Chapter 12

Non right angled triangle trigonometry

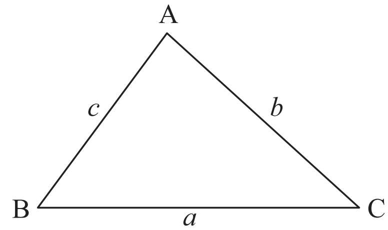
Contents:

A THE COSINE RULE

The **cosine rule** involves the sides and angles of a triangle.

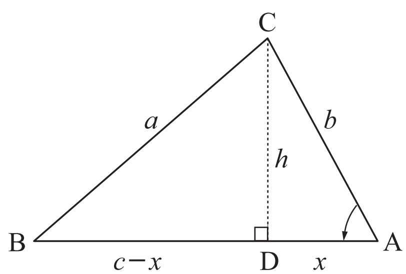
In any ΔABC:

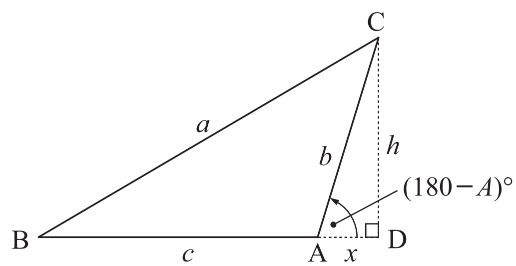
$$\boldsymbol{a^2 = b^2 + c^2 - 2bc\cos A}$$
or $$\boldsymbol{b^2 = a^2 + c^2 - 2ac\cos B}$$
or $$\boldsymbol{c^2 = a^2 + b^2 - 2ab\cos C}$$



We will develop the first formula for both an acute and an obtuse triangle.

Proof:





In both triangles drop a perpendicular from C to meet AB (extended if necessary) at D.

Let AD $= x$ and let CD $= h$.

Apply the theorem of Pythagoras in ΔBCD:

$$a^2 = h^2 + (c-x)^2$$
$$\therefore \quad a^2 = h^2 + c^2 - 2cx + x^2$$

$$a^2 = h^2 + (c+x)^2$$
$$\therefore \quad a^2 = h^2 + c^2 + 2cx + x^2$$

In both cases, applying Pythagoras to ΔADC: $h^2 + x^2 = b^2$ and substitute for h^2.

$$\therefore \quad a^2 = b^2 + c^2 - 2cx$$

In ADC: $\cos A = \dfrac{x}{b}$

$$\therefore \quad b\cos A = x$$
$$\therefore \quad a^2 = b^2 + c^2 - 2bc\cos A$$

$$\therefore \quad a^2 = b^2 + c^2 + 2cx$$

Now $\cos(180 - A) = \dfrac{x}{b}$

$$\therefore \quad b\cos(180 - A) = x$$

But, $\cos(180 - A) = -\cos A$

$$\therefore \quad -b\cos A = x$$
$$\therefore \quad a^2 = b^2 + c^2 - 2bc\cos A$$

The other variations of the cosine rule could be developed by rearranging the vertices of ΔABC.

Note that if $A = 90^o$, $\cos A = 0$ and $a^2 = b^2 + c^2 - 2bc\cos A$ reduces to $a^2 = b^2 + c^2$, the Pythagoras' Rule.

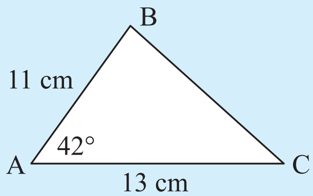
The **cosine rule** can be used to solve triangles given:

- two sides and an included angle
- three sides.

There is no ambiguity possible using the cosine rule.

Example 1

Find, correct to 2 decimal places, the length of BC.



By the cosine rule:

$$BC^2 = 11^2 + 13^2 - 2 \times 11 \times 13 \times \cos 42^o$$

$\therefore$ $BC \doteqdot \sqrt{(11^2 + 13^2 - 2 \times 11 \times 13 \times \cos 42^o)}$

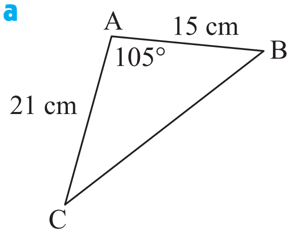
$\therefore$ $BC \doteqdot 8.801.....$

$\therefore$ BC is 8.80 cm in length.

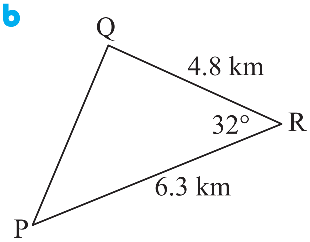
EXERCISE 12A

1 Find the length of the remaining side in the given triangle:

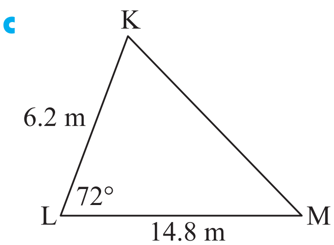
a



b



c

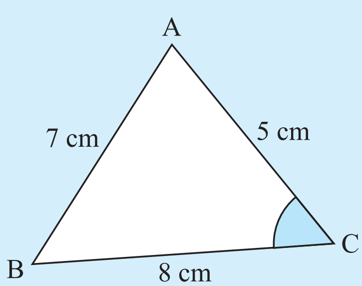


Rearrangement of the original cosine rule formulae can be used for angle finding if we know all three sides. The formulae for finding the angles are:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \qquad \cos B = \frac{c^2 + a^2 - b^2}{2ca} \qquad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Example 2

In triangle ABC, if AB = 7 cm, BC = 8 cm and CA = 5 cm, find the measure of angle BCA.



By the cosine rule:

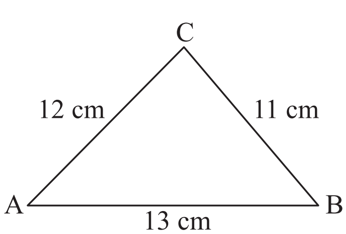
$$\cos C = \frac{(5^2 + 8^2 - 7^2)}{(2 \times 5 \times 8)}$$

$\therefore$ $C = \cos^{-1}\left(\frac{(5^2 + 8^2 - 7^2)}{(2 \times 5 \times 8)}\right)$

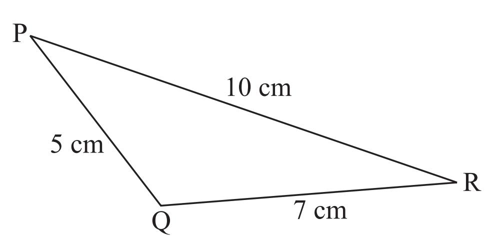
$\therefore$ $C = 60$

So, angle BCA measures 60^o.

