Chapter 6

Linear Function

6.1. Idea of function as a rule

Suppose Romeo is two years younger than Juliet. You would have no trouble telling Romeo how old Juliet will be when he is 13 years old. If Romeo wanted Juliet's age when he is 14, you would say she will be 16. And if he asks, "How old will Juliet be when I am 15?", you would say, "17". If Romeo asks for her age when he is 16, you might begin to get fed up and make a chart for him so that he could just look up Juliet's age without bothering you. You provide the handy pocket chart shown below.

Romeo's age in years	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Juliet's age in years	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**

TABLE 6.1. Romeo and Juliet Ages

All is well with Romeo, until one night you get a call because he wishes to know how old will Juliet be when he is 31 years old. Instead of continuing the table, you might simply tell him a rule for finding Juliet's age. "Just add 2 to your age in years to get Juliet's age in years."

Rule for finding Juliet's age, j (years), given Romeo's age, r (years):

$$j = r + 2. \tag{6.1}$$

Table (6.1), Equation (6.1), and the instruction "Just add 2 to Romeo's age in years to get Juliet's age in years," all express the same idea. Each provides Juliet's age, given Romeo's age. In mathematics, we say each tells Juliet's age *as a function of* Romeo's age.

The idea of a function is one of the most important ideas in mathematics. There is more to the idea of a function than will be said in this book. For the time being, you should think of a function as a rule that tells how a number is computed from a given number. You will never go wrong by thinking that.

Example 6.1
Several functions.

(1) $y = 3x$. The function is "multiply by 3".
(2) $s = 5t + 2$. The function is "multiply by 5 then add 2".
(3) $s = 5(t + 2)$. The function is "add 2 then multiply by 5".
(4) $z = \frac{x-7}{8}$. The function is "subtract 7 then divide by 8".
(5) $y = x$. The function is "do nothing".

6.2. Terminology

The letters in a function are *variables*, because they take a variety of different values. In other words, their values vary. In the example about Romeo and Juliet, the variable "r" took a variety of values and the variable "j" took corresponding values. Some of those pairs of values appear in Table (6.1).

6.2.1. Variable, argument, value

In the Romeo and Juliet example, the variable j would be called the *dependent* variable and the variable r would be called the *independent* variable, the idea being that the value of j is determined by the value of r according to the rule of the function. Alternative phrases are the *value* of the the function (for the dependent variable) and *argument* of the function (for the independent variable). The word "argument" is widely used.

6.2.2. Domain

The set of numbers whose members can serve as the arguments of the function is called the *domain* of the function. Often, the domain of the function is stated. If it is not, then the domain is taken to be the largest set of numbers for which the function makes mathematical sense. If a number would cause a division by zero, that number is excluded from the domain.

If the function describes a physical phenomena, the domain might be restricted to only those numbers for which the phenomena occurs. In the Romeo-Juliet example, it would be silly for the domain to include the number 1000, since unless Romeo and Juliet have discovered the Fountain of Youth, they will not be around that long.

6.2.3. Range

The set of numbers that the function takes as values is called the *range* of the function.

Example 6.2
What is the domain of the function $d = 40t$?
Answer. All numbers.

Example 6.3
What is the domain of the function $d = 40t$ if d is the distance Jane traveled and the total time, t, of the trip was 3 hours?
Answer. All numbers from 0 to 3.

Example 6.4
What is the range of the function $d = 40t$ if d is the distance Jane traveled and the total time, t, of the trip was 3 hours?
Answer. All numbers from 0 to 120.

Example 6.5
A taxi company charges $0.72 per mile and rounds up the number of miles to the nearest whole number. Write the cab fare y dollars as a function of x miles and state the domain of the function.
Answer. $y = 0.72x$. Domain is $\{1, 2, 3, \cdots\}$.

Example 6.6
What is the range of the function in Example (6.5)?
Answer. : Range is $\{0.72, 1.44, 2.16, \cdots\}$.

Example 6.7
What is the value of the function $y = 3 + 2x$ when $x = 6$?
Answer. 15.

Example 6.8
For the function $y = 2 + 5x$ domain $\{-3, -2, -1, 0, 1, 2, 3\}$, make a table that shows the pairings of arguments and values.
Answer.

x	-3	-2	-1	0	1	2	3
y	-13	-8	-3	2	7	12	17

6.3. Idea of function as correspondence

It is true that a function can be thought of as a rule. It is also helpful to think of a function as a *correspondence* between the elements of one set, the domain, and the elements of another set, the range. Suppose that x represents an element in the domain of a function and y represents an element in the range. Then the notation

$$x \longrightarrow y$$

says that "y corresponds to x."

Example 6.9
Suppose that

$$\begin{aligned} x &\longrightarrow y \\ 0 &\longrightarrow 1 \\ 1 &\longrightarrow 3 \\ 2 &\longrightarrow 5 \\ 3 &\longrightarrow 7 \\ 4 &\longrightarrow 9 \end{aligned}$$

Write the function as a rule that pairs members of the domain, represented by x, with members of range, represented by y.
Answer. $y = 1 + 2x$. Domain $= \{0, 1, 2, 3, 4\}$ and Range $= \{1, 3, 5, 7, 9\}$

6.4. Well, which is it – a rule or a correspondence?

In section (6.1) the idea is that a function is a rule. But, in section (6.3) the idea is that a function is a correspondence. Which is it?

The idea of a function that mathematicians finally settled on is neither of these. Now, before you conclude that the author has made a mess of the whole topic, you should know that the ideas of function as a rule and function as a correspondence each had its day when it was considered *the* idea of a function. But more important is that, to this day, mathematicians use each of these ideas when they work with functions. The ultimate idea of a function is great for writing proofs, but it is not always as illuminating as the two ideas we have discussed so far.

