1. At a small midwestern:college

31 female seniors were on the dean's list
62 non-senior women were on the dean's list
45 male seniors were on the dean's list
87 female seniors were not on the dean's list
96 male seniors were not on the dean's list
275 women were not seniors and were not on the dean's list
89 non-senior men were on the dean's list
227 men . . were not seniors and were not on the dean's list.

a) Draw a Venn diagram to represent the above data.
b) How many of these students were seniors?
c) How many were women?
d) How many were on the dean's list?
e) How many were men or seniors?
f) How many were on the dean's but were not seniors?
g) How many students were there altogether?

2. A survey of 75 UWW students found

27 MCS majors
22 seniors
11 female seniors
2 male senior MCS majors
24 males who were neither seniors nor MCS majors
7 female senior MCS majors
17 non-senior male MCS majors.

a) How many of these students were females?
b) How many were seniors or MCS majors?
c) How many were female MCS majors?
d) How many were senior MCS majors?
e) How many were senior males but not MCS majors?

3. Guess a formula for $1 + 3 + 5 + \dots + (2n-1)$ and prove it by induction.

4. Use induction to prove that $1^2 - 2^2 + 3^2 - 4^2 + \dots + (-1)^{n+1}n^2 = \frac{(-1)^{n+1}\, n\,(n+1)}{2}$

5. Prove by induction that 3 divides $n^3 - n$ for every positive integer n.

6. Guess a formula for $F_1 + F_3 + F_5 + \dots + F_{2n-1}$ and prove it by induction.

7. Prove by induction that 8 divides $3^{2n} - 1$ for every positive integer n.

8. Prove by induction that 5 divides $3^{4n} - 1$ for every positive integer n.

9. Prove by induction that $F_0^2 + F_1^2 + F_2^2 + \dots + F_n^2 = F_n F_{n+1}$ for every nonnegative integer n.

10. Prove by induction that $7^n + 15^n$ is a multiple of 22 for every positive integer n.

11. An animal tamer marches 5 lions and 4 tigers into the arena. How many ways can he line up the animals if a tiger must not be followed by another tiger?

12. There are 15 different books on a bookshelf. How many ways can 6 of these books be selected if a selection must not include two neighboring books?

13. A meeting is to be addressed by 5 speakers: A, B, C, D, and E. How many ways can the speakers be ordered if

a) B must not precede A?
b) B is to speak immediately after A?

14. How many anagrams of MISSISSIPPI have no adjacent I's?

15. How many anagrams of MISSOURI have no adjacent vowels?

16. In how many anagrams of JUPITER do the vowels appear in alphabetical order?

17. In how many anagrams of BIVOUAC do the vowels appear in their original order?

18. In how many anagrams of POSTER are there exactly two consonants between the two vowels? E.g., RTEPSO .

19. In how many anagrams of ILLINOIS are the two L's not adjacent?

20. In how many anagrams of ANDROMEDA do both the vowels and the consonants appear (separately) in alphabetical order?

21. Count the anagrams of TRIANNUAL in which the consonants and vowels alternate.

22. Count the anagrams of MUHAMMADAN which do not have 3 consecutive identical letters.

23. In how many ways can 5 men and 5 women sit at a round table so that no two men sit next to each other?

24. a) In how many ways can an American, a Dane, an Egyptian, a Russian, and a Swede sit at round table?

b) In how many ways can an amethyst, a diamond, an emerald, a ruby, and a sapphire be arranged on a gold necklace?

25. In how many ways can 5 men and 8 women sit at a round table if the men sit in consecutive seats?

26. In how many ways can 8 people, including Dow and Jones, sit at a round table with Dow next to Jones?

27. a) In how many ways can 8 people sit at a lunch counter with 8 stools?
    b) In how many ways can 8 people sit at a round table?
    c) In how many ways can 4 couples sit at the lunch counter if each wife sits next to her husband?
    d) In how many ways can 4 couples sit at the round table if each husband sits next to his wife?

28. A store has in stock one copy of a certain book, 2 copies of another book, and 4 copies of a third book. How many different purchases can a customer make of these books? (A purchase can be anything from one copy of one book to the entire stock of the store.)

29. Consider code words consisting of 6 characters each. If the characters are chosen from a, b, c, d, 2, 3, 4, 5, and 6 (with no repetitions) how many words can be formed
    a) altogether?
    b) if the third and fourth characters must be digits and the other characters must be letters?
    c) if there must be 3 digits and 3 letters?
    d) if no two digits and no two letters may be adjacent?

