac{r^2}{4}$ on the LHS. Dividing by another equation like Eqn (1) looks like a promising strategy. But where will the second equation come form?

Since it is given that $\frac{r^2}{4}(3x)^r\left(\frac{2}{9x^2}\right)^{6-r}$ simplifies to $\frac{k}{x^3}$ with *no* restrictions on the values x may take, Eqn (1) must hold for all values of x. This means we can easily produce two equations like Eqn (1) merely by substituting two different values for x. Any two values will work, but it is easy to compute with 1 and 2, so those will be our choices.

When $x=1$, Eqn (1) becomes Eqn (2). When $x=2$, Eqn (1) becomes Eqn (3).

$$\frac{r^2}{4}(3)^r\left(\frac{2}{9}\right)^{6-r}=\frac{k}{1} \tag{2}$$

$$\frac{r^2}{4}(6)^r\left(\frac{2}{36}\right)^{6-r}=\frac{k}{8} \tag{3}$$

Dividing Eqn (2) by Eqn (3),

$$\frac{\frac{r^2}{4}(3)^r\left(\frac{2}{9}\right)^{6-r}}{\frac{r^2}{4}(6)^r\left(\frac{2}{36}\right)^{6-r}}=\frac{k}{\frac{k}{8}}$$

$$\frac{3^r\left(\frac{2}{9}\right)^{6-r}}{6^r\left(\frac{2}{36}\right)^{6-r}}=8$$

$$\frac{\left(\frac{2}{9}\right)^{6-r}}{2^r\left(\frac{2}{36}\right)^{6-r}} = 8$$

$$\frac{\left(\frac{2}{9}\right)^{6}\left(\frac{2}{9}\right)^{-r}}{2^r\left(\frac{2}{36}\right)^{6}\left(\frac{2}{36}\right)^{-r}} = 8$$

$$\frac{\left(\frac{2}{9}\right)^{6}\left(\frac{2}{9}\right)^{-r}}{2^r\left(\frac{2}{9}\right)^{6}\left(\frac{1}{4}\right)^{6}\left(\frac{2}{9}\right)^{-r}\left(\frac{1}{4}\right)^{-r}} = 8$$

$$\frac{1}{2^r\left(\frac{1}{4}\right)^{6}4^r} = 8$$

$$\frac{1}{2^{3r}\left(\frac{1}{4}\right)^{6}} = 8$$

$$\frac{1}{2^{3r}} = 2^{-9}$$

$$2^{3r} = 2^9$$

So $r = 3$. Substituting $r = 3$ into Eqn (2),

$$\frac{3^2}{4}(3)^3\left(\frac{2}{9}\right)^3 = k.$$

Therefore, $r = 3$ and $k = \frac{2}{3}$.