Chapter 5

Applications

5.1. Introduction

Some people believe that the reason to study mathematics is that it is useful for accomplishing practical tasks. The author of this book believes that the reason to study mathematics is the intellectual and aesthetic satisfaction that human beings find in it. He hopes you agree with him. That said, it is a fact that mathematics is enormously useful. And its utility ranges from baking a cake to positioning a space vehicle on Mars. There is not a bridge you cross that does not owe its existence to applied mathematics. Additionally, it is kind of fun to discover how easily certain problems are solved once a little mathematics is used. Admittedly, the problems you will solve in this book are much more humble than designing the Brooklyn Bridge, but you have got to start someplace, right?

Translating facts given in words or pictures into mathematical expressions and equations is an acquired skill. Like most skills, it takes practice. Sometimes you will be frustrated. But, persevere. There is no other path to success.

5.2. Words to equations

Example 5.1
Suppose there are 10 more girls than boys enrolled at a certain school. If there are 252 students, can you say how many boys and girls are enrolled?

It is easy to draw a picture that represents the information provided.

Once the bar representing the number of boys is drawn, the bar representing the number of girls can be drawn in only one way.

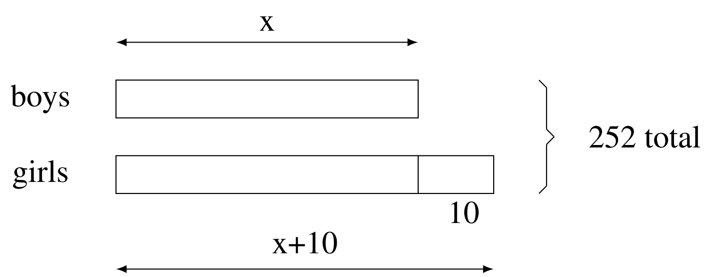

FIGURE 5.1. Boys and girls at a school.

From the picture, we can see that the quantity $x+(x+10)$ must be equal to 252. We could write

$$x+(x+10)=252.$$

Solving for x,

$$\begin{aligned} 2x+10 &= 252 \\ x &= 121. \end{aligned}$$

So, we conclude that at this school there are 121 boys and 131 girls.

■

We can work the problem of example (5.1) without the picture, but we will have to be careful to say exactly what the letter "x" represents. Here goes.

Example 5.2
Suppose there are 10 more girls than boys enrolled at a certain school. If there are 252 students, can you say how many boys and girls are enrolled?

Solution.

Once you declare that "x = the number of boys", the rest of the story can only unfold one way.

Let x represent the number of boys.
Then $x+10$ represents the number of girls.
Hence, $x+(x+10)=252$.
So, $x=121$.
Therefore, there are 121 boys and 131 girls. ■

Example 5.3
Suppose there are twice as many girls as boys in a certain class. If there are 27 students, can you say how many boys and girls are in the class?

We can, as before, draw a picture.

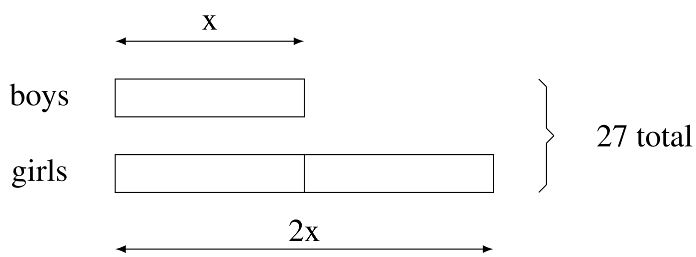

FIGURE 5.2. Boys and girls at a school.

From the picture, we can see that the quantity $x+2x$ must be equal to 27. We could write

$$x+2x=27.$$

Solving for x,

$$\begin{aligned} 3x &= 27 \\ x &= 9. \end{aligned}$$

So, we would conclude that there at this school there are 9 boys and 18 girls. ■

We can work the problem of example (5.3) without the picture. We do so in the next example.

Example 5.4
Suppose there twice as many girls as boys in a certain class. If there are 27 students, can you say how many boys and girls are in the class?