2 Find the measure of all angles of:



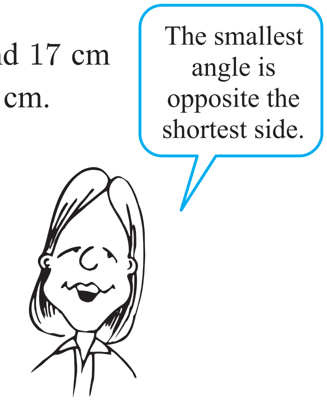
3 Find the measure of obtuse angle PQR.



4 Find:

a the smallest angle of a triangle with sides 11 cm, 13 cm and 17 cm

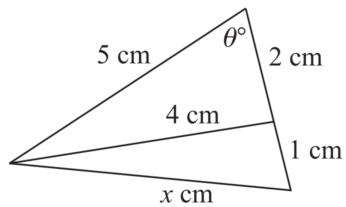
b the largest angle of a triangle with sides 4 cm, 7 cm and 9 cm.



5 Find:

a $\cos\theta$ but not θ

b the value of x.

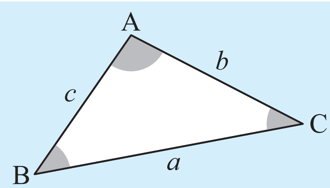


B THE SINE RULE

The **sine rule** is a set of equations which connects the lengths of the sides of any triangle with the sines of the angles of the triangle. The triangle does not have to be right angled for the sine rule to be used.

In any triangle ABC with sides a, b and c units in length, and opposite angles A, B and C respectively,

$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c} \quad \text{or} \quad \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$



Proof: The area of any triangle ABC is given by

$\frac{1}{2}bc\sin A=\frac{1}{2}ac\sin B=\frac{1}{2}ab\sin C$.

Dividing each expression by $\frac{1}{2}abc$ gives

$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}.$$

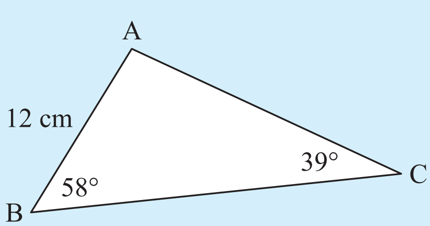
Note: The sine rule is used to solve problems involving triangles given either:

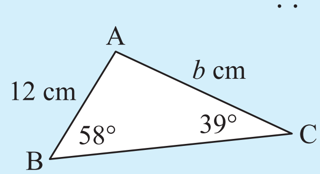
- **two angles** and **one side**, or
- **two sides** and a **non-included** angle.

FINDING SIDES

Example 3

Find the length of AC correct to two decimal places.





By the sine rule

$\therefore \quad \dfrac{b}{\sin 58^o}=\dfrac{12}{\sin 39^o}$

$\therefore \quad b=\dfrac{12\times\sin 58^o}{\sin 39^o}$

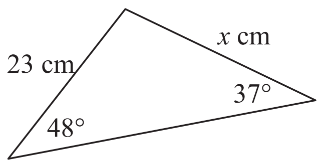
$\therefore \quad b \doteqdot 16.170\,74$

$\therefore$ AC is 16.2 cm long.

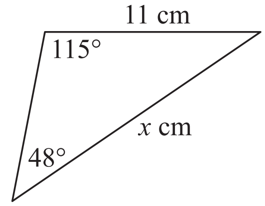
EXERCISE 12B.1

1 Find the value of x:

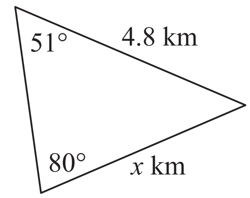
a



b



c



2 In triangle ABC find:

a a if $\text{A} = 63^o$, $\text{B} = 49^o$ and $b = 18$ cm

b b if $\text{A} = 82^o$, $\text{C} = 25^o$ and $c = 34$ cm

c c if $\text{B} = 21^o$, $\text{C} = 48^o$ and $a = 6.4$ cm

FINDING ANGLES

The problem of finding angles using the sine rule is more complicated because there may be two possible answers.

INVESTIGATION — THE AMBIGUOUS CASE

You will need a blank sheet of paper, a ruler, a protractor and a compass for the tasks that follow. In each task you will be required to construct triangles from given information. You could also do this using a computer package such as Geometer Sketch Pad.

Task 1: Draw AB = 10 cm. At A construct an angle of 30^o. Using B as centre, draw an arc of a circle of radius 6 cm. Let the arc intersect the ray from A at C. How many different positions may C have and therefore how many different triangles ABC may be constructed?

Task 2: As before, draw AB = 10 cm and construct a 30^o angle at A. This time draw an arc of radius 5 cm based on B. How many different triangles are possible?

Task 3: Repeat, but this time draw an arc of radius 3 cm on B. How many different triangles are possible?

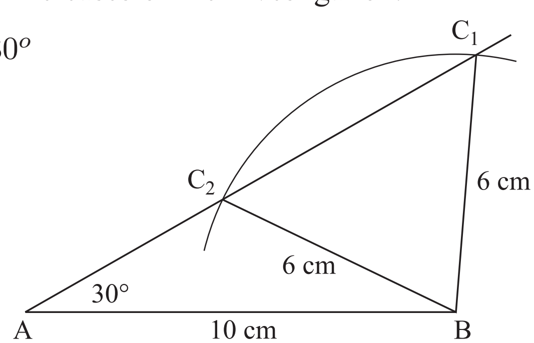
Task 4: Repeat with an arc of radius 12 cm from B. How many possible triangles?

In this investigation you should have discovered that when you are given two sides and a non-included angle there are a number of different possibilities. You could get two triangles, one triangle or it may be impossible to draw any triangles from the given data.

