[12-04-12-T8]

Procedure for completing the square

Completing the square is a technique that always proceeds in the same way. Below are the several steps.

EXAMPLE 1

Rewrite $f(x) = x^2 + 3x - 2$ in the form $f(x) = a(x-h)^2 + k$.

Step [1] $f(x) = (x^2 + 3x) - 2$

Step [2] $f(x) = \left(x^2 + 3x + \mathbf{\frac{9}{4}}\right) - 2 - \frac{9}{4}$

COMMENT: $x^2 + 3x + \frac{9}{4}$ is a perfect square trinomial. Therefore, if factors into $(x + \frac{3}{2})^2$. The term added within the parenthesis, in this example $\frac{9}{4}$ is said to "complete the square". The number that completes the square will *always* be (one-half the coefficient of the variable)^2. Always!

COMMENT: By adding $\frac{9}{4}$ the value of the function is increased by $\frac{9}{4}$. This we do not wish. By subtracting $\frac{9}{4}$, we cancel the unwanted increase in the value of the function.

Step [3] $f(x) = (x + \frac{3}{2})^2 - \frac{17}{4}$. We have rewritten $(x^2 + 3x + \frac{9}{4})$ as $(x + \frac{3}{2})^2$. The goal of completing the square is now accomplished. The number in the second position in $(x + \frac{3}{2})$, which in this example is $\frac{3}{2}$, will *always* be one-half the coefficient of the variable. Always!

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EXAMPLE 2

Rewrite $f(x) = x^2 - 5x + 1$ in the form $f(x) = a(x-h)^2 + k$.

Step [1] $f(x) = (x^2 - 5x) + 1$

Step [2] $f(x) = \left(x^2 - 5x + \mathbf{\frac{25}{4}}\right) - 1 - \frac{25}{4}$. Because, (one-half the coefficient of the variable)$^2 = \frac{25}{4}$.

Step [3] $f(x) = (x - \frac{5}{2})^2 - \frac{29}{4}$. We have rewritten $x^2 - 5x + 1$ as$(x - \frac{5}{2})^2 - \frac{29}{4}$.

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The next example makes an important point

EXAMPLE 3

Rewrite $f(x) = 3x^2 - 5x + 1$ in the form $f(x) = a(x - h)^2 + k$.

Step [1] $f(x) = 3(x^2 - \frac{5}{3}x) + 1$

COMMENT: Always begin by "factoring out" the coefficient of the squared variable. Check this step by mentally performing the multiplication $3(x^2 - \frac{5}{3}x) = 3x^2 - 5x$. Check!

Step [2] $f(x) = 3\left(x^2 - 5x + \frac{25}{4}\right) - 1 - \frac{75}{4}$. Here is where people make a mistake. Remember that the $\frac{25}{4}$ inside the parenthesis is multiplied by 3. So, you have actually added $\frac{75}{4}$ to the value of the function. Therefore, subtract $\frac{75}{4}$ to maintain the value of the function.

Step [3] $f(x) = 3(x - \frac{5}{2})^2 - \frac{79}{4}$. We have rewritten $x^2 - 5x + 1$ as $3(x - \frac{5}{2})^2 - \frac{79}{4}$.

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Exercises

Write each of the following in the form $f(x) = a\,(x - h)^2 + k$.

[1] $f(x) = x^2 + 8\,x + 17$

[2] $f(x) = x^2 - 12\,x + 34$

[3] $f(x) = x^2 - 7\,x + \frac{37}{4}$

[4] $f(x) = 3\,x^2 - 9\,x + \frac{7}{4}$

[5] $f(x) = 5\,x^2 + 30\,x + 55$

[6] $f(x) = \frac{x^2}{2} - 3\,x + \frac{5}{2}$

Answers

Write each of the following in the form $f(x) = a\,(x-h)^2 + k$.

[1] $f(x) = (x+4)^2 + 1$

[2] $f(x) = (x-6)^2 - 2$

[3] $f(x) = (x - \frac{7}{2})^2 - 3$

[4] $f(x) = 3\,(x - \frac{3}{2})^2 - 5$

[5] $f(x) = 5\,(x+3)^2 + 10$

[6] $f(x) = \frac{1}{2}\,(x-3)^2 - 2$