Supplementary Problems

[11-04-02-S]

SETS AND SUBSETS

1.41. Which of the following sets are equal?

$A = \{x : x^2 - 4x + 3 = 0\}$ $\quad C = \{x : x \in \mathbf{P}, x < 3\}$ $\quad E = \{1, 2\}$ $\quad G = \{3, 1\}$

$B = \{x : x^2 - 3x + 2 = 0\}$ $\quad D = \{x : x \in \mathbf{P}, x \text{ is odd}, x < 5\}$ $\quad F = \{1, 2, 1\}$ $\quad H = \{1, 1, 3\}$

1.42. List the elements of the following sets if the universal set is $\mathbf{U} = \{a, b, c, \dots, y, z\}$. Furthermore, identify which of the sets, if any, are equal.

$A = \{x : x \text{ is a vowel}\}$ $\quad C = \{x : x \text{ precedes f in the alphabet}\}$

$B = \{x : x \text{ is a letter in the word "little"}\}$ $\quad D = \{x : x \text{ is a letter in the word "title"}\}$

1.43. Let

$$A = \{1, 2, \dots, 8, 9\}, \quad B = \{2, 4, 6, 8\}, \quad C = \{1, 3, 5, 7, 9\}, \quad D = \{3, 4, 5\}, \quad E = \{3, 5\}$$

Which of the above sets can equal a set X under each of the following conditions?

(*a*) X and B are disjoint. (*c*) $X \subseteq A$ but $X \not\subseteq C$.

(*b*) $X \subseteq D$ but $X \not\subseteq B$. (*d*) $X \subseteq C$ but $X \not\subseteq A$.

1.44. Consider the following sets:

$$\varnothing, \quad A = \{a\}, \quad B = \{c, d\}, \quad C = \{a, b, c, d\}, \quad D = \{a, b\}, \quad E = \{a, b, c, d, e\}.$$

Insert the correct symbol, $\subseteq$ or $\not\subseteq$, between each pair of sets:

(*a*) $\varnothing, A$ (*c*) A, B (*e*) B, C (*g*) C, D

(*b*) D, E (*d*) D, A (*f*) D, C (*h*) B, D

SET OPERATIONS

1.45. Let $\mathbf{U} = \{1, 2, 3, \dots, 8, 9\}$ be the universal set and let:

$$A = \{1, 2, 5, 6\}, \quad B = \{2, 5, 7\}, \quad C = \{1, 3, 5, 7, 9\}$$

Find: (*a*) $A \cap B$ and $A \cap C$, (*b*) $A \cup B$ and $A \cup C$, (*c*) A^c and C^c.

1.46. For the sets in Problem 1.45, find: (*a*) $A \backslash B$ and $A \backslash C$, (*b*) $A \oplus B$ and $A \oplus C$.

1.47. For the sets in Problem 1.45, find: (*a*) $(A \cup C) \backslash B$, (*b*) $(A \cup B)^c$, (*c*) $(B \oplus C) \backslash A$.

1.48. Let $A = \{a, b, c, d, e\}$, $B = \{a, b, d, f, g\}$, $C = \{b, c, e, g, h\}$, $D = \{d, e, f, g, h\}$. Find:

(*a*) $A \cup B$ (*c*) $B \cap C$ (*e*) $C \backslash D$ (*g*) $A \oplus B$

(*b*) $C \cap D$ (*d*) $A \cap D$ (*f*) $D \backslash A$ (*h*) $A \oplus C$

1.49. For the sets in Problem 1.48, find:

(*a*) $A \cap (B \cup D)$ (*c*) $(A \cup D) \backslash C$ (*e*) $(C \backslash A) \backslash D$ (*g*) $(A \cap D) \backslash (B \cup C)$

(*b*) $B \backslash (C \cup D)$ (*d*) $B \cap C \cap D$ (*f*) $(A \oplus D) \backslash B$ (*h*) $(A \backslash C) \cap (B \cap D)$

1.50. Let A and B be any sets. Prove $A \cup B$ is the disjoint union of $A \backslash B$, $A \cap B$, and $B \backslash A$.