Let us consider the calculations involved in each of the cases of the investigation.

Task 1: Given: $c = 10$ cm, $a = 6$ cm, $A = 30^o$

Finding C:
$$\frac{\sin C}{c} = \frac{\sin A}{a}$$
$$\therefore \quad \sin C = \frac{c \sin A}{a}$$
$$\therefore \quad \sin C = \frac{10 \times \sin 30^o}{6} = 0.8333$$



Because $\sin\theta = \sin(180^o - \theta)$ there are two possible angles:

$$C = 56.44^o \quad or \quad 180^o - 56.44^o = 123.56^o$$

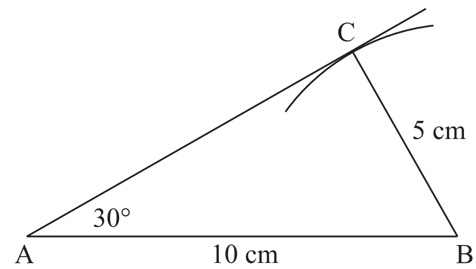
On your calculator check that the sine ratio of both of these angles is 0.8333.

Task 2: Given: $c = 10$ cm, $a = 5$ cm, $A = 30^o$

Finding C: $\dfrac{\sin C}{c} = \dfrac{\sin A}{a}$

$\therefore \quad \sin C = \dfrac{c \sin A}{a}$

$\therefore \quad \sin C = \dfrac{10 \times \sin 30^o}{5} = 1$



There is only one possible solution for C in the range from 0^o to 180^o and that is $C = 90^o$. So only one triangle (i.e., one set of solutions) is possible. Complete the solution of the triangle yourself.

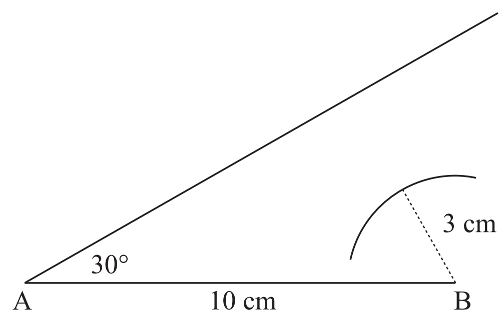
Task 3: Given: $c = 10$ cm, $a = 3$ cm, $A = 30^o$

Finding C: $\dfrac{\sin C}{c} = \dfrac{\sin A}{a}$

$\therefore \quad \sin C = \dfrac{c \sin A}{a}$

$\therefore \quad \sin C = \dfrac{10 \times \sin 30^o}{3}$

$\therefore \quad \sin C = 1.6667$



There is no angle that has a sine ratio > 1. Therefore there is *no solution* for this given data, i.e., *no possible* triangle can be drawn.

Task 4: Given: $c = 10$ cm, $a = 12$ cm, $A = 30^o$

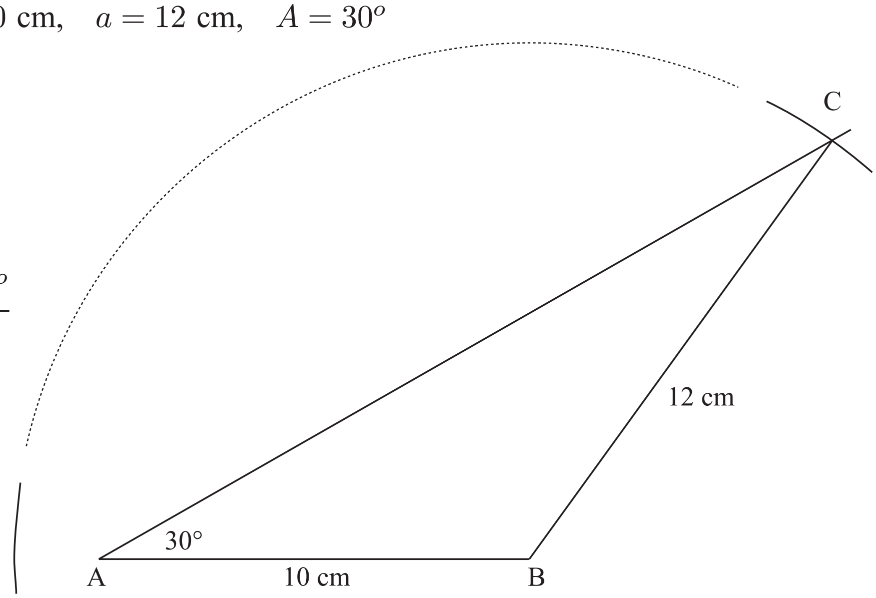
Finding C:

$\dfrac{\sin C}{c} = \dfrac{\sin A}{a}$

$\therefore \quad \sin C = \dfrac{c \sin A}{a}$

$\therefore \quad \sin C = \dfrac{10 \times \sin 30^o}{12}$

$\therefore \quad \sin C = 0.4167$



Two angles have a sine ratio of 0.4167

$C = 24.62^o$ *or* $180^o - 24.62^o$

$C = 24.62^o$ *or* 155.38^o

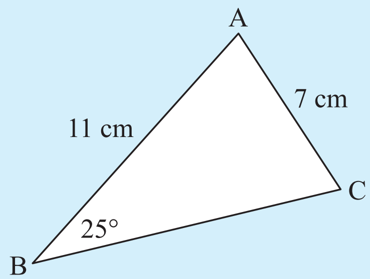
However, in this case only one of these two angles is valid. If $A = 30^o$ then C cannot possibly equal 155.38^o because $30^o + 155.38^o > 180^o$.

Therefore, there is only one solution, $C = 24.62^o$. Once again, you may wish to carry on and complete the solution.

Conclusion: Each situation using the sine rule with two sides and a non-included angle must be examined very carefully.

Example 4

Find the measure of angle C in triangle ABC if AC is 7 cm, AB is 11 cm and angle B measures 25^o.