Solution.
Let x represent the number of boys.
Then,
$2x$ represents the number of girls,
$x+2x=27$,
$x=9$.
Therefore, there are 9 boys and 18 girls. ■

Example 5.5
Janice sold $\frac{2}{5}$ as many cupcakes as did Brian. Together they sold 70 cupcakes. How many cupcakes did each person sell?

Solution.
Let x represent the number of cupcakes Brian sold.
Then,
$\frac{2}{5}x$ represents the number of cupcakes Janice sold.

$x+\frac{2}{5}x=70.$

$x=50.$

Therefore, Brian sold 50 cupcakes and Janice sold 20 cupcakes. ■

Example 5.6
One bag of oranges contained 8 less than 3 times the quantity in another bag. If together the bags contained 44 oranges, how many oranges were contained in each bag?

Solution.
Let $x=$ the number of oranges in the other bag.
Then,
$3x-8=$ the number of oranges in the one bag,
$x+3x-8=44.$ So, $x=13$, and $3x-8=31$.
$\therefore$ One bag held 31 and the other 13 oranges. ■

Example 5.7
Jim had 8 times as many dimes as quarters and 12 times as many nickels as dimes. If he had \$40.95, how many quarters did he have?

Solution.
Let $x=$ the number of quarters.
Then,
$\$.25x=$ the *v*alue of the quarters,
$8x=$ the quantity of dimes,
$(\$.10)8x=$ the *v*alue of the dimes,
$12\cdot 8x=$ the quantity of nickels,
$(\$.05)(12\cdot 8x)=$ the *v*alue of nickels.

Since the *v*alue of all the coins is \$40.95,

$$\begin{aligned}
\$.25x+\$(.10)8x+\$(.05)(12\cdot 8x) &= \$40.95\\
25x+(10)8x+(5)(12\cdot 8x) &= 4095\\
25x+80x+480x &= 4095\\
585x &= 4095\\
x &= 7
\end{aligned}$$

$\therefore$ Jim had 7 quarters. ■

The purpose of the following exercise is to provide practice discovering and writing solutions. The process of solving is the focus in this exercise, not the answer. Your work should be similar to that shown in the preceding examples and to the solutions provided in the appendix. The answers to the problems in this exercise are intended to be almost obvious. In the future, when the answers are not obvious, you might be glad you practiced this.

Exercise 5.1

1. Josh earned $10 more than Joe. Their total earnings were $1200. How much did each earn?
2. The capacity of a tank is 10 gallons less than the capacity of another tank. If together they hold 152 gallons, how many gallons does each tank hold?
3. A rectangle is 10 inches longer than it is wide and half its perimeter is 152 inches. How wide and long is the rectangle?
4. Mrs. Lu gave her children $2.40 in quarters, dimes, and nickels. The number of nickels was twice the number of quarters. The number of quarters was twice the number of dimes. How many of each coin did she give out?
5. Two inches are cut off one side of a square paper and 3 inches are added to an adjacent side. The resulting rectangle has a perimeter of 54 inches. What are the dimensions of the original square?
6. Linda has a certain number of nickels and Betsy has the same number of pennies. The sum of the amounts of money possessed by the two girls is $2.10. How much money has each girl?
7. John is three times as old as Dick, and three years ago the sum of their ages was 22 years. How old is each now?

Although no law says you have to draw a picture, sometimes it can be very helpful to do so.

Example 5.8
Alex, Bill, and Carla share a sum of money in the ratio 4 : 5 : 6. If Carla receives $80 more than Alex, find the sum of money shared by the three children?

Solution.

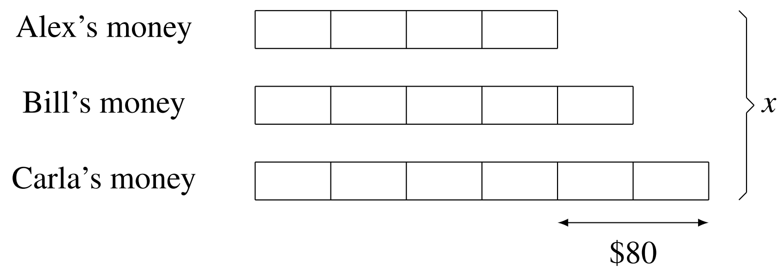

FIGURE 5.3. A very helpful picture.

