

Teacher needs help

1.1 Oops

Well, it appears your teacher has got himself in a tough spot. Maybe you can help him out of it.

The basic idea seemed sound. It was this. If the difference of two numbers is positive, then the first number is greater than the second. On the other hand, if the difference is negative, then the second number is greater than the first. The phrases “first number” and “second number” are a little vague, so let us put it as follows.

If $A - B > 0$, then $A > B$. If $A - B < 0$, then $B > A$.

Wanting to be sure we all agreed with the statements above, the teacher set out to illustrate them with a few examples. But that is where the trouble began. Here is what he had on the board.

$-3 - 1$ Negative
conclusion
 $-1 > -3$

$-1 - 3$ Negative
conclusion
 $-3 > -1$

What went wrong?

1.2 Multiplication of inequality by a negative number

Assuming the trouble outlined above gets straightened out, we can take up the issue that started your teacher down that path. We wondered if we could prove the following theorem.

Theorem 1.1. *Let a, b and c represent integers. If $a < b$ and $c < 0$ then $ac > bc$. If $a > b$ and $c < 0$ then $ac < bc$.*

Proof.

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