By the sine rule

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\therefore \quad \frac{\sin C}{11} = \frac{\sin 25^o}{7}$$

$$\therefore \quad \sin C = \frac{11 \times \sin 25^o}{7}$$

$$\therefore \quad C = \sin^{-1}\left(\frac{11 \times \sin 25^o}{7}\right) \quad \text{or its supplement}$$

$$\therefore \quad C \doteqdot 41.6^o \text{ or } 180^o - 41.6^o$$

{as C may be obtuse}

$$\therefore \quad C \doteqdot 41.6^o \text{ or } 138.4^o$$

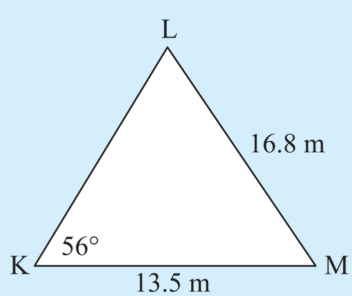
$\therefore$ C measures 41.6^o if angle C is acute
or C measures 138.4^o if angle C is obtuse.

In this example there is insufficient information to determine the actual shape of the triangle.

Note: Sometimes there is information in the question which enables us to **reject** one of the answers.

Example 5

Find the measure of angle L in triangle KLM given that angle LKM measures 56^o, LM = 16.8 m and KM = 13.5 m.



$$\frac{\sin L}{13.5} = \frac{\sin 56^o}{16.8} \quad \text{\{the sine rule\}}$$

$$\therefore \quad \sin L = \frac{13.5 \times \sin 56^o}{16.8}$$

$$\therefore \quad L = \sin^{-1}\left(\frac{13.5 \times \sin 56^o}{16.8}\right) \quad \text{or its supplement}$$

$$\therefore \quad L \doteqdot 41.8^o \quad \text{or} \quad 180^o - 41.8^o$$

$$\therefore \quad L \doteqdot 41.8^o \quad \text{or} \quad 138.2^o$$

But reject $L = 138.2^o$ as $138.2^o + 56^o > 180^o$ which is impossible. $\therefore$ $\measuredangle L \doteqdot 41.8^o$.

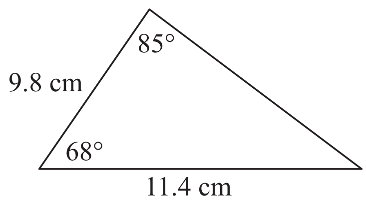
EXERCISE 12B.2

1 Triangle ABC has $\measuredangle B = 40^o$, $b = 8$ cm and $c = 11$ cm. Find the two possible values for angle C.

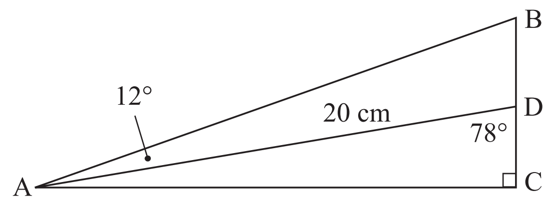
2 In triangle ABC, find the measure of:

- **a** angle A if $a = 14.6$ cm, $b = 17.4$ cm and $\measuredangle$ABC $= 65^o$
- **b** angle B if $b = 43.8$ cm, $c = 31.4$ cm and $\measuredangle$ACB $= 43^o$
- **c** angle C if $a = 6.5$ km, $c = 4.8$ km and $\measuredangle$BAC $= 71^o$.

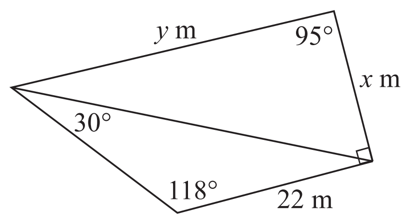
3 Is it possible to have a triangle with measurements as shown? Explain!



4 Find the magnitude of the angle ABC and hence BD in the given figure.



5 Find x and y in the given figure.



6 Triangle ABC has $\widehat{A} = 58^o$, AB $= 10$ cm and AC $= 5.1$ cm. Find:

- **a** $\widehat{C}$ correct to the nearest tenth of a degree using the sine rule
- **b** $\widehat{C}$ correct to the nearest tenth of a degree using the cosine rule.
- **c** Copy and complete: "When faced with using either the sine rule or the cosine rule it is better to use the .... as it avoids ...."

C USING THE SINE AND COSINE RULES

First decide which rule to use.

If the triangle is right angled then the trigonometric ratios or Pythagoras' Theorem can be used, and for some problems adding an extra line or two to the diagram may result in a right triangle.

However, if you have to choose between the sine and cosine rules, the following checklist may assist you.

Use the **cosine rule** when given
- three sides
- two sides and an included angle.

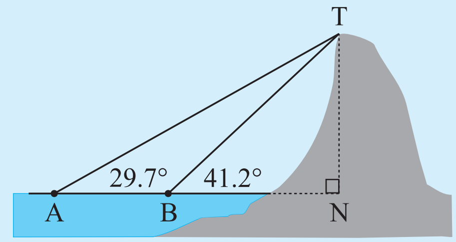
Use the **sine rule** when given
- one side and two angles
- two sides and a non-included angle (but beware of the *ambiguous case* which can occur when the smaller of the two given sides is opposite the given angle).

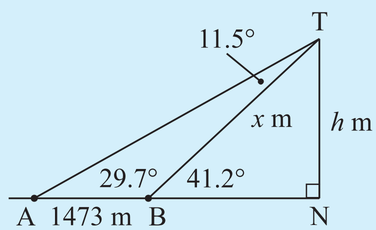
Example 6

The angles of elevation to the top of a mountain are measured from two beacons A and B, at sea.

These angles are as shown on the diagram.

If the beacons are 1473 m apart, how high is the mountain?





$\measuredangle ATB = 41.2^o - 29.7^o$ {exterior angle of Δ}

$= 11.5^o$

We can now find x in ΔABT using the sine rule,

i.e., $\dfrac{x}{\sin 29.7} = \dfrac{1473}{\sin 11.5}$

$\therefore \quad x = \dfrac{1473}{\sin 11.5} \times \sin 29.7$

$\doteqdot 3660.62.....$

Now, in ΔBNT, $\sin 41.2^o = \dfrac{h}{x} = \dfrac{h}{3660.62....}$

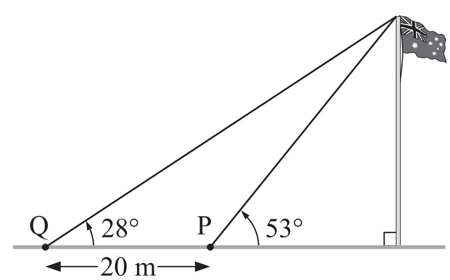
$\therefore \quad h = \sin 41.2^o \times 3660.62....$

$\therefore \quad h \doteqdot 2410$

So, the mountain is about 2410 m high.