The picture makes plain that each unit represents \$40 and the total, represented by "x", is 15 such units. So, the total amount of money shared by the three children is $15 \cdot \$40 = \$600$.

The solution could be written as follows.

Let $x =$ the sum of money (\$).
Then,

$\frac{6}{15}x =$ Carla's share (\$),

$\frac{4}{15}x =$ Alex's share,

$\frac{6}{15}x - \frac{4}{15}x = 80.$

$6x - 4x = 1200. \qquad x = 600.$

$\therefore$ The sum of money shared by the three people is \$600. ∎

Exercise 5.2

1. The area of one circle is $\frac{7}{8}$ that of a rectangle. If the sum of the areas is 105 square inches, what is the area of the rectangle?
2. Jim had \$4.40 in nickels, dimes, and quarters. If he had the same number of each denomination of coin, how many coins did he have?
3. In a collection of stamps, the number of Canadian stamps is 14 less than three times the number of United States stamps. The total number of Canadian and United States stamps is 78. How many United States stamps are there?
4. Alice had 18 times as many nickels as Bob had quarters. Together they had \$18.40. How many coins did each child have?
5. Barb is 4 times as old as Cindy. Four years ago the sum of their ages was 37 years. How old is Cindy?
6. Russel though he would be clever by paying a debt using an equal number of of nickels, dimes, and quarters? How many coins did he use to pay the debt of \$25.60?
7. One tank contains 5 gallons of water and is being filled at the rate of 12 gallons per minute. Another tank contains 61 gallons and is being drained at the rate of 4 gallons per minute. After how many minutes do the tanks contain the same volume of water?
8. Abigail has \$142 and Barbara has \$20. How much money should Abigail give to Barbara so that Abigal would have five times as much money as Barbara?
9. A farmer used $\frac{7}{8}$ as much fuel in July as he used in June. If he used 310 gallons of fuel less in July than in June, find the quantity of fuel he used in June?
10. Sue had three lame horses. The veterinarian's charge for the second second horse was $\frac{2}{3}$ the charge for the first horse and the charge for the third was $1\frac{3}{4}$ the charge for the second. If the total charge was \$408, how much was the charge for the first horse?
11. Arthur and Jill had equal amounts of money. After Arthur spent \$30 and Jill spent \$15, the ratio of Arthur's money to Jill's money was 4 : 5. How much did each person have originally?
12. In January, Ace had twice as much money as Sam. In February, Ace increased his money by 40% and Sam increased his by 30%. At the end of February, Ace had \$12 more than Sam. How much money did each boy have in January?

13. In a school, 70% of the teachers are female. If there are 36 more female than male teachers, how many teachers does the school have?
14. Alice had $\frac{3}{5}$ as many marbles as Bridget. After Bridget gave $\frac{1}{4}$ of her marbles to Alice, Alice had 170 marbles. How many marbles did Alice have originally?
15. Roger spent 30% of his money on a book and $\frac{3}{5}$ of the remainder on a pen. He had $28 left. How much money did he have at first?

5.2.1. Units of measure

Practical applications of mathematics *always* include units of measure such as minutes, hours, feet, meters, kilograms, pounds, hogsheads and many others. Knowing the distance from here to there is 612, is not especially helpful. If it is 612 feet, you will be there in a jiffy, but let it be 612 miles and you will need to pack several lunches unless you go by jet plane.

Many of the units with which you will work will be in the form of a fraction. An example of this is miles per hour which we write as $\frac{m}{h}$ or $\frac{mi}{h}$. We could write this as $\frac{1 \cdot m}{1 \cdot h}$. We usually do not bother writing the "1". When you see $\frac{m}{h}$, it is helpful to know that the numerator is a product of two factors, "1" and "m" and the denominator is a product of two factors, "1" and h". Similarly, the numerator of $\frac{60m}{h}$ is the product of factors 60 and m. Just as always, factors common to the numerator and denominator may be canceled.

Example 5.9

$$\frac{60m}{m} = \frac{60\cancel{m}}{\cancel{m}} = 60.$$

Example 5.10

$$\frac{\frac{60m}{h}}{m} = \frac{60m}{h} \cdot \frac{1}{m} = \frac{60\cancel{m}}{h} \cdot \frac{1}{\cancel{m}} = \frac{60}{h}.$$

Example 5.11

$$\frac{20m}{\frac{10m}{h}} = 20m \cdot \frac{h}{10m} = \cancel{20m} \cdot \frac{h}{\cancel{10m}} = 2h.$$

Exercise 5.3

Simplify the following.