"Well, which notion, rule or correspondence, should I use?" The answer is *both*. Depending on the context, one idea may be more illuminating or helpful than the other. Whichever you use, your thinking will in no sense be "wrong".

Exercise 6.1

1. Write the function that gives d miles as a function of t hours, if the constant speed is 60 $^{\text{mi}}/_{\text{hr}}$.

2. A pail that initially contains 2 gallons of water is filled at a rate of 3 $^{\text{gal}}/_{\text{min}}$.
   a) Write the volume V gallons as a function of time t minutes.
   b) Make a table that shows the volume at increments of 2 minutes from 0 minutes up to 12 minutes.
3. Gary's combine harvests soybeans at the rate of 7 $^{\text{acres}}/_{\text{hr}}$.
   a) Write the number of acres A harvested as a function of time t hours.
   b) Find the value of this function at $t = 7$.
4. Jill walks to school at a constant rate of 4 $^{\text{mi}}/_{\text{hr}}$. The distance from her house to school is 0.75 mi.
   a) Write the time remaining, t, in Jill's walk to school as a function of the distance, d, she has walked.
   b) State the domain of the function.
   c) State the range of the function.
5. Suppose that the correspondence between two variables, x and y is as shown below.

$$x \longrightarrow y$$
$$0 \longrightarrow 0$$
$$1 \longrightarrow 5$$
$$2 \longrightarrow 10$$
$$3 \longrightarrow 15$$
$$4 \longrightarrow 20$$

   a) Write the function as a rule that shows how the value of y is determined by the value of x.
   b) State the domain of the function.
   c) State the range of the function.

6.5. Graphs

One of the best ways to acquire an understanding of a particular function it to create a picture of it. That picture is referred to as the graph of the function. We will return to our discussion of functions in a little while. But for now, the topic is graphs.

6.5.1. The coordinate plane

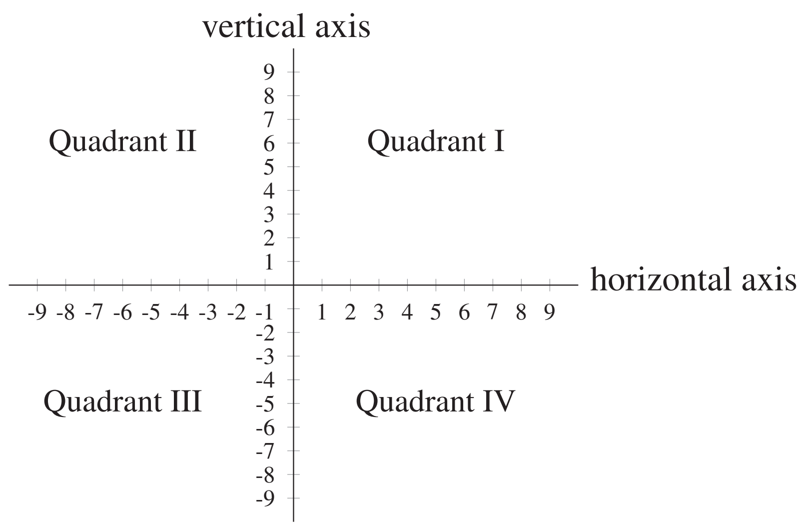


FIGURE 6.1. The coordinate plane

Figure (6.1) shows the coordinate plane. The point at which the axes intersect is called the *origin* and the axes intersect at a right angle. It is understood that each axis continues in both directions in spite of the fact that in this book we do not put arrows on the ends of the axes. The two axes divide the plane into four regions called quadrants. The quadrants are named with roman numerals as in the figure. Other names for the coordinate plane are the "Cartesian plane", the "Cartesian coordinate system", or the "rectangular coordinate system'.

The exact location of any of any point in the plane can be given by a pair of numbers that are always written in parenthesis. These numbers are called the *coordinates* of the point. The first number of the pair is the horizontal coordinate of the point, the second number the vertical coordinate.

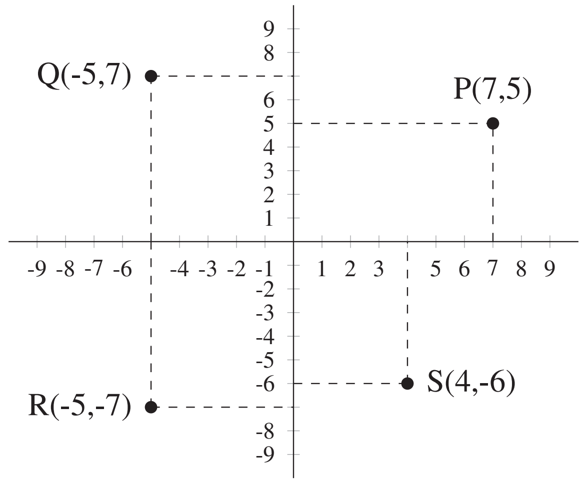


FIGURE 6.2. The coordinate plane

Figure (6.2) shows several points in the plane labeled with their coordinates. The dashed lines show how the coordinates of the points are obtained.

Exercise 6.2

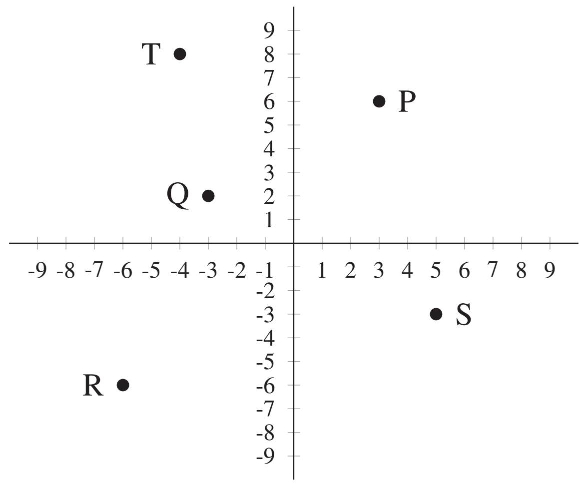


FIGURE 6.3.