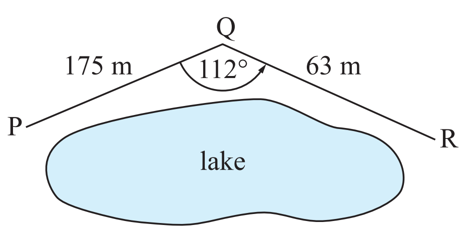
EXERCISE 12C

1 Manny wishes to determine the height of a flag pole. He takes a sighting of the top of the flagpole from point P. He then moves further away from the flagpole by 20 metres to point Q and takes a second sighting. The information is shown in the diagram alongside. How high is the flagpole?

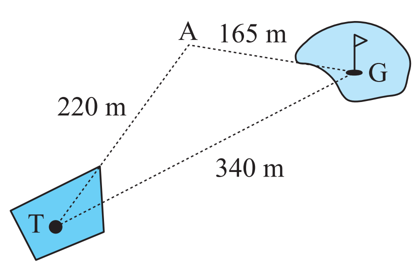


2 To get from P to R, a park ranger had to walk along a path to Q and then to R as shown.

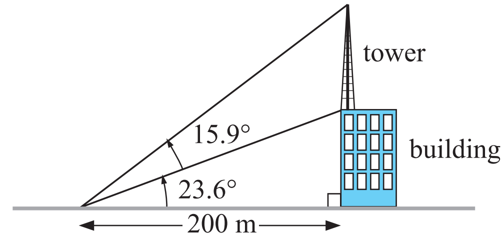
What is the distance in a straight line from P to R?



3 A golfer played his tee shot a distance of 220 m to a point A. He then played a 165 m six iron to the green. If the distance from tee to green is 340 m, determine the number of degrees the golfer was off line with his tee shot.



4 A Communications Tower is constructed on top of a building as shown. Find the height of the tower.



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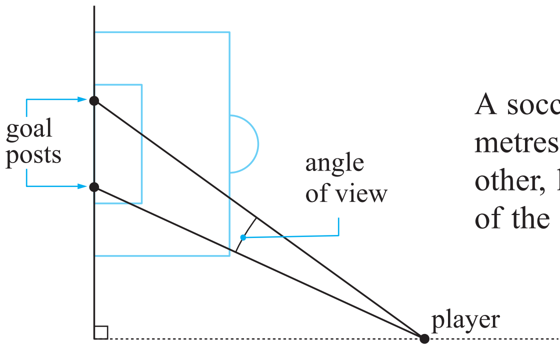


A soccer goal is 5 metres wide. When a player is 21 metres from one goal post and 19 metres from the other, he shoots for goal. What is the angle of view of the goals that the player sees?

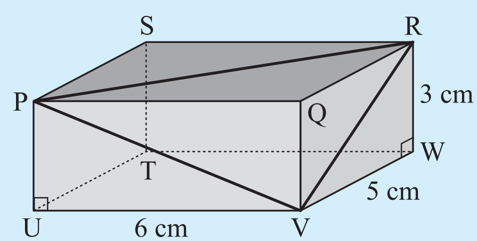
6 A tower 42 metres high, stands on top of a hill. From a point some distance from the base of the hill, the angle of elevation to the top of the tower is 13.2^o. From the same point the angle of elevation to the bottom of the tower is 8.3^o. Find the height of the hill.

7 From the foot of a building I have to look upwards at an angle of 22^o to sight the top of a tree. From the top of the building, 150 metres above ground level, I have to look down at an angle of 50^o below the horizontal to sight the tree top.

a How high is the tree? **b** How far from the building is this tree?

Example 7

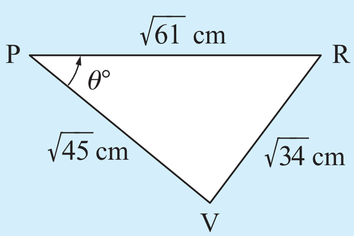
Find the measure of angle RPV.



In ΔRVW, $\text{RV} = \sqrt{5^2+3^2} = \sqrt{34}$ cm. {Pythagoras}

In ΔPUV, $\text{PV} = \sqrt{6^2+3^2} = \sqrt{45}$ cm. {Pythagoras}

Likewise in ΔPQR, $\text{PR} = \sqrt{6^2+5^2} = \sqrt{61}$ cm.



$$\cos\theta = \frac{(\sqrt{61})^2+(\sqrt{45})^2-(\sqrt{34})^2}{2\sqrt{61}\sqrt{45}}$$

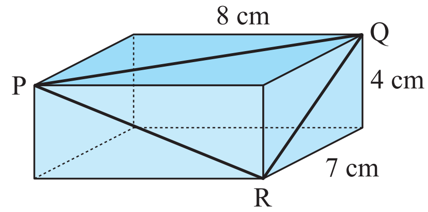
$$= \frac{61+45-34}{2\sqrt{61}\sqrt{45}}$$

$$= \frac{72}{2\sqrt{61}\sqrt{45}}$$

$$\therefore \quad \theta = \cos^{-1}\left(\frac{36}{\sqrt{61}\sqrt{45}}\right) \doteqdot 46.6$$

i.e., angle RPV measures 46.6^o.

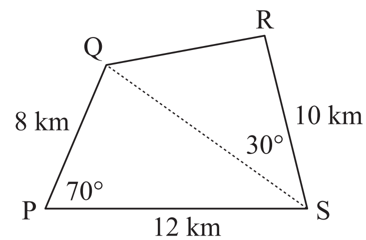
8 Find the measure of angle PQR in the rectangular box shown.



9 Two observation posts are 12 km apart at A and B. From A, a third observation post C is located such that angle CAB is 42^o while angle CBA is 67^o. Find the distance of C from both A and B.