1. $\dfrac{42m}{\frac{6m}{h}}$
2. $\dfrac{12\,\text{gallon}}{\frac{6\,\text{gallon}}{\text{minute}}}$
3. $\dfrac{15\,\text{hogshead}}{\frac{3\,\text{hogshead}}{h}}$
4. Divide 180 miles by 30 miles per hour.
5. Divide 15 miles by 10 miles per hour.
6. Divide 24 liters by 5 liters per minute.

5.2.1.1. Conversion of units

To rewrite a of measure, we can use the fact that multiplying by the number 1 changes the name but not the quantity. Along with this, it is helpful to know facts such as the following.

$$\begin{aligned} 1 \text{ minute} &= 60 \text{ seconds}, & 1 \text{ hour} &= 60 \text{ minutes} \\ 1 \text{ foot} &= 12 \text{ inches}, & 1 \text{ mile} &= 5280 \text{ feet} \\ 1 \text{ meter} &= 100 \text{ centimeters}, & 1 \text{ kilometer} &= 1000 \text{ meters} \\ 1 \text{ gram} &= 1000 \text{ milligrams}, & 1 \text{ kilogram} &= 1000 \text{ grams} \end{aligned}$$

Does $\frac{60 \text{ minutes}}{\text{hour}} = 1$? Yes, because,

$$60 \text{ minutes} = 1 \text{ hour} \iff \frac{60 \text{ minutes}}{\text{hour}} = 1.$$

Example 5.12
Rewrite 30 miles per hour as miles per minute.
Solution.

$$\left(\frac{30 \text{ mile}}{\cancel{\text{hour}}}\right) \cdot \left(\frac{1 \cancel{\text{hour}}}{60 \text{ minute}}\right) = 0.5 \frac{\text{mile}}{\text{minute}} \quad \blacksquare$$

Example 5.13
Rewrite 4 meters per second as kilometers per hour.
Solution.

$$\left(\frac{4 \cancel{\text{m}}}{\cancel{\text{s}}}\right) \cdot \left(\frac{60 \cancel{\text{s}}}{1 \cancel{\text{min}}}\right) \cdot \left(\frac{60 \cancel{\text{min}}}{1 \text{ h}}\right) \cdot \left(\frac{1 \text{ km}}{1000 \cancel{\text{m}}}\right) = 14.4 \frac{\text{km}}{\text{h}} \quad \blacksquare$$

Exercise 5.4

1. Rewrite 2 miles per minute as miles per hour.

2. Rewrite 1200 meters per minute as kilometers per hour.

3. Rewrite 2200 centimeters per second as meters per minute.

4. Rewrite 1200 feet per second as miles per hour.

5. Rewrite 33000 feet per minute as miles per hour.

5.2.2. Distance, rate, and time

It is a fact of physics that the distance one travels is directly proportional to the constant rate at which one travels and to the duration of time one travels at that rate. This is not hard to believe, because your own experience tells you it is true.

You know that if you travel at a certain rate for two hours you will travel twice as far as you would in one hour; in four hours, four times as far as in one hour, in six hours, six times as far as in one hour. This is summed up by saying that "distance traveled is *directly proportional to the time* of travel".

Similarly, you know that if you travel for a three hours at twice the rate of your friend who also travels three hours, you will travel twice as far as she. Travel at four times her rate and you will cover four times the distance she does in a given period of time. And this means "distance traveled is *directly proportional to the rate* of travel".

The fact that distance traveled, at a constant rate, is directly proportional to time of travel at that rate and to the rate itself is captured in the following equation

$$d = rt, \tag{5.1}$$

where d = distance traveled, r = constant rate of travel, and t = time of travel.

Equation (5.1) is humble in appearance, but important precisely because of its simplicity.

