Miscellaneous Exercises [B]

Factor:

1. $m^2 + m - n^2 - n$
2. $2\,pq + 2\,pr + r^2 - q^2$
3. $n(n+1)(n+2) - 3\,n(n+1)$
4. $x^3 + y^3 - xy(x+y)$
5. $(a-b)^3 + 4\,ab(a-b)$
6. $ax + 2\,ay - 3\,x^2 + 12\,y^2$
7. $2\,x + 3\,y + 5(2\,x + 3\,y)^2$
8. $x^3 - 3\,x^2 - 6\,x + 8$
9. $x^3 - 3\,x^2 + 3\,x - 1$
10. $a^2 - 2\,ab + b^2 - 3\,a + 3\,b$

11. Show that $x(x+y)^3 - x^2(x+y)^2 = xy(x+y)^2$, for all x and y.
12. If $a = m^2 - n^2$, $b = 2\,mn$, $c = m^2 + n^2$, show that, for all values of m and n, $c^2 - b^2 = a^2$.
13. Show that the factored form of $(a^2 - 3\,a + 1)^2 - 1$ is
$$a(a-1)(a-2)(a-3).$$
14. Factor: $4\,b^2c^2 - (b^2 + c^2 - a^2)^2$.
15. Factor: $n(n-1)(n+1) - 2(n+1)$.
16. Show that $(a+b)^2 + (b+c)^2 + (c+a)^2 = (a+b+c)^2 + a^2 + b^2 + c^2$. Use the result to express $4^2 + 9^2 + 7^2$ as the sum of the squares of four integers.
17. Verify that $12^2 - 2(11)^2 + 10^2 = 2$, and that $21^2 - 2(20)^2 + 19^2 = 2$. If n is the smallest of three consecutive integers, express in terms of n the relationship of which the above are specific instances. Show that the relationship is true for all values of n.
18. If a and b represent any unequal numbers in our present field of operation (positive and negative integers and fractions), is it possible for $(a-b)^2$ to have zero or negative value?
What may be deduced about the relative values of $a^2 + b^2$ and $2\,ab$? Use the result to show that if a, b, c are unequal then the value of $a^2 + b^2 + c^2$ exceeds the value of $ab + bc + ac$.
19. (a) Show that if $x^6 - 1$ is factored as the difference of two cubes, the factored form is $(x+1)(x-1)(x^4 + x^2 + 1)$.
(b) Show that if $x^6 - 1$ is factored as the difference of two squares, the factored form is $(x+1)(x-1)(x^2 + x + 1)(x^2 - x + 1)$.
(c) Factor $x^4 + x^2 + 1$ by writing it in the form $x^4 + 2\,x^2 + 1 - x^2$, and so demonstrate that the factored forms in (a) and (b) are equivalent.
20. Factor $4\,x^4 + 1$, by writing it as $4\,x^4 + 4\,x^2 + 1 - 4\,x^2$, and noting that this is the difference of two squares.
21. Factor $a^4 + 4\,b^4$ by expressing it as the difference of two squares.
22. Factor: $x^4 + x^2y^2 + y^4$.
23. Factor: $x^4 - 12\,x^2y^2 + 16\,y^4$.
24. Factor: $4\,t^4 + 11\,t^2u^2 + 9\,u^4$.
25. Factor: $4\,a^4 + 625\,b^4$.
26. Show that the value of $(x^4 + x^2 + 1) \div (x^3 + 1)$ when $x = 1.5$ is 1.9.

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1. $(m - n)(m + n + 1)$
2. $(r + q)(2p + r - q)$
3. $n(n + 1)(n - 1)$
4. $(x + y)(x - y)^2$
5. $(a - b)(a + b)^2$
6. $(x + 2y)(a - 3x + 6y)$
7. $(2x + 3y)(1 + 10x + 15y)$
8. $(x + 2)(x - 1)(x - 4)$
9. $(x - 1)^3$
10. $(a - b)(a - b - 3)$

14. $(b + c + a)(b + c - a)(a + b - c)(a - b + c)$
15. $(n + 1)^2(n - 2)$
16. $10^2 + 1^2 + 3^2 + 6^2$
17. $(n + 2)^2 - 2(n + 1)^2 + n^2 = 2$
18. No; $a^2 + b^2 > 2ab$
19. c. $(x^2 + x + 1)(x^2 - x + 1)$
20. $(2x^2 + 2x + 1)(2x^2 - 2x + 1)$
21. $(a^2 + 2ab + 2b^2)(a^2 - 2ab + 2b^2)$
22. $(x^2 + xy + y^2)(x^2 - xy + y^2)$
23. $(x^2 + 2xy - 4y^2)(x^2 - 2xy - 4y^2)$
24. $(2t^2 + tu + 3u^2)(2t^2 - tu + 3u^2)$
25. $(2a^2 + 10ab + 25b^2)(2a^2 - 10ab + 25b^2)$