1. Based on the appearance of Figure (6.3), state the coordinates of each point. (The first question is answered as an example.)
   a) T. Answer: $(-4,8)$.
   b) P.
   c) Q.
   d) R.
   e) S.
2. Plot each of the following points.
   a) $P(3,5)$.
   b) $Q(3,-5)$.
   c) $R(-3,5)$.
   d) $S(-3,-5)$.
3. In which quadrant does each point in Figure (6.3) lie?
   a) T.
   b) P.
   c) Q.
   d) R.
   e) S.

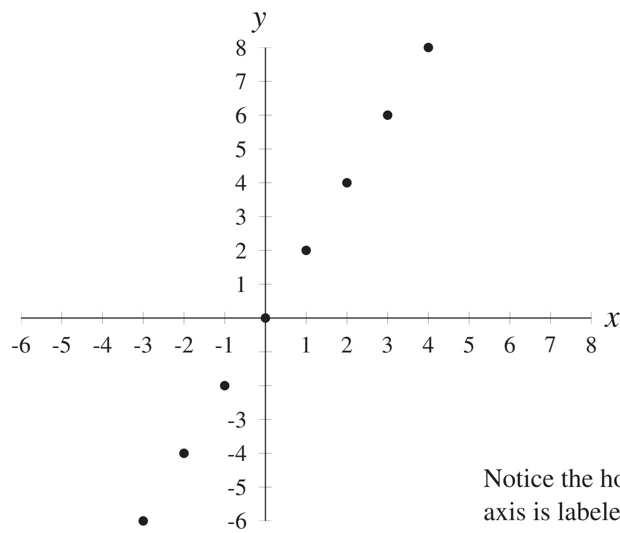
6.5.2. The graph of a function

Below, we represent the function

$$y = 2x, \quad \text{domain} = \{-3, -2, -1, 0, 1, 2, 3, 4\} \tag{6.2}$$

as a correspondence and a graph.

$$\begin{aligned} x &\longrightarrow y \\ -3 &\longrightarrow -6 \\ -2 &\longrightarrow -4 \\ -1 &\longrightarrow -2 \\ 0 &\longrightarrow 0 \\ 1 &\longrightarrow 2 \\ 2 &\longrightarrow 4 \\ 3 &\longrightarrow 6 \\ 4 &\longrightarrow 8 \end{aligned}$$



Notice the horizontal axis is labeled "x" and the vertical axis "y". When so labeled, it is common to use the phrases "x direction" and "y direction" in place of "horizontal" and "vertical" respectively.

Who can help but think, "The points make a straight line!"?

So it seems. But, looks can deceive. Maybe there is some number between $x = 0$ and $x = 1$ that produces a point that is not in line with the other points.

Question: Do the points produced by the function $y = 2x$ all lie on the same straight line?

We might try to get a finer view of how the function behaves at values of "x" between, for example, 0 and 1. We consider the same rule, but with a different domain.

$$y = 2x, \text{ domain} = \{-0.5, -0.375, -0.25, \cdots, 1\}. \tag{6.3}$$

Figure (6.4) shows the graph. Note that the horizontal (x-axis) is marked every 0.125, the vertical (y-axis) every 0.25.

The evidence that graph of the function $y = 2x$ is a straight line of points is pretty compelling. But we can do better. We can be certain. But to achieve such certainty, we need a few more ideas.

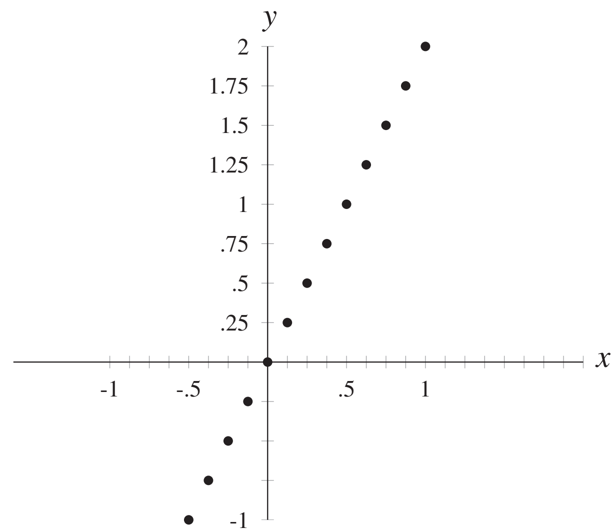


FIGURE 6.4. $y = 2x$, domain $= \{-0.5, -0.375, \cdots, 1\}$

6.5.3. Rise and run

If you study Figure (6.5), you will learn what is meant when we say “rise” and “run”.

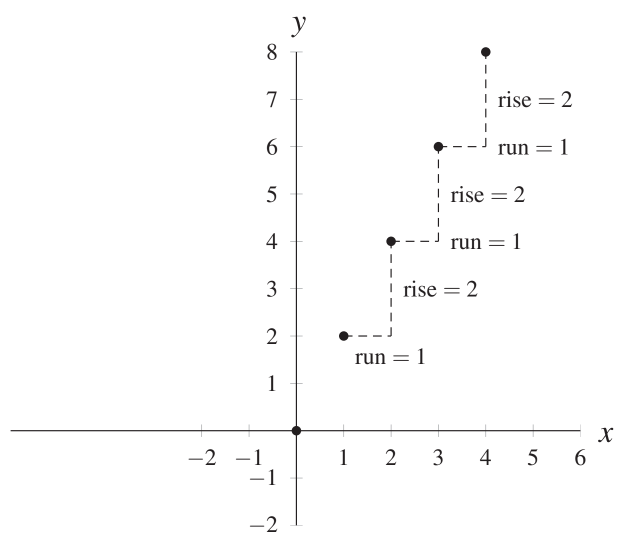


FIGURE 6.5. Graph of $y = 2x$, domain $= \{0, 1, 2, 3, 4\}$

If you have the idea that the run is “how far to the right” and the rise is “how far up”, then you are on the correct track. There is an additional detail that you will discover when you study Figure (6.6).