The phrase "constant rate" has now appeared several times. It means that the rate is unvarying. A car that travels at a constant rate of speed, neither speeds up nor slows down. When the driver presses the accelerator, the car speeds up and for that time its speed is *not* constant. From your experience, you know that you can feel the change in speed. Another phrase that means constant rate is "uniform rate". Instead of saying "rate of travel", we usually just say "speed". It is also a fact that when traveling at a constant rate, the average rate of travel equals the rate of travel. We are careful to note that Equation (5.1) holds only for cases of constant rate.

Equation (5.1) can be rewritten in two other forms

$$r = \frac{d}{t} \tag{5.2a}$$

$$t = \frac{d}{r} \tag{5.2b}$$

Equation (5.2a) tells us that in all cases of travel at a constant rate, the ratio of distance to time is constant.

Example 5.14
The following table provides information about a one hour journey by automobile. Assume the speed was constant in each time interval. Was the speed constant throughout this journey?

Distance traveled for several time intervals				
time (minutes)	0 – 15	15 – 30	30 – 45	45 – 60
distance (miles)	25	25	30	25

TABLE 5.1. A journey by automobile

The speed was not constant for the whole trip. From 30 to 45 minutes the ratio $\frac{\text{distance}}{\text{time}}$ was $\frac{30 \text{ mile}}{15 \text{ minute}}$, 2 miles per minute, but this differs from the speed in the other time intervals which was $\frac{25 \text{ mile}}{15 \text{ minute}}$, about 1.6 miles per minute. ■

Example 5.15
Sally traveled by car at a constant speed of $50\frac{m}{h}$. If she traveled 450 miles, for what duration of time did she travel?

Indenting the two lines where d and t are

Solution.

$$d = rt$$
$$t = \frac{d}{r}$$
$$d = 450\,m$$
$$r = 50\,\frac{m}{h}$$
$$t = \frac{450\cancel{m}}{50\frac{\cancel{m}}{h}}$$
$$t = 9\,h$$

$\therefore$ Sally traveled by car for 9 hours. ■

Example 5.16
Ali took 3 hours to travel from Place A to Place B at an average speed of 80 km/h. On the way back, he drove at an average speed of 60 km/h. How long did he take to travel from Place B to Place A?

Solution.
Let $d = \text{distance}_{B\to A}$, $r = \text{speed}_{B\to A}$, $t = \text{time}_{B\to A}$.

Begin by stating the idea you will use to solve the problem. Here the idea is that distance is directly proportional to the time taken and to the rate of travel; that is, $d = rt$.

$$d = rt$$
$$t = \frac{d}{r}$$
$$r = 60\,\frac{km}{h}$$
$$d = \frac{80\text{ km}}{h} \cdot 3h = 240\text{ km}$$
$$t = \frac{240\cancel{km}}{60\frac{\cancel{km}}{h}}$$
$$t = 4\,h.$$

$\therefore$ Ali took 4 hours to return to Place A from Place B. ■

Example 5.17
Emily took 8 hours to travel from Place A to Place B at an average speed of 40 km/h. Dylan took only 5 hours for the trip. What was Dylan's average speed for the trip from A to B?

Solution.
Let d = distance A→B, r = Dylan's speed A→B, t = Dylan's time A→B.

$$d = rt$$

$$r = \frac{d}{t}$$

$$t = 5h$$

$$d = \frac{40 \text{ km}}{h} \cdot 8h = 320 \text{ km}$$

$$r = \frac{320\, km}{5\, h}$$

$$r = 64\, \frac{km}{h}.$$

∴ Dylan's average speed from A to B was 64 km/h. ■

Example 5.18
Becky drove for 1 hour at 70 mph. Then she drove for 6 hours at 56 mph. What was her average speed for the entire journey?

"mph" is a common way to write "miles per hour".