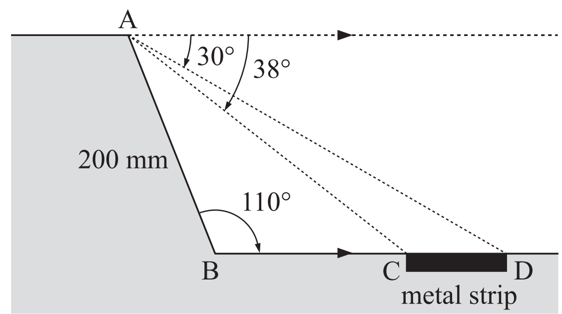
10 Stan and Olga are considering buying a sheep farm and the land agent supplies them with the given accurate sketch. Find the area of the property giving your answer in:

a km^2 **b** hectares.



11 Thabo and Palesa start at point A. They each walk in a straight line at an angle of 120^o to each other. Thabo walks at 6 kmph and Palesa walks at 8 kmph. How far apart are they after 45 minutes?

12 The design of the kerbing cross-section for a driverless-bus roadway is given. The metal strip is inlaid into the concrete and is used to control the direction of travel and speed of the bus. Find the width of the metal strip.



13 An orienteer runs for $4\frac{1}{2}$ km and then turns through an angle of 32^o and runs another 6 km. How far is she from her starting point?

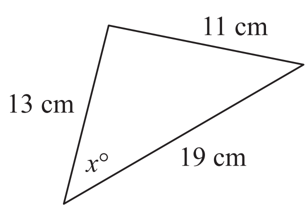
14 Sam and Markus are standing on level ground 100 metres apart. A large tree is due North of Markus and on a bearing of 065^o from Sam. The top of the tree appears at an angle of elevation of 25^o to Sam and 15^o to Markus. Find the height of the tree.

15 A helicopter A, flying at 4000 m, observes two ships B and C. B is 23.8 km from the helicopter and C is 31.9 km from it. The angle of view from the helicopter to B and C (angle BAC) is 83.6^o. How far are the ships apart?

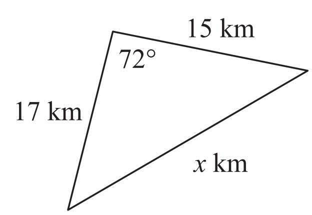
REVIEW SET 12

1 Determine the value of x:

a



b



g 18^o **h** 27^o **i** 150^o **j** 22.5^o

4 **a** 114.59^o **b** 87.66^o **c** 49.68^o **d** 182.14^o **e** 301.78^o

5 **a**

Degrees	0	45	90	135	180	225
Radians	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$

Degrees	270	315	360
Radians	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π

b

Degrees	0	30	60	90	120	150	180
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π

Degrees	210	240	270	300	330	360
Radians	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π

EXERCISE 11E.1

1 **a** 0, 1 **b** 1, 0 **c** 0, -1 **d** 0, -1

2 **a** $\cos\theta = \pm\frac{\sqrt{3}}{2}$ **b** $\cos\theta = \pm\frac{2\sqrt{2}}{3}$ **c** $\cos\theta = \pm 1$
d $\cos\theta = 0$

3 **a** $\sin\theta = \pm\frac{3}{5}$ **b** $\sin\theta = \pm\frac{\sqrt{7}}{4}$ **c** $\sin\theta = 0$ **d** $\sin\theta = \pm 1$

4 **a**

Quad-rant	*Degree measure*	*Radian measure*	$\cos\theta$	$\sin\theta$
1	$0 < \theta < 90$	$0 < \theta < \frac{\pi}{2}$	+ve	+ve
2	$90 < \theta < 180$	$\frac{\pi}{2} < \theta < \pi$	$-$ve	+ve
3	$180 < \theta < 270$	$\pi < \theta < \frac{3\pi}{2}$	$-$ve	$-$ve
4	$270 < \theta < 360$	$\frac{3\pi}{2} < \theta < 2\pi$	+ve	$-$ve

b **i** 1 and 4 **ii** 2 and 3 **iii** 3 **iv** 2

5 **a** $\sin\theta = \frac{\sqrt{5}}{3}$ **b** $\cos\theta = -\frac{\sqrt{21}}{5}$ **c** $\cos\theta = \frac{4}{5}$
d $\sin\theta = -\frac{12}{13}$

EXERCISE 11E.2

1

	a	**b**	**c**	**d**	**e**
$\sin\theta$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	0	$-\frac{1}{\sqrt{2}}$
$\cos\theta$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	-1	$-\frac{1}{\sqrt{2}}$

2

	a	**b**	**c**	**d**	**e**
$\sin\beta$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$
$\cos\beta$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$

3 **a** $\frac{3}{4}$ **b** $\frac{1}{4}$ **c** 3 **d** $\frac{1}{4}$ **e** $-\frac{1}{4}$ **f** 1 **g** $\sqrt{2}$ **h** $\frac{1}{2}$ **i** $\frac{1}{2}$

4 **a** 30^o, 150^o **b** 60^o, 120^o **c** 45^o, 315^o **d** 120^o, 240^o
e 135^o, 225^o **f** 240^o, 300^o

5 **a** 30^o, 330^o, 390^o, 690^o **b** 210^o, 330^o, 570^o, 690^o
c 270^o, 630^o

6 **a** $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$ **b** $\theta = \frac{\pi}{3}, \frac{2\pi}{3}$ **c** $\theta = \pi$ **d** $\theta = \frac{\pi}{2}$
e $\theta = \frac{3\pi}{4}, \frac{5\pi}{4}$ **f** $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ **g** $\theta = 0, \pi, 2\pi$
h $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

EXERCISE 11F

1 **a** 28.9 cm^2 **b** 384 km^2 **c** 26.7 cm^2 **2** $x = 19.0$

3 18.9 cm^2 **4** 137 cm^2 **5** 374 cm^2 **6** 7.49 cm

7 11.9 m **8** **a** 48.6^o or 131.4^o **b** 42.1^o or 137.9^o

9 $\frac{1}{4}$ is not covered

10 **a** **i** and **ii** 6 cm^2 **b** **i** $\doteqdot$ 21.3 cm^2 **ii** 30.7 cm^2