The additional idea is that rise can be “how far down” in which case it is a negative number. The ratio of rise to run, $\frac{\text{rise}}{\text{run}}$, is a number that plays an

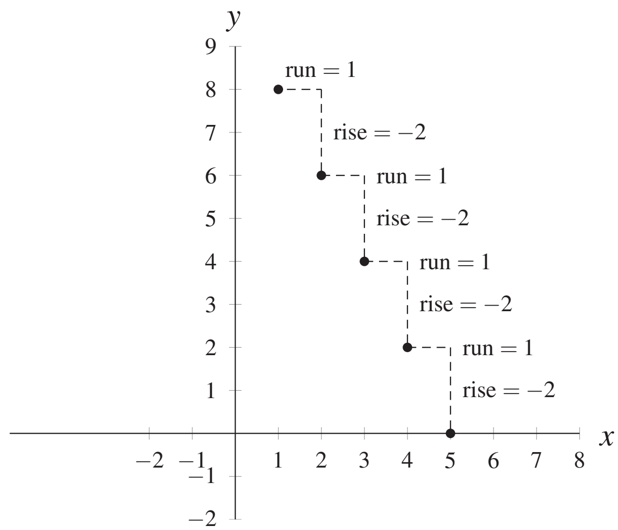


FIGURE 6.6. Graph of $y = 2x$, domain $= \{1, 2, 3, 4, 5\}$

important part in the study of functions. If you examine Figure (6.5), you will see that the $\frac{\text{rise}}{\text{run}} = \frac{2}{1} = 2$. In Figure (6.6), $\frac{\text{rise}}{\text{run}} = \frac{-2}{1} = -2$.

Exercise 6.3

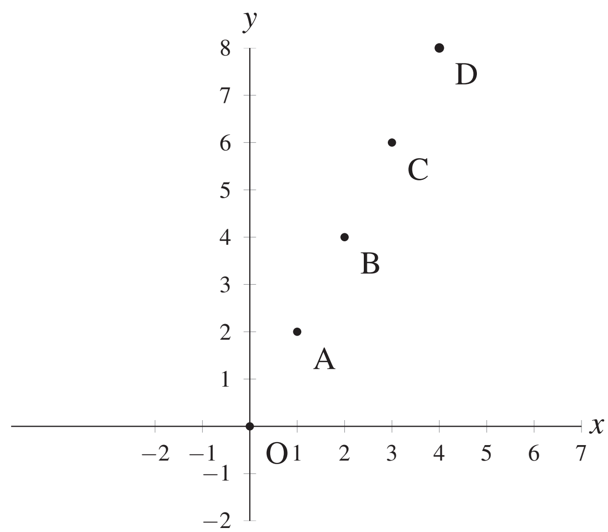


FIGURE 6.7. Graph of $y = 2x$, domain $= \{0, 1, 2, 3, 4\}$

1. Based on the appearance of Figure (6.7), complete the following table of the ratio $\frac{\text{rise}}{\text{run}}$ for each pair of points. (The first row is completed as an example.)

	0	**A**	**B**	**C**	**D**
0	undefined	$\frac{2}{1} = 2$	$\frac{4}{2} = 2$	$\frac{6}{3} = 2$	$\frac{8}{4} = 2$
A		undefined			
B			undefined		
C				undefined	
D					undefined

2. What is notable about the $\frac{\text{rise}}{\text{run}}$ that you filled in?
3. Why does the word "undefined" appear in certain cells of the table?

6.5.4. The question reconsidered

On page 159 we wondered if the points produced by the function $y = 2x$ all lie on the same straight line.

If you worked Exercise (6.3), you have likely noticed that for every pair of points on the graph of $y = 2x$, the ratio $\frac{rise}{run}$ is equal to 2. The ratio $\frac{rise}{run}$ gives the direction from one point to the next point. Figure (6.8) illustrates this idea.

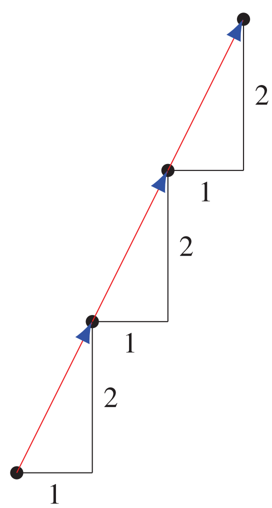


FIGURE 6.8. $\frac{rise}{run}$ shows direction.

Think of the map to buried treasure so popular among the pirates of old.

In Figure (6.8), the path from one point to the next point is always the same, "1 right, 2 up". We can say more, that the path from *any point* to *any other point* is in the *ratio* $\frac{\text{2 up}}{\text{1 right}}$. The consequence is that the four points lie on the same line. This idea is general and we state it in Definition (6.1).

Definition 6.1 (Collinear points)
For a given collection of points, if the ratio $\frac{rise}{run}$ is the same for every pair of points in the collection, then all the points of the collection are collinear (lie on the same line). ■

We have considered $y = 2x$ for only a few domains. How can we be certain that $y = 2x$ will produce collinear points on every possible domain? Is there some quality of the rule $y = 2x$ that guarantees its graph will *always* be a collection of collinear points? The answer to this is "Yes". Here is why.

$$y = 2x \iff \frac{y}{x} = 2 = \frac{2}{1}, \quad x \neq 0.$$

The ratio $\frac{\text{rise}}{\text{run}}$ is built into the rule $y = 2x$!

On page 159 we asked, "Do the points produced by the function $y = 2x$ all lie on the same straight line?" The answer is "Yes". And not merely for several domains, but for every possible domain.