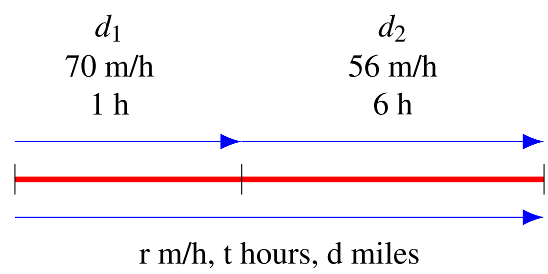

FIGURE 5.4

Solution.
Let $d =$ total distance, $r =$ average speed, $t =$ total time.

$$d = rt$$

$$r = \frac{d}{t}$$

$$t = 1h + 6h = 7h$$

$$d = d_1 + d_2$$

$$= \frac{70\,m}{h} \cdot 1h + \frac{56\,m}{h} \cdot 6h = 406 \text{ m}$$

$$r = \frac{406\;m}{7\;h}$$

$$r = 58\,\frac{m}{h}.$$

∴ Becky's average speed for the entire journey was 58 mph. ■

Example 5.19
Tom traveled from Town A to Town B. He drove the first 30 miles at 50 mph. He drove the remaining 80 miles at 75 mph. What was Tom's average speed for the trip from Town A to Town B?

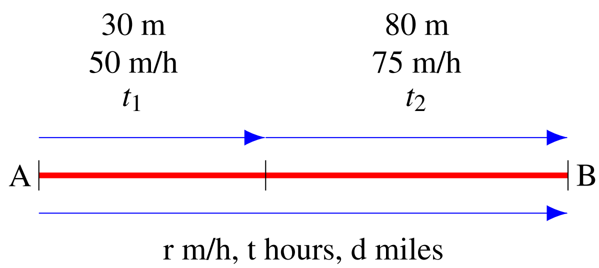

FIGURE 5.5

Solution.
Let $d =$ total distance, $r =$ average speed, $t =$ total time.

$$d = rt$$

$$r = \frac{d}{t}$$

$$d = 30\,m + 80\,m = 110\,m$$

$$t = t_1 + t_2$$

$$= \frac{30\,m}{\frac{50\,m}{h}} + \frac{80\,m}{\frac{75\,m}{h}} = \frac{5}{3}\,\text{h}$$

$$r = \frac{110\,m}{\frac{5}{3}h}$$

$$r = 66\,\frac{m}{h}.$$

∴ Tom's average speed driving from Town A to Town B was 66 mph. ■

Example 5.20
Al traveled 2/3 of the way from Town A to Town B in 4 hours. He traveled the remaining 90 km in 2 hours. What was his average speed for the whole trip?

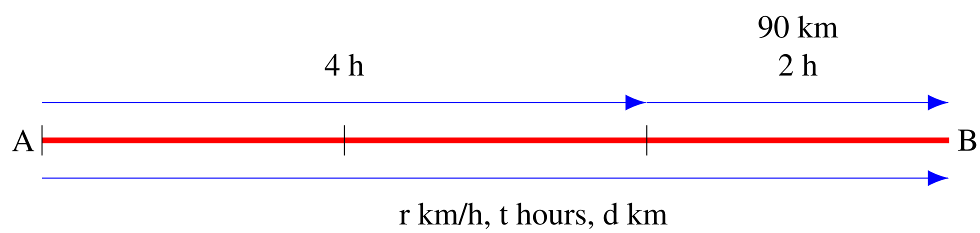

FIGURE 5.6

Solution.
Let $d =$ total distance, $r =$ average speed, $t =$ total time.

$$d = rt$$

$$r = \frac{d}{t}$$

$$d = 3 \cdot 90\ km = 270\ km$$

$$t = 2\ h + 4\ h$$

$$r = \frac{270\ km}{6\ h}$$

$$r = 45\,\frac{km}{h}.$$

$\therefore$ Al's average speed driving from Town A to Town B was 45 km/h. ■

Remark 5.1
Each of the examples (5.15)-(5.20) required the key idea that $d = rt$. Every problem you work that concerns distance, rate, and time will rely on this key idea. You might as well begin, as in the examples, by stating this idea. As soon as you have done so, your focus shifts to two well defined tasks. Specifically, you must find values for the two quantities in the equation $d = rt$ that determine the quantity you wish to know. Devote your effort to those two tasks.

Remark 5.2
The pictures in examples (5.17)-(5.20) were useful. It will often be advantageous to draw such pictures when working problems involving distance, rate, and time.

Exercise 5.5

1. Jack spent 3 hours driving from Town A to Town B. His average speed was 45 mph. Find the distance of Town B from Town A.

2. A train traveled at a constant speed of 90 mph. How long did it take to travel 75 miles?

3. A boy ran at a constant speed of 8 mph for 15 minutes. Then he walked at 3 mph for 90 minutes. How far did the boy walk?

