COMBINATIONS

2.55. A class contains 9 boys and 3 girls. (i) In how many ways can the teacher choose a committee of 4? (ii) How many of them will contain at least one girl? (iii) How many of them will contain exactly one girl?

2.56. A woman has 11 close friends. (i) In how many ways can she invite 5 of them to dinner? (ii) In how many ways if two of the friends are married and will not attend separately? (iii) In how many ways if two of them are not on speaking terms and will not attend together?

2.57. There are 10 points $A, B, \ldots$ in a plane, no three on the same line. (i) How many lines are determined by the points? (ii) How many of these lines do not pass through A or B? (iii) How many triangles are determined by the points? (iv) How many of these triangles contain the point A? (v) How many of these triangles contain the side AB?

2.58. A student is to answer 10 out of 13 questions on an exam. (i) How many choices has he? (ii) How many if he must answer the first two questions? (iii) How many if he must answer the first or second question but not both? (iv) How many if he must answer exactly 3 of the first 5 questions? (v) How many if he must answer at least 3 of the first 5 questions?

2.59. A man is dealt a poker hand (5 cards) from an ordinary playing deck. In how many ways can he be dealt (i) a straight flush, (ii) four of a kind, (iii) a straight, (iv) a pair of aces, (v) two of a kind (a pair)?

2.60. The English alphabet has 26 letters of which 5 are vowels.

(i) How many 5 letter words containing 3 different consonants and 2 different vowels can be formed?

(ii) How many of them contain the letter b?

(iii) How many of them contain the letters b and c?

(iv) How many of them begin with b and contain the letter c?

(v) How many of them begin with b and end with c?

(vi) How many of them contain the letters a and b?

(vii) How many of them begin with a and contain b?

(viii) How many of them begin with b and contain a?

(ix) How many of them begin with a and end with b?

(x) How many of them contain the letters a, b and c?

2.55. (i) $\binom{12}{4} = 495$, (ii) $\binom{12}{4} - \binom{9}{4} = 369$, (iii) $3 \cdot \binom{9}{3} = 252$

2.56. (i) $\binom{11}{5} = 462$, (ii) $\binom{9}{3} + \binom{9}{5} = 210$, (iii) $\binom{9}{5} + 2 \cdot \binom{9}{4} = 378$

2.57. (i) $\binom{10}{2} = 45$, (ii) $\binom{8}{2} = 28$, (iii) $\binom{10}{3} = 120$, (iv) $\binom{9}{2} = 36$, (v) 8

2.58. (i) $\binom{13}{10} = \binom{13}{3} = 286$

(ii) $\binom{11}{8} = \binom{11}{3} = 165$

(iii) $2 \cdot \binom{11}{9} = 2 \cdot \binom{11}{2} = 110$

(iv) $\binom{5}{3}\binom{8}{7} = 80$

(v) $\binom{5}{3}\binom{8}{7} + \binom{5}{4}\binom{8}{6} + \binom{5}{5}\binom{8}{5} = 276$

2.59. (i) $4 \cdot 10 = 40$, (ii) $13 \cdot 48 = 624$, (iii) $10 \cdot 4^5 - 40 = 10{,}200$. (We subtract the number of straight flushes.) (iv) $\binom{4}{2}\binom{12}{3} \cdot 4^3 = 84{,}480$, (v) $13 \cdot \binom{4}{2}\binom{12}{3} \cdot 4^3 = 1{,}098{,}240$

2.60. (i) $\binom{21}{3}\binom{5}{2} \cdot 5! = 1{,}596{,}000$

(ii) $\binom{20}{2}\binom{5}{2} \cdot 5! = 228{,}000$

(iii) $19 \cdot \binom{5}{2} \cdot 5! = 22{,}800$

(iv) $19 \cdot \binom{5}{2} \cdot 4! = 4560$

(v) $19 \cdot \binom{5}{2} \cdot 3! = 1140$

(vi) $4 \cdot \binom{20}{2} \cdot 5! = 91{,}200$

(vii) $4 \cdot \binom{20}{2} \cdot 4! = 18{,}240$

(viii) 18,240 (same as (vii))

(ix) $4 \cdot \binom{20}{2} \cdot 3! = 4560$

(x) $4 \cdot 19 \cdot 5! = 9120$