EXERCISE 11G

1 **a** **i** 6.53 cm **ii** 29.4 cm^2 **b** **i** 10.5 cm **ii** 25.9 cm^2

2 **a** 3.14 m **b** 9.30 m^2

3 **a** 5.91 cm **b** 18.9 cm **4** **a** 39.3^o **b** 34.4^o

5 **a** 0.75^c **b** 1.68^c **c** 2.32^c

6 **a** 8.75 cm^2 **b** 36.2 cm^2 **c** 62.8 cm^2

7 10 cm, 25 cm^2 **8** 65 cm^2

9 **a** 11.7 cm **b** 11.7 **c** 37.7 cm **d** 185^o

10 **a** $\alpha = 18.43$ **b** $\theta = 143.1$ **c** 387.3 m^2

11 **b** 2 h 24 min **12** 227 m^2

13 **a** $\alpha = 5.739$ **b** $\theta = 168.5$ **c** $\phi = 191.5$ **d** 71.62 cm

REVIEW SET 11A

1 **a** $\sin 70^o \doteqdot 0.94$ **b** $\cos 35^o \doteqdot 0.82$

2 M($\cos 73^o$, $\sin 73^o$) $\doteqdot$ (0.292, 0.956)
N($\cos 190^o$, $\sin 190^o$) $\doteqdot$ (-0.985, -0.174)
P($\cos 307^o$, $\sin 307^o$) $\doteqdot$ (0.602, -0.799)

3 $\theta \doteqdot 102.8^o$ **4** **a** 60^o **b** 15^o **c** 85^o

5 **a** 133^o **b** 172^o **c** 94^o

6 **a** 0.358 **b** -0.035 **c** 0.259 **d** -0.731

7 **a** 1, 0 **b** -1, 0 **8** **a** 79^o **b** 53^o **c** 12^o

9 **a** 84^o **c** 62^o **c** 3^o

10 **a** 0.961 **b** -0.961 **c** -0.961 **d** -0.961

11 **a** -0.743 **b** -0.743 **c** 0.743 **d** -0.743

REVIEW SET 11B

1 21.1 km^2 **2** **a** 118 cm^2 **b** 44.9 cm^2

3 perimeter = 34.1 cm, area = 66.5 cm^2

4 $r = 8.79$ cm, area = 81.0 cm^2 **5** 67.4^o or 112.6^o

6 $x = 47.5$, AC = 14.3 cm **or** $x = 132.5$, AC = 28.1 cm

7 36.8 cm^2 **8** **a** 10 600 m^2 **b** 1.06 ha

10 **a** $\frac{2\pi}{3}$ **b** $\frac{5\pi}{4}$ **c** $\frac{5\pi}{6}$ **d** 3π

11 **a** 1.239^c **b** 2.175^c **c** -2.478^c **d** -0.4416^c

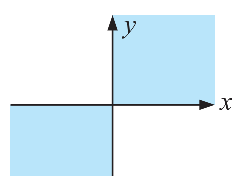
12 **a** 72^o **b** 225^o **c** 140^o **d** 330^o

13 **a** 171.89^o **b** 83.65^o **c** 24.92^o **d** -302.01^o

REVIEW SET 11C

1 **a** (0.766, -0.643) **b** (-0.956, 0.292)

2



3 **a** $\sin(\frac{2\pi}{3}) = \frac{\sqrt{3}}{2}$,
$\cos(\frac{2\pi}{3}) = -\frac{1}{2}$
b $\sin(\frac{8\pi}{3}) = \frac{\sqrt{3}}{2}$,
$\cos(\frac{8\pi}{3}) = -\frac{1}{2}$

4 **Hint:** $\tan\theta = -1$ when $\cos\theta = -\sin\theta$

5 **a** 0, -1 **b** 0, -1 **6** $\pm\frac{\sqrt{7}}{4}$ **7** $\frac{\sqrt{7}}{4}$ **8** **a** $\frac{3}{4}$ **b** $-\sqrt{2}$

9 **a** $\frac{\sqrt{3}}{2}$ **b** $-\frac{1}{2}$ **c** $\frac{1}{2}$ **10** **a** 150^o, 210^o **b** 45^o, 135^o

11 **a** $\theta = \pi + k2\pi$ **b** $\theta = \left.\begin{matrix}\frac{\pi}{3}\\ \frac{2\pi}{3}\end{matrix}\right\} + k\pi$

EXERCISE 12A

1 **a** 28.8 cm **b** 3.38 km **c** 14.2 m

2 $\measuredangle$A = 52.0^o, $\measuredangle$B = 59.3^o, $\measuredangle$C = 68.7^o **3** 112^o

4 **a** 40.3^o **b** 107^o **5** **a** $\cos\theta = 0.65$ **b** $x = 3.81$

EXERCISE 12B.1

1 **a** $x = 28.4$ **b** $x = 13.4$ **c** $x = 3.79$

2 **a** $a = 21.25$ cm **b** $b = 76.9$ cm **c** $c = 5.09$ cm

EXERCISE 12B.2

1 $\measuredangle C = 62.1^o$ or $\measuredangle C = 117.9^o$

2 **a** $\measuredangle A = 49.5^o$ **b** $\measuredangle B = 72.05^o$ or 107.95^o **c** $\measuredangle C = 44.3^o$

3 No, $\dfrac{\sin 85^o}{11.4} \neq \dfrac{\sin 27^o}{9.8}$ **4** $\measuredangle ABC = 66^o$, BD $= 4.55$ cm **5** $x = 17.7$, $y = 33.1$

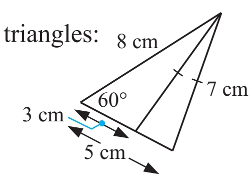
6 **a** 88.7^o or 91.3^o **b** 91.3^o
c .... *cosine rule* as it avoids the *ambiguous case*.