4. Janice walked 18 km. For the first 12 km, she walked at a rate of 8 km/h. Then she walked the rest of the way at 6 km/h. Find the time it took Janice to complete her walk.

5. Mr. James bicycled 2 hours covering $\frac{5}{6}$ of the total distance to home. The last 10 km of his journey took 2 hours. What was his average speed for the journey home?

6. Mrs. Joyce drove from her house to her office at an average speed of 60 mph. It took her 1/2 hour. On the way home, her average speed was 50 mph. How long did it take her to drive home?

7. Jack and Jill each drove from Place A to Place B, a distance of 140 miles. Jack's average speed was 40 mph. Jill left Place A 60 minutes later than Jack, but she arrived at Place B 30 minutes earlier than Jack. What was Jill's average speed?

8. Sally drove a round trip from Place A to Place B. The distance from A to B was 30 miles. She drove from Place A to Place B at an average speed of 60 mph. She drove back from Place B to Place A at an average speed of 30 mph. What was her average speed for the entire

round trip?

9. A train traveled 3/5 of the way from Station A to Station B at an average speed of 60 mph. If the distance from Station A to Station B was 175 miles and the total time to travel from A to B was $3\frac{1}{2}$ hours, what was the average speed of the train for the remaining portion of the trip?

10. A train traveled 1/6 of the way from Station A to Station B in 1/2 hour at an average speed of 60 mph. The average speed of the train for the whole journey was 80 mph. How long did it take the train to travel the remaining portion of the journey?

11. Trains A and B traveling in the same direction on parallel tracks at 90 mph and 60 mph respectively pass the same station at the same time. How many minutes later will the distance between the trains be 10 miles?

12. Alice and Bill walk towards each other on path. Alice walks at a constant speed of 4 feet/second. Bill walks at a constant speed of 6 feet/second. If they are 3600 feet apart at noon, at what time will they meet?

13. Fred, who is walking at a constant speed of 0.1 feet/minute, passes point A at noon. Gladys, running at a constant speed to 0.3 feet/minute to catch Fred, passes point A at 1:00 PM. At what time does Gladys catch Fred?

14. Trains A and B are 264 miles apart and traveling towards each other on parallel tracks at 72 mph and 60 mph. What will be the distance between the two trains 12 hours later?

Exercise 5.6

Answer by writing the equation you would solve to obtain the numeric answer. You do not need to actually solve the equation.

1. Twelve oranges remain when each student is given 4 oranges. If 2 additional oranges were given to each student, there would be 10 oranges too few. How many students are there?

2. We wish to form a rectangle whose perimeter is 11 inches and whose length is 5 inches longer than its width. How many inches should the width be?

3. The sum of three consecutive integers is 195. Find these three integers.

4. A child is presently 12 years old and his father is 44 years old. In how many years will the age of the father be 3 times that of his child?

5. A store bought a certain number of eggs at \$0.20 each. During shipment to the store, eight of the eggs were broken. If the rest of the eggs were sold at \$0.23 each, the store will make a profit of \$25.16. How many eggs did the store buy originally?

6. The number of students at a school has increased 5% over last year and is now 336. How many students were there last year?

7. A man could arrive on time for an appointment if he drove his car at 40 miles per hour; however, he leaves 15 minutes late, he drives his car at 50 miles per hour and arrives 3 minutes early for his appointment. Find the distance between the starting point and his destination?

8. It takes 12 minutes to fill a tank with water using pipe A and 18 minutes using pipe B. First pipe A was opened, and with the tank still filling, pipe A was closed and pipe B was opened. When pipe B had been open for 3 minutes longer than pipe A, the tank was full. How many minutes was pipe A open?

9. When graduating seniors were being seated at graduation, three people were put on each bench, but then there were 25 people who could not be seated. When 4 people were put on each bench, there were exactly 4 benches remaining. How many graduates were there?

10. An express train that passed through station A at 48 miles per hour had to slow down to 32 miles per hour because of deep snow 30 minutes after passing through station A. It then reached station B 30 minutes behind schedule. How many miles apart are station A and station B?