6.5.5. The function *y* = *ax* unbounded

The author admits that the real numbers have not yet been introduced. No harm is done in speaking of the "continuous" line, since it is continuous, even though the student cannot know why until several years from now.

We have just discovered in Section (6.5.4) that the graph of $y = 2x$ is a set of collinear points for every possible domain. If we allow the domain to be all the numbers, then the points of the graph fill in to produce a solid continuous straight line. The graph is in Figure (6.9). It is understood that the graph, like the axes, continues in both directions.

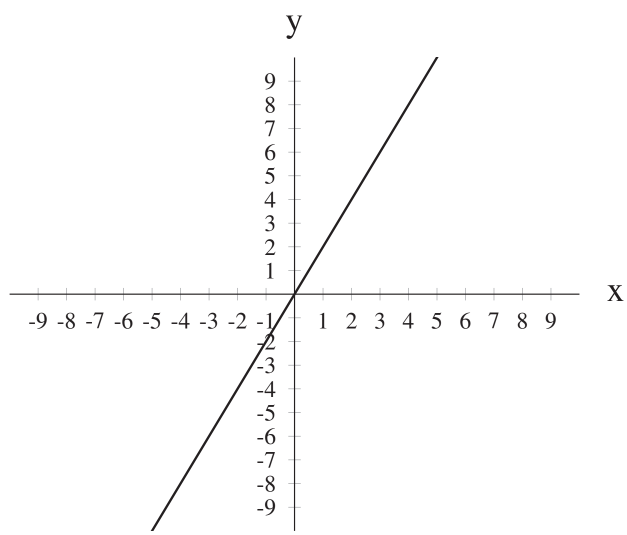


FIGURE 6.9. Graph of $y = 2x$, domain all numbers.

Remark 6.1

There is a difference between plotting points and graphing. We might plot points in science class to make a chart of, for example, temperature and number of cricket chips per hour. The chart would consist of carefully plotted points on carefully laid out axes. When we graph a function in mathematics, we typically avoid plotting anymore than a few strategic points. The graph is drawn based on our knowledge of the function. Our goal is to display the mathematically interesting features of the function. The axes are typically not carefully marked into units.

The process by which we arrived at the graph of $y = 2x$ in Figure (6.9) is a good example of how we graph in mathematics. The graph includes the key point $(0,0)$ and the ${}^{\text{rise}}/_{\text{run}}$ ratio of ${}^{2}/_{1}$. These are inherent qualities of the function that our careful analysis of the function uncovered.

The student may be thinking "Great, now how am I supposed to know what you call the 'mathematically interesting features of the function'?" The answer is, do not worry, that is what you will be learning over a period of years. No one expects you to be an expert now or anytime soon. So, relax and enjoy!

6.6. The function $y = ax$

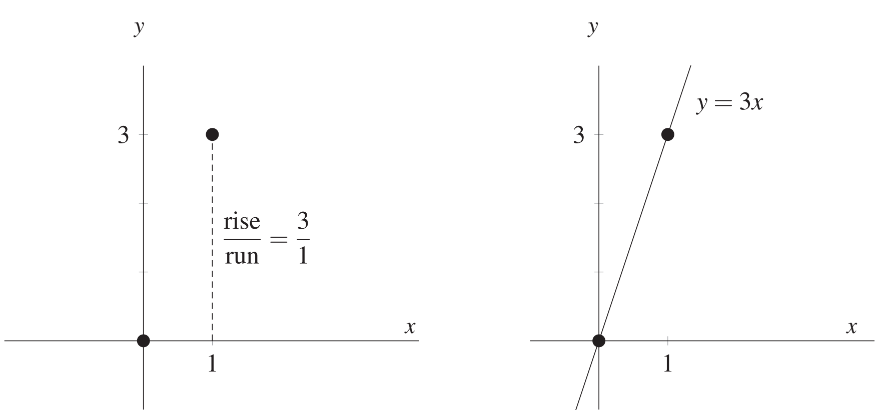
The function $y = 2x$ is but one example of $y = ax$ where a is a constant. This kind of function is called a "linear function". By now we know quite well that the number a is the ${}^{\text{rise}}/_{\text{run}}$ ratio. The name to use for the number in the role of a is "slope". In our example function $y = 2x$, the slope was ${}^{2}/_{1}$ or more simply 2. Slope is worth a definition.

Definition 6.2 (Slope)
The ratio $\dfrac{\text{rise}}{\text{run}}$ is called the slope. ■

6.6.1. Graphing y = ax

Example 6.10
Graph the function $y = 3x$, domain all numbers.

Solution.
Figure (6.10) shows the steps. (a) The point $(0,0)$ is on the graph, because



(a) Plot $(0,0)$. Then use slope. (b) Draw the line.

FIGURE 6.10. Graphing $y = 3x$

$0 = 3 \cdot 0$. The slope is $3 = \frac{3}{1}$ and we use it to locate another point on the line thinking "1 right, 3 up,". (b) Then draw the line through points $(0,0)$ and $(1,3)$.

Remark 6.2
Several notes about our first graph.

(1) Since the domain was not stated, we assumed the largest domain for for which $y = 3x$ is meaningful.
(2) We determined the slope of $y = 3x$ *by inspection*. That is, we eye-balled it. The slope will always be the coefficient of the independent variable.
(3) On the axes, we showed only the numbers that are necessary to identify the point $(3,1)$. The origin, $(0,0)$, is considered obvious.
(4) We include "$y = 3x$" on the graph.
(5) Figure (6.10)(a), shows what we think as we make Figure (6.10)(b). Part (a) would not be shown.

The several examples that follow should answer some of your questions about graphing.

Example 6.11
Graph $y = x$.

Solution The graph goes through the origin. The slope is 1, think "$y = 1 \cdot x$". So another point on the line is $(1,1)$.