1.51. Prove the following:

(*a*) $A \subseteq B$ if and only if $A \cap B^c = \varnothing$ (*c*) $A \subseteq B$ if and only if $B^c \subseteq A^c$

(*b*) $A \subseteq B$ if and only if $A^c \cup B = U$ (*d*) $A \subseteq B$ if and only if $A \backslash B = \varnothing$

(Compare with Theorem 1.3.)

1.52. Prove the absorption laws: (*a*) $A \cup (A \cap B) = A$, (*b*) $A \cap (A \cup B) = A$.

1.53. The formula $A \backslash B = A \cap B^c$ defines the difference operation in terms of the operations of intersection and complement. Find a formula that defines the union $A \cup B$ in terms of the operations of intersection and complement.

1.54. (*a*) Prove: $A \cap (B \backslash C) = (A \cap B) \backslash (A \cap C)$.
(*b*) Give an example to show that $A \cup (B \backslash C) \neq (A \cup B) \backslash (A \cup C)$.

1.55. Prove the following properties of the symmetric difference:

(*a*) $A \oplus (B \oplus C) = (A \oplus B) \oplus C$ (Associative law)
(*b*) $A \oplus B = B \oplus A$ (Commutative law)
(*c*) If $A \oplus B = A \oplus C$, then $B = C$ (Cancellation law)
(*d*) $A \cap (B \oplus C) = (A \cap B) \oplus (A \cap C)$ (Distributive law)

Answers to Supplementary Problems

1.41. $B = C = E = F$; $A = D = G = H$

1.42. $A = \{\mathrm{a, e, i, o, u}\}$; $B = D = \{\mathrm{l, i, t, e}\}$; $C = \{\mathrm{a, b, c, d, e}\}$

1.43. (*a*) C and E; (*b*) D and E; (*c*) A, B, D; (*d*) none

1.44. (*a*) $\subseteq$; (*b*) $\subseteq$; (*c*) $\not\subseteq$; (*d*) $\not\subseteq$; (*e*) $\subseteq$; (*f*) $\subseteq$; (*g*) $\not\subseteq$; (*h*) $\not\subseteq$

1.45. (*a*) $A \cap B = \{2, 5\}$, $A \cap C = \{1, 5\}$; (*b*) $A \cup B = \{1, 2, 5, 6, 7\}$, $A \cup C = \{1, 2, 3, 5, 6, 7, 9\}$;
(*c*) $A^c = \{3, 4, 7, 8, 9\}$, $C^c = \{2, 4, 6, 8\}$

1.46. (*a*) $A \backslash B = \{1, 6\}$, $A \backslash C = \{2, 6\}$; (*b*) $A \oplus B = \{1, 6, 7\}$, $A \oplus C = \{2, 3, 6, 7, 9\}$

1.47. (*a*) $\{1, 3, 6, 7, 9\}$, (*b*) $\{3, 4, 8, 9\}$, (*c*) $\{3, 9\}$

1.48. (*a*) $\{a, b, c, d, e, f, g\}$; (*b*) $\{e, g, h\}$; (*c*) $\{b, g\}$; (*d*) $\{d, e\}$; (*e*) $\{b, c\}$; (*f*) $\{f, g, h\}$;
(*g*) $\{c, e, f, g\}$; (*h*) $\{a, d, g, h\}$

1.49. (*a*) $\{a, b, d, e\}$; (*b*) $\{a\}$; (*c*) $\{a, d, f\}$; (*d*) $\{g\}$; (*e*) $\varnothing$; (*f*) $\{c, h\}$; (*g*) $\varnothing$; (*h*) $\{a, d\}$

1.53. $A \cup B = (A^c \cap B^c)^c$

1.54. (*b*) $A = \{a\}$; $B = \{b\}$; $C = \{c\}$, $A \cup (B \backslash C) = \{a\}$, $(A \cup B) \backslash (A \cup C) = \{b\}$