11. A police officer is pursuing a robber. The officer is driving at a uniform speed of 120 mph, the robber at a uniform speed of 90 mph. When observed at 1:00 PM, the robber was 30 miles ahead of the police officer. At what time does the police officer catch up to the robber?

Answers to Exercise 5.1

(1)
Let $x =$ amount Joe earned (\$).
Then,
$x + 10 =$ amount Josh earned,
$x + x + 10 = 1200.$
$x = 595.$
∴ Joe earned \$595 and Josh earned \$605.

(2)
Let $x =$ the capacity of one tank (gallons).
Then,
$x - 10 =$ the capacity of the other tank,
$x + x - 10 = 152.$
$x = 81.$
∴ one tank holds 71 gallons and the other holds 81 gallons.

(3)
Let $x =$ the the width of the rectangle (inches).
Then,
$x + 10 =$ the length of the rectangle,
$2x + 2(x + 10) =$ perimeter,
$x + (x + 10) =$ half of perimeter,
$x + (x + 10) = 152.$
$x = 71.$
∴ The width of the rectangle is 71 inches and the length is 81 inches.

(4)
Let $x =$the number of dimes.
Then,
$2x =$ the number of quarters,
$4x =$ the number of nickels,
$10x =$ the value of the dimes (cents),
$50x =$ the value of the quarters (cents),
$20x =$ the value of the nickels (cents),
$10x + 50x + 20x = 240.$
$x + 5x + 2x = 24.$
$x = 3.$
∴ She had 3 dimes, 6 quarters, and 12 nickels.

(**5**)
Let x =length of side of original square paper (inches).
Then,
$x-2=$ the width of rectangular piece,
$x+3=$ the length of rectangular piece,
$2(x-2)+2(x+3)=54.$
$x=13.$
$\therefore$ The original paper was 13 inches by 13 inches.

(**6**)
Let $x=$ the number of Linda's nickels.
Then,
$x=$ the number of Betsy's pennies (cents),
$5x=$ the value of Linda's nickels (cents),
$x=$ the value of Betsy's pennies.
$x+5x=210.$
$x=35.$
$\therefore$ Betsy has \$0.35 and Linda has \$1.75.

(**7**)
Let $x=$ Dick's age now (years).
Then,
$3x=$ John's age now,
$x-3=$ Dick's age three years ago,
$3x-3=$ John's age three years ago.
$x-3+(3x-3)=22.$
$x=7.$
$\therefore$ Dick is 7 years old now. John is 21 years old now.

Answers to Exercise 5.2

(**1**) The area of the rectangle is 56 square inches. (**2**) Jim had 11 of each denomination of coin. (**3**) There are 23 U.S. stamps. (**4**) Bob had 16 quarters and Alice had 288 nickels. (**5**) Cindy is 9 years old. (**6**) Russel used 192 coins. (**7**) The tanks contain the same volume of water after $3\frac{1}{2}$ minutes. (**8**) Abigail should give \$7 to Barbara. (**9**) He used 2480 gallons of fuel in June. (**10**) The veterinarian charged \$144 for the first lame horse. (**11**) Each person originally had \$90. (**12**) In January, Ace had \$16 and Sam had \$8. (**13**) The school has 90 teachers. (**14**) Alice had 120marbles to start with. (**15**) He had \$100 at first.

Answers to Exercise 5.3

(**1**) $7h$. (**2**) 2 minute. (**3**) $5h$. (**4**) 6 hours. (**5**) 1.5 hours. (**6**) 4.8 minutes.

Answers to Exercise 5.4

(**1**) $120\frac{m}{h}$. (**2**) $72\frac{km}{h}$. (**3**) $1320\frac{m}{min}$. (**4**) $818\frac{m}{h}$. (**5**) $375\frac{m}{h}$.

Answers to Exercise 5.5

(**1**) 135 miles. (**2**) $\frac{5}{6}h$ or 50 minutes. (**3**) 6.5 miles. (**4**) 2.5 hours. (**5**) 15 km/h. (**6**) 0.6 hour. (**7**) 70 mph. (**8**) 40 mph. (**9**) 40 mph. (**10**) 1.75 hours. (**11**) 20. (**12**) 12:06 PM. (**13**) 1:30 PM. (**14**) 1320 miles.