EXERCISES 2.5

1–4 ■ Find dy/dx and $dy/dx]_{x=1}$ in two ways: (a) using the Chain Rule and (b) without using the Chain Rule, as in Example 2.

1. $y = u^2, \quad u = x^2 + 2x + 3$
2. $y = u^2 - 2u + 3, \quad u = 5 - 6x$
3. $y = u^3, \quad u = x + (1/x)$
4. $y = u - u^2, \quad u = \sqrt{x} + \sqrt[3]{x}$

5–48 ■ Find the derivative of the function.

5. $F(x) = (x^2 + 4x + 6)^5$
6. $F(x) = (x^3 - 5x)^4$
7. $G(x) = (3x - 2)^{10}(5x^2 - x + 1)^{12}$
8. $g(t) = (6t^2 + 5)^3(t^3 - 7)^4$
9. $f(t) = (2t^2 - 6t + 1)^{-8}$
10. $f(t) = \dfrac{1}{(t^2 - 2t - 5)^4}$
11. $g(x) = \sqrt{x^2 - 7x}$
12. $k(x) = \sqrt[3]{1 + \sqrt{x}}$
13. $h(t) = \left(t - \dfrac{1}{t}\right)^{3/2}$
14. $F(s) = \sqrt{s^3 + 1}\,(s^2 + 1)^4$
15. $F(y) = \left(\dfrac{y - 6}{y + 7}\right)^3$
16. $s(t) = \sqrt[4]{\dfrac{t^3 + 1}{t^3 - 1}}$
17. $f(z) = \dfrac{1}{\sqrt[5]{2z - 1}}$
18. $f(x) = \dfrac{x}{\sqrt{7 - 3x}}$
19. $y = (2x - 5)^4(8x^2 - 5)^{-3}$
20. $y = (x^2 + 1)\sqrt[3]{x^2 + 2}$
21. $y = \tan 3x$
22. $y = 4 \sec 5x$
23. $y = \cos(x^3)$
24. $y = \cos^3 x$
25. $y = (1 + \cos^2 x)^6$
26. $y = \tan(x^2) + \tan^2 x$
27. $y = \cos(\tan x)$
28. $y = \sin(\sin x)$
29. $y = \sec^2 2x - \tan^2 2x$
30. $y = \sqrt{1 + 2\tan x}$
31. $y = \csc(x/3)$
32. $y = \cot \sqrt[3]{1 + x^2}$
33. $y = \sin^3 x + \cos^3 x$
34. $y = \sin^2(\cos 4x)$
35. $y = \sin\dfrac{1}{x}$
36. $y = \dfrac{\sin^2 x}{\cos x}$
37. $y = \dfrac{1 + \sin 2x}{1 - \sin 2x}$
38. $y = x \sin\dfrac{1}{x}$
39. $y = \tan^2(x^3)$
40. $y = (\sin \sqrt{x^2 + 1}\,)^{\sqrt{2}}$
41. $y = \cos^2(\cos x) + \sin^2(\cos x)$
42. $y = \sin(\sin(\sin x))$
43. $y = \sqrt{x + \sqrt{x}}$
44. $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$
45. $f(x) = [x^3 + (2x - 1)^3]^3$
46. $g(t) = \sqrt[4]{(1 - 3t)^4 + t^4}$
47. $y = \sin(\tan \sqrt{\sin x}\,)$
48. $y = \sqrt{\cos(\sin^2 x)}$

49–52 ■ Find the equation of the tangent line to the curve at the given point.