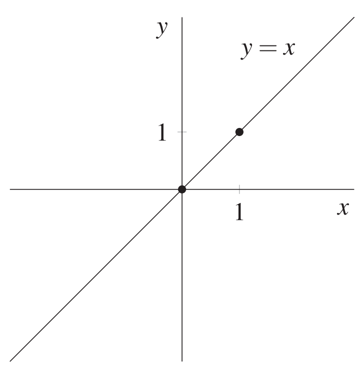


FIGURE 6.11. Example (6.11)

Example 6.12

Graph $y = \frac{2}{3}x$.

Solution The graph goes through the origin. The slope is $\frac{2}{3}$, think "three right, two up". So another point on the line is $(3,2)$.

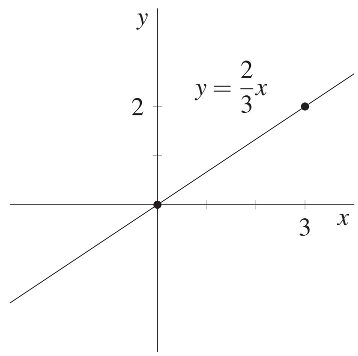


FIGURE 6.12. Example (6.12)

Example 6.13

Graph $y = \frac{-2}{3}x$.

Solution The graph goes through the origin. The slope is $\frac{-2}{3}$, think "three right, two down". So another point on the line is $(3,-2)$.

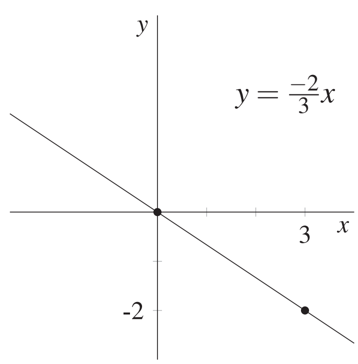


FIGURE 6.13. Example (6.13)

Example 6.14

Graph $y = \dfrac{-1}{4}x$.

Solution The graph goes through the origin. The slope is $\frac{-1}{4}$, think "four right, one down". So another point on the line is $(-1,4)$.

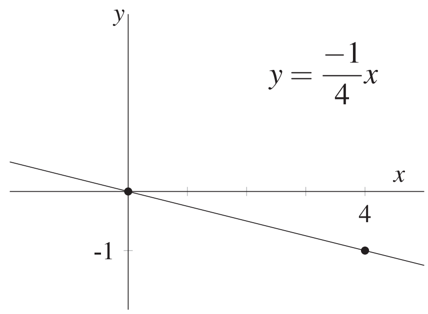


FIGURE 6.14. Example (6.14)

Example 6.15

Graph $y = 3x$.

Solution The graph goes through the origin. The slope is 3, so another point on the line is $(1,3)$.

In Figure (6.15), we write the coordinates of the point $(1,3)$ beside the point, since there are several numbers shown on the axes.

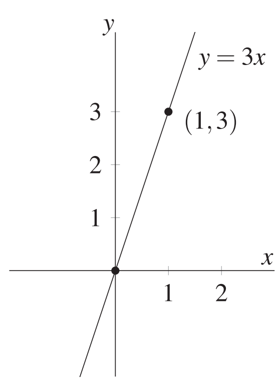


FIGURE 6.15. Example (6.15)

■

Exercise 6.4

1. Graph the function $y = 4x$.
2. Graph the function $y = \frac{1}{4}x$.
3. Graph the function $y = -2x$.
4. Graph the function $y = -3x$.
5. Graph the function $y = \dfrac{-2}{3}x$.
6. Graph the function $y = 8x$.

6.6.2. Graphs to equations

Sometimes we wish to write the function, given its graph. For example, consider the graph of Figure (6.16).