EXERCISE 12C

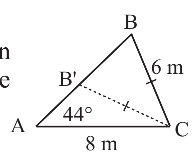
1 17.7 m **2** 207 m **3** 23.9^o **4** 77.5 m **5** 13.2^o
6 69.1 m **7** **a** 38.0 m **b** 94.0 m **8** 55.1^o
9 AC $= 11.7$ km BC $= 8.49$ km
10 **a** 74.9 km^2 **b** 7490 hectares **11** 9.12 km
12 $\doteqdot 85$ mm **13** 10.1 km **14** 29.2 m **15** 37.6 km

REVIEW SET 12

1 **a** $x = 34.1$ **b** $x = 18.9$ **2** **a** $x = 41.5$ **b** $x = 15.4$
3 AC $= 12.55$ cm, $\measuredangle A = 48.6^o$, $\measuredangle C = 57.4^o$
4 113 cm^2 **5** 7.32 m **6** 204 m
7 530 m, bearing 077.2^o **8** 179 km, bearing 352^o
9 If the unknown is an angle, use the cosine rule to avoid the ambiguous case.
10 **a** $x = 3$ or 5 **b** Kady can draw 2 triangles:



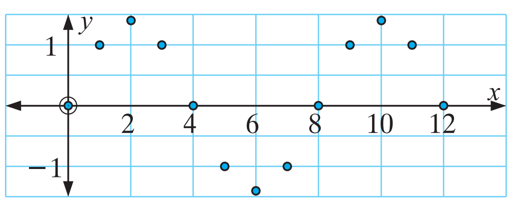
11 **a** The information given could give two triangles:



b $\doteqdot 2.23$ m^3

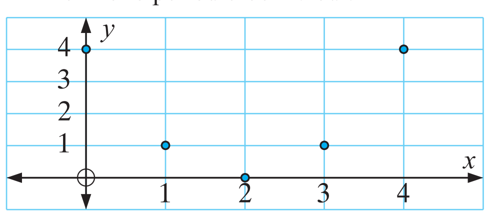
EXERCISE 13A

1 **a**



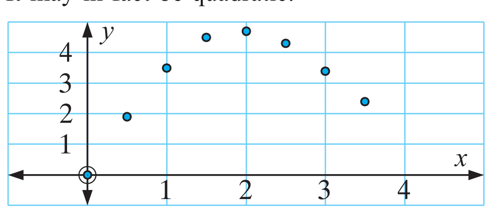
Data exhibits periodic behaviour.

b



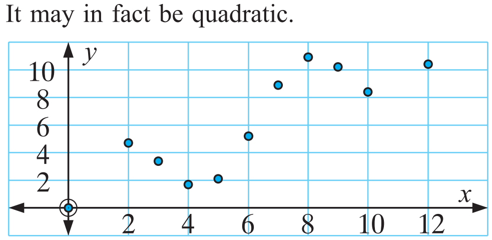
Not enough information to say data is periodic.
It may in fact be quadratic.

c



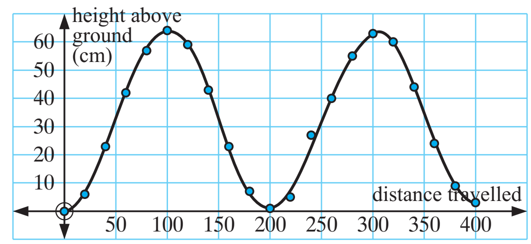
Not enough information to say data is periodic.
It may in fact be quadratic.

d



Not enough information to say data is periodic.

2 **a**

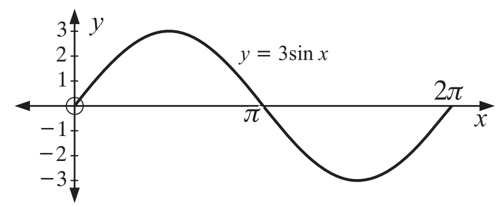


b The data is periodic. **i** $y = 32$ (approx.)
ii $\doteqdot 64$ cm **iii** $\doteqdot 200$ cm **iv** $\doteqdot 32$ cm
c A curve can be fitted to the data.

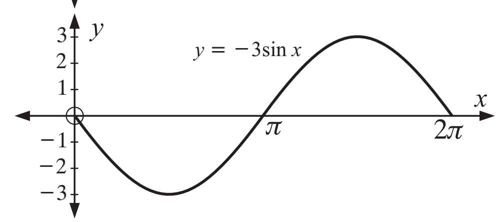
3 **a** periodic **b** periodic **c** periodic **d** not periodic
e periodic **f** periodic

EXERCISE 13B.1

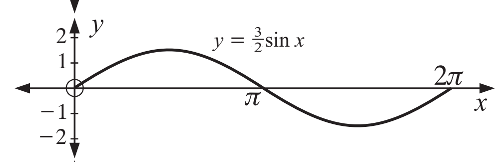
1 **a**



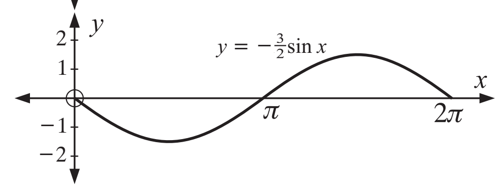
b



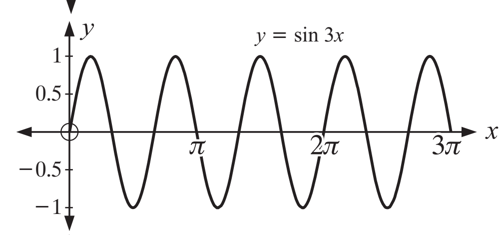
c



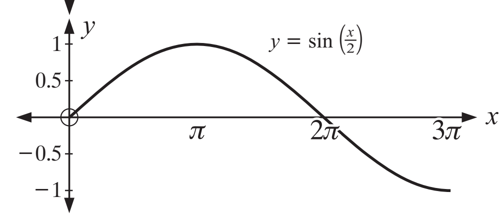
d



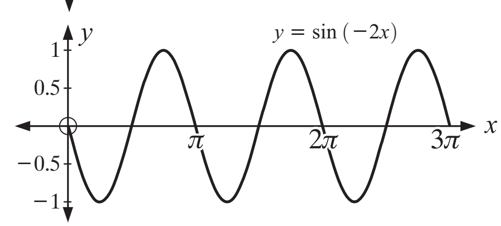
2 **a**



b



c



3 **a** $\frac{\pi}{2}$ **b** $\frac{\pi}{2}$ **c** 6π **d** $\frac{10\pi}{3}$

4 **a** $B = \frac{2}{5}$ **b** $B = 3$ **c** $B = \frac{1}{6}$ **d** $B = \frac{\pi}{2}$ **e** $B = \frac{\pi}{50}$