49. $y = (x^3 - x^2 + x - 1)^{10}, \quad (1, 0)$
50. $y = \sqrt{x + (1/x)}, \quad (1, \sqrt{2}\,)$
51. $y = \dfrac{8}{\sqrt{4 + 3x}}, \quad (4, 2)$
52. $y = \sin x + \cos 2x, \quad (\pi/6, 1)$
53. (a) Find an equation of the tangent line to the curve $y = \tan(\pi x^2/4)$ at the point $(1, 1)$.
    (b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.
54. (a) The curve $y = |x|/\sqrt{2 - x^2}$ is called a **bullet-nose curve.** Find an equation of the tangent line to this curve at the point $(1, 1)$.
    (b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.
55. (a) If $f(x) = \sqrt{1 - x^2}/x$, find $f'(x)$.
    (b) Check to see that your answer to part (a) is reasonable by comparing the graphs of f and f'.
56. (a) If $f(x) = 1/(\cos^2 \pi x + 9 \sin^2 \pi x)$, find $f'(x)$.
    (b) Check to see that your answer to part (a) is reasonable by comparing the graphs of f and f'.
57. Find all points on the graph of the function $f(x) = 2 \sin x + \sin^2 x$ at which the tangent line is horizontal.

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1. $4u(x + 1)$, 48 **3.** $3u^2(1 - 1/x^2)$, 0

5. $F'(x) = 10(x^2 + 4x + 6)^4(x + 2)$

7. $G'(x) = 6(3x - 2)^9(5x^2 - x + 1)^{11}(85x^2 - 51x + 9)$

9. $f'(t) = -16(2t^2 - 6t + 1)^{-9}(2t - 3)$

11. $g'(x) = (2x - 7)/(2\sqrt{x^2 - 7x})$

13. $h'(t) = \frac{3}{2}(t - 1/t)^{1/2}(1 + 1/t^2)$

15. $F'(y) = 39(y - 6)^2/(y + 7)^4$

17. $f'(z) = -\frac{2}{5}(2z - 1)^{-6/5}$

19. $y' = 8(2x - 5)^3(8x^2 - 5)^{-4}(-4x^2 + 30x - 5)$

21. $y' = 3\sec^2 3x$ **23.** $y' = -3x^2 \sin(x^3)$

25. $y' = -12 \cos x \sin x\,(1 + \cos^2 x)^5$

27. $y' = -\sin(\tan x)\sec^2 x$ **29.** $y' = 0$

31. $y' = -(1/3)\csc(x/3)\cot(x/3)$

33. $y' = 3 \sin x \cos x\,(\sin x - \cos x)$

35. $y' = -\cos(1/x)/x^2$ **37.** $y' = 4(\cos 2x)/(1 - \sin 2x)^2$

39. $y' = 6x^2 \tan(x^3)\sec^2(x^3)$ **41.** $y' = 0$

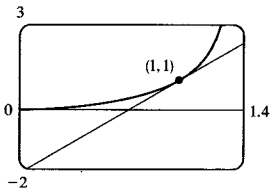
43. $y' = [1 + 1/(2\sqrt{x})]/(2\sqrt{x + \sqrt{x}})$

45. $f'(x) = 9[x^3 + (2x - 1)^3]^2(9x^2 - 8x + 2)$

47. $y' = \cos(\tan\sqrt{\sin x})\,(\sec^2\sqrt{\sin x})\,[1/(2\sqrt{\sin x})]\,(\cos x)$

49. $y = 0$ **51.** $3x + 16y = 44$

53. (a) $y = \pi x - \pi + 1$ (b)



55. (a) $-1/(x^2\sqrt{1 - x^2})$

57. $((\pi/2) + 2n\pi, 3)$, $((3\pi/2) + 2n\pi, -1)$, n any integer

59. 28 **61.** $v(t) = (5\pi/2)\cos(10\pi t)$ cm/s

63. (a) $dB/dt = (7\pi/54)\cos(2\pi t/5.4)$ (b) 0.16

65. (a) On $(0, \infty)$ (b) $G'(x) = h'(\sqrt{x})/(2\sqrt{x})$

67. (a) $F'(x) = -\sin x\, f'(\cos x)$ (b) $G'(x) = -\sin(f(x))f'(x)$

69. 0 **75.** $f'(x) = x/|x|$

77. $h'(x) = |2x - 1| + 2x(2x - 1)/|2x - 1|$