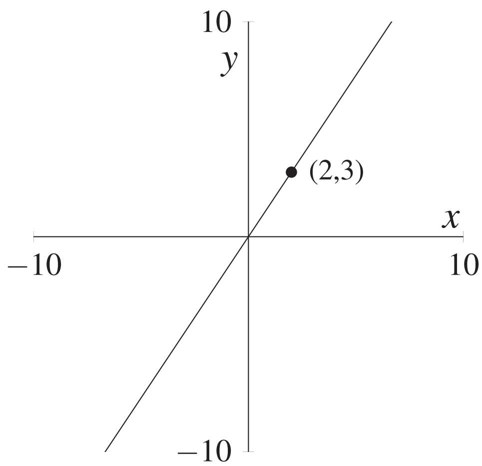


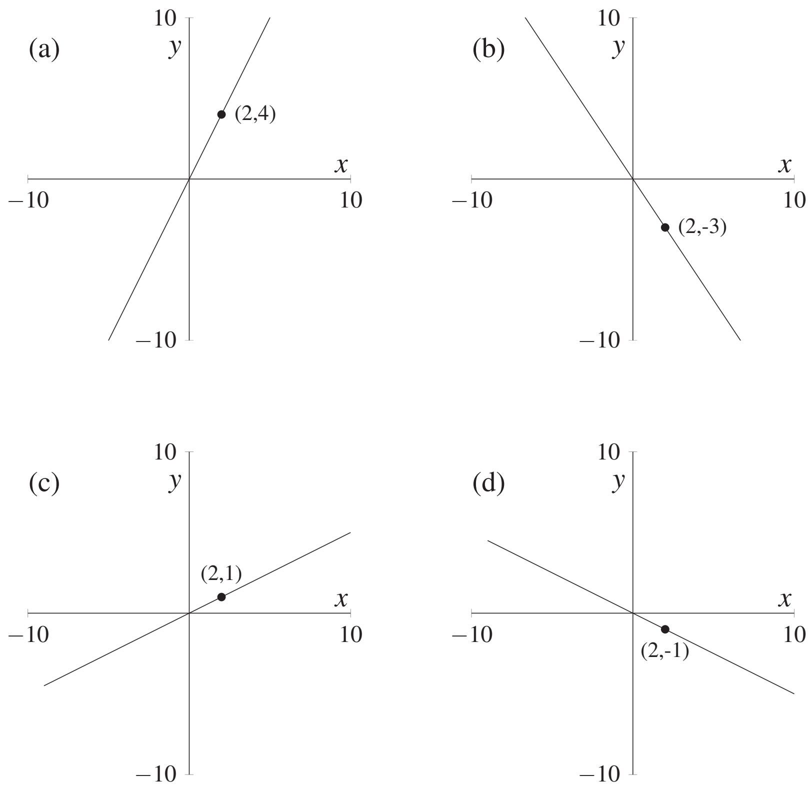
FIGURE 6.16

The graph includes the origin, $(0,0)$, so the function must have the form $y = ax$ where a is the slope of the line. The slope equals $\frac{3}{2}$, so the function must be $y = \frac{3}{2}x$.

Exercise 6.5

For each of the following graphs, write the function.

1.



6.7. Linear functions and rate of change

In a linear function, the coefficient of the independent variable is constant. This quality defines the character of the linear function. In $y = ax$, a measures the rate at which y is changing as a function of x. In each of the following examples, the rate of change in one quantity with respect to another is constant.

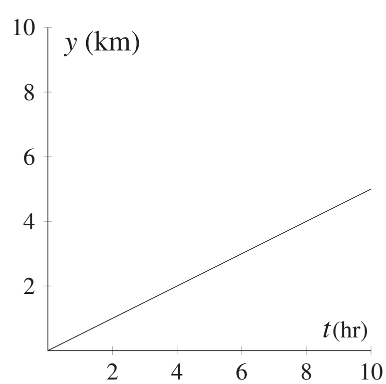
Example 6.16
Sue drives at a uniform speed of 40 miles per hour. The distance she travels increases at the constant rate of 40 $^{\text{mi}}/_{\text{hr}}$. If $d =$ distance traveled (miles) and t = time (minutes), then $d = 40t$.

Example 6.17
A pool is filled from a hose at the rate of 5 $^{\text{gal}}/_{\text{min}}$. The rate of change in the volume of water in the pool is constant. If $V =$ water in pool (gallons) and t = time (minutes), then $V = 5t$.

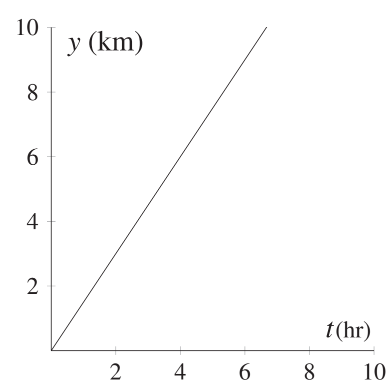
Example 6.18
The area of a circle is a constant, π, times the diameter. The area (A) of a circle diameter (D) is a linear function of D, $A = \pi D$.

Example 6.19
Figure (6.17) shows the distance, y, Tom traveled as a function of time and the distance, y, Sue traveled as a function of time, t. Which person's speed was greater?

The slope of the line is greater for Sue than for Tom. So, the rate of change



(a) Tom's journey.



(b) Sue's journey.

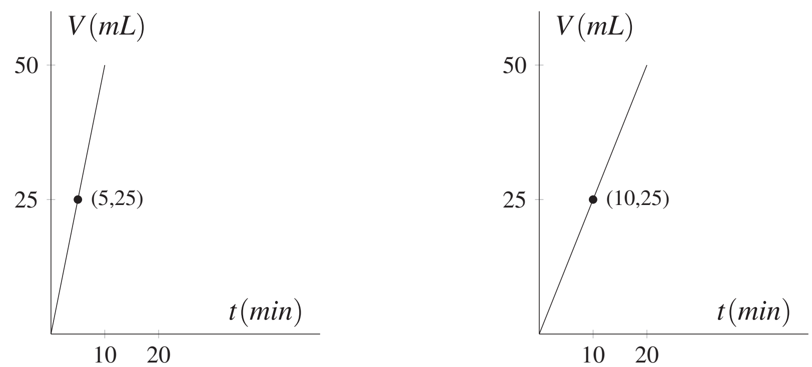
FIGURE 6.17. Graphing $y = 3x$

in distance with respect to time is greater for Sue than for Tom. Sue traveled at the greater speed.

Example 6.20
Beaker A is being filled twice as fast as Beaker B. Both 50 mL beakers began empty and are filled until full. After 5 minutes, Beaker A contains 25 milliliters of water. Write the volume, V, of each breaker as a function of time, t, and graph each function.

Solution.

$$\text{For A: } V = 5t.$$
$$\text{For B: } V = \frac{5}{2}t.$$



(a) Beaker A: $V = 5t$. (b) Beaker B: $V = \frac{5}{2}t$.

FIGURE 6.18. Filling beakers

6.8. Summary for the function y = ax

A function of the form $y = ax$, where a is constant but not 0, is a linear function. The quantity y increases or decreases in direct proportion to x. The constant a is sometimes called the "constant of proportionality". For a linear function, a is always equal to $\frac{y}{x}$. The graph of the linear function $y = ax$ is always a straight line through the point $(0,0)$. The constant of proportionality a is the slope of that line.

6.9. Summary for function

A rule that shows how to compute values of y given values of x from a set of numbers (the "domain of the function") is said to show y as a function of x. Also valuable, is the idea that a function shows a correspondence between the members of one set of numbers (the domain of the function) and another set of numbers (the "range of the function"). If y is a function of x, x is called the independent variable (or argument) and y is called the dependent variable (or value).

Although everything you have learned here about functions is true, there is more to be learned. You have been given "nothing but the truth", but you have not been given "the whole truth".