30. Work Problem 29 with the modification that characters may be used more than once.

31. a) How many five-person committees can be chosen from a club with 15 members?
    b) In how many ways can a five-person committee consisting of a chair, a secretary, and three other members be chosen from the club?
    c) In how many ways can the club be divided into 3 committees of 5 members each, with no member serving on more than one committee, if each committee has different duties?
    d) In how many ways can the club be divided into 3 committees, each consisting of a chair, a secretary, and 3 other members, if the committees have different duties and no one serves on more than once committee?

32. How many ways are there to choose 3 letters from the phrase

    MISS MISSISSIPPI NEVER EVER SIMPERS ignoring the order of selection?

33. a) In how many ways can 11 A's and 8 B's be arranged in a row so that no two B's are adjacent?
    b) In how many ways can 11 men and 8 women stand in a row so that no two women are adjacent?

34. How many collections of letters, in no particular order, can be formed by choosing one or more letters from the phrase MISSISSIPPI RIVER ?

35. a) A disgruntled mail carrier has to place 10 different letters into 7 mail boxes. He pays no attention to the addresses on the letters. In how many ways can he distribute the letters into the boxes?
    b) Suppose he has 10 identical circulars, instead of the letters. In how many ways can he distribute the circulars?

36. Each of two novice collectors has 20 stamps and 10 postcards. We call an exchange fair if they exchange a stamp for a stamp or a postcard for a postcard. How many ways are there to carry out one fair exchange between these two collectors?

37. How many six-digit numbers have all their digits of equal parity (all odd or all even)?

38. In how many ways can we send six urgent letters if we can use three messengers and each letter can be given to any of them?

39. How many ways are there to choose four cards of different suits and different values from a deck of 52 cards?

40. There are five books on a shelf. How many ways are there to arrange some (or all) of them in a stack? The stack may consist of a single book.

41. The rules of a chess tournament say that each contestant must play every other contestant exactly once. How many games will be played if there are 16 participants?

42. There are three rooms in a dormitory: one single, one double, and one for four students. How many ways are there to house seven students in these rooms?

43. How many code words can be written useing exactly five A's and no more than three B's (and no other letters)?

44. How many ten-digit numbers have at least two equal digits?

45. Do seven-digit numbers with no 1's in their decimal representations constitute more than 50% of all seven-digit numbers?

46. The Hermetian alphabet consists of only three letters: A, B, and C. A word in this language is an arbitrary sequence of no more than four letters. How many words does the Hermetian language contain?

47. How many six-digit numbers have at least one even digit?

48. There are six letters in the Hermetian language. A word is any sequence of six letters, some pair of which are the same. How many words are there in the Hermetian language?

49. A six-digit number is given. How many seven-digit numbers are there which will produce that number if one digit is crossed out?

50. Prove that $1/(1\cdot 2) + 1/(2\cdot 3) + \dots + 1/((n-1)n) = (n-1)/n$.

51. Prove that $(1 - 1/4)(1 - 1/9) \dots (1 - 1/n^2) = (n + 1)/(2n)$.

52. Prove that $n^3 + (n+1)^3 + (n+2)^3$ is divisible by 9 for any positive integer n.

53. Prove that $3^{2n+2} + 8n - 9$ is divisible by 16 for any positive integer n.

54. Prove that $4^n + 15n - 1$ is divisible by 9 for any positive integer n.

55. Prove that $2^{3^n} + 1$ is divisible by 3^{n+1} for any positive integer n.

56. One student has 6 different math books, and another has 8 different books. How many ways are there to exchange 3 books belonging to the first student with 3 books belonging to the second?

57. There are 2 girls and 7 boys in a chess club. A team of four persons must be chosen for a tournament, and there must be at least 1 girl on the team. In how many ways can this be done?

58. Ten points are marked on a straight line, and 11 points are marked on another line, parallel to the first one. How many triangles are there with vertices at these points?

59. There are 4 married couples in a club. How many ways are there to choose a committee of 3 members so that no two members of the committee are married to each other?

60. There are 31 students in a class, including Dow and Jones. How many ways are there to choose a soccer team (11 players) so that Dow and Jones are not on the team together?

61. How many anagrams of ASUNDER have the vowels in alphabetical order and the consonants in alphabetical order? (E.g., DANERUS).

62. How many six-digit numbers have 3 even and 3 odd digits?

63. We must choose a 5-member team from 12 girls and 10 boys. How many such teams include no more than 3 boys?

64. How many ways are there to arrange 5 red, 5 blus, and 5 green balls in a row so that no two blue balls lie next to each other?

65. Find the number of integers from 0 through 999,999 inclusive that have no two equal adjacent digits.