

Example 10

What is the converse of “if $x > 10$, then $x > 4$ ”?

p : $x > 10$, and q : $x > 4$. The statement is $p \Rightarrow q$.

The converse is $q \Rightarrow p$, and is ‘If $x > 4$ then $x > 10$ ’.

Notice that the converse is false.

THE INVERSE

The **inverse** statement of $p \Rightarrow q$ is the statement $\neg p \Rightarrow \neg q$.

The inverse has truth table:

p	q	$\neg p$	$\neg q$	$\neg p \Rightarrow \neg q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

Notice that this is the same truth table as $q \Rightarrow p$. (See the table on page 507.)

So, the converse and inverse of an implication are logically equivalent.

THE CONTRAPOSITIVE

The **contrapositive** of the statement $p \Rightarrow q$ is the statement $\neg q \Rightarrow \neg p$.

The **contrapositive** has truth table:

p	q	$\neg q$	$\neg p$	$\neg q \Rightarrow \neg p$
T	T	F	F	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

Notice that the truth table for $\neg q \Rightarrow \neg p$ is the same as that for $p \Rightarrow q$, (See **Table 2** on page 506)

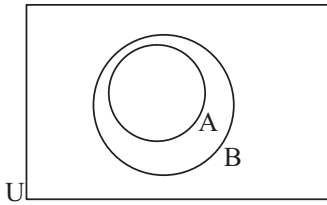
i.e., the implication and its contrapositive are logically equivalent.

EXERCISE 15F

1 Write the converse and inverse for:

- a If $5x - 2 = 13$, then $x = 3$.
- b If two triangles are similar, then they are equiangular.
- c If $2x^2 = 12$, then $x = \pm\sqrt{6}$.
- d If a figure is a parallelogram, then its opposite sides are equal in length.
- e If a triangle is equilateral, then its three sides are equal in length.

2



a Which of these implications are true?

$$\text{i } a \in A \Rightarrow a \in B \quad \text{ii } b \in B \Rightarrow b \in A$$

$$\text{iii } c \in B \Rightarrow c \notin A \quad \text{iv } c \notin B \Rightarrow c \notin A$$

b Which if **ii**, **iii** or **iv** is the contrapositive of **i**?

3 Write down the contrapositives of these statements:

- a All rose bushes have thorns.
- b No umpire makes correct decisions all the time.
- c No good soccer player has poor kicking skills.
- d Liquids always take the shape of the container in which they are placed.
- e If a person is fair and clever then the person is a doctor.

4 a State the contrapositive of: “All high school students study Mathematics.”

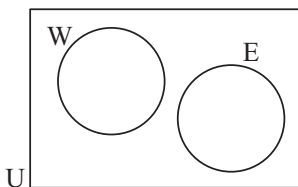
b What can be deduced (if anything) about these statements?

- i Keong, who is a high school student
- ii Tamra, who does not study Mathematics
- iii Eli, who studies English and Mathematics.

5 Write down the contrapositive of:

- a x is divisible by 3 $\Rightarrow x^2$ is divisible by 9
- b x is a number ending in 2 $\Rightarrow x$ is even
- c PQRS is a rectangle $\Rightarrow PQ \parallel SR$ and $PS \parallel QR$
- d KLM is an equilateral triangle $\Rightarrow \angle KML$ measures 60° .

6



W represents all weak students and E represents all Year 11 students.

a Copy and complete:

i No weak students are....

ii No Year 11 students are....

b Copy and complete:

i If $x \in W$ thenii If $x \in E$ thenc What is the relationship between the implications in **b**?

G

VALID ARGUMENTS

An **argument** is made up of premises (propositions) that lead to a conclusion. An argument is usually indicated by the words ‘therefore’ or ‘hence’.

Example 11

What conclusion results from: “Emus cannot fly. Jane is an Emu”?

Conclusion is: Jane cannot fly.

Let p : Emus cannot fly

q : Jane is an emu

The argument above can be written as

$$p$$

$$\frac{q}{p} \text{ premise}$$

$$q \text{ conclusion}$$

Everything above the line is the premise. Below the line is the conclusion.