Answers to Exercise 5.3

(**1**) $7h$. (**2**) 2 minute. (**3**) $5h$. (**4**) 6 hours. (**5**) 1.5 hours. (**6**) 4.8 minutes.

Answers to Exercise 5.4

(**1**) $120\frac{m}{h}$. (**2**) $72\frac{km}{h}$. (**3**) $1320\frac{m}{min}$. (**4**) $818\frac{m}{h}$. (**5**) $375\frac{m}{h}$.

Answers to Exercise 5.5

(**1**) 135 miles. (**2**) $\frac{5}{6}h$ or 50 minutes. (**3**) 6.5 miles. (**4**) 2.5 hours. (**5**) 15 km/h. (**6**) 0.6 hour. (**7**) 70 mph. (**8**) 40 mph. (**9**) 40 mph. (**10**) 1.75 hours. (**11**) 20. (**12**) 12:06 PM. (**13**) 1:30 PM. (**14**) 1320 miles.

Answers to Exercise 6.1

(**1**) $d = 60t$. (**2**) (**a**) $V = 2 + 3t$.

(**b**)

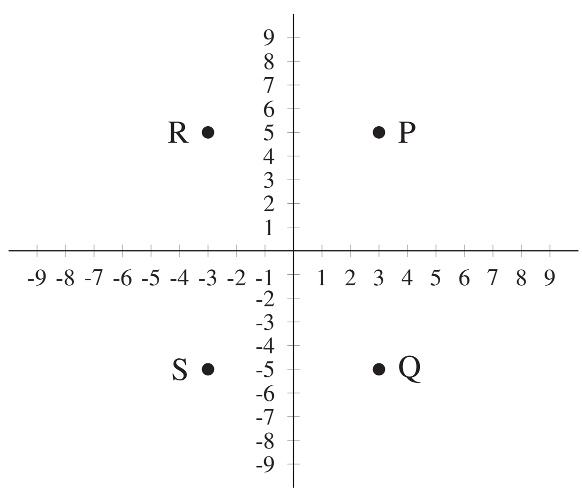
t (min)	0	2	4	6	8	10	12
V (gal)	2	8	14	20	26	32	38

(**3**) (**a**) $A = 7t$. (**b**) 49. (**4**) (**a**) $t = \frac{0.75-d}{4}$. (**b**) All numbers between 0 and 0.75. (**c**) All numbers between 0 and 0.1875. (**5**) (**a**) $y = 5x$. (**b**) $\{0,1,2,3,4\}$. (**c**) $\{0,5,10,15,20\}$.

Answers to Exercise 6.2

(**1**) (**a**) Done as example. (**b**) $(3,6)$. (**c**) $(-3,2)$. (**d**) $(-6,-6)$. (**e**) $(5,-3)$.

(**2**)



(**3**) (**a**) II. (**b**) I. (**c**) II. (**d**) III. (**e**) IV.

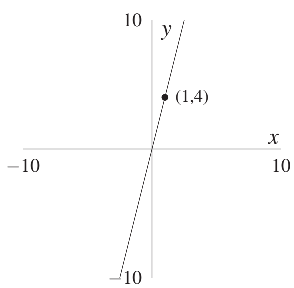
Answers to Exercise 6.3

(**1**)

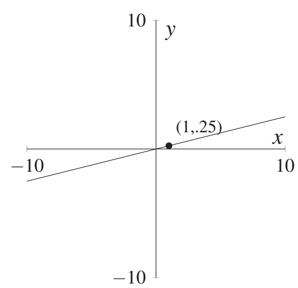
	0	**A**	**B**	**C**	**D**
0	undefined	$\frac{2}{1}=2$	$\frac{4}{2}=2$	$\frac{6}{3}=2$	$\frac{8}{4}=2$
A		undefined	$\frac{2}{1}=2$	$\frac{4}{2}=2$	$\frac{6}{3}=2$
B			undefined	$\frac{2}{1}=2$	$\frac{4}{2}=2$
C				undefined	$\frac{2}{1}=2$
D					undefined

(**2**) The ratio $\frac{\text{rise}}{\text{run}}$ equals 2 whenever it is defined. (**??**) Done as example.
(**3**) The run is 0, so the ratio $\frac{\text{rise}}{\text{run}}$ is undefined because division by zero s undefined.

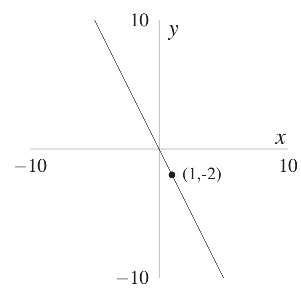
Answers to Exercise 6.4



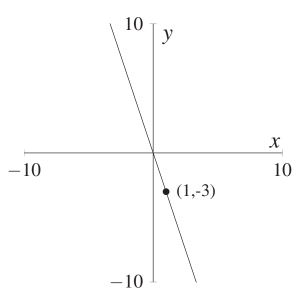
(**1**)



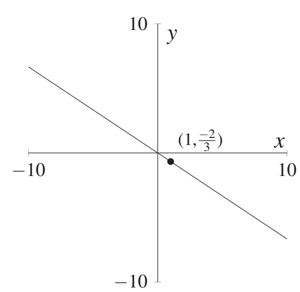
(**2**)



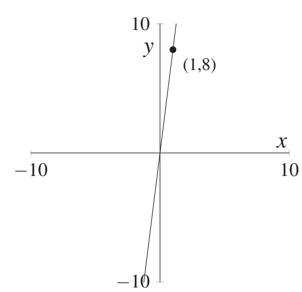
(**3**)



(**4**)



(**5**)



(**6**)

Answers to Exercise 6.5

(**1**) (a) $y = 2x$. (b) $y = \dfrac{-3}{2}x$. (c) $y = \dfrac{1}{2}x$. (d) $y = \dfrac{-1}{2}x$.