[10-09-09-T11]

■ Solving $|A| = |B|$, ctd.

Well, we made a project out of this in class, but part of the point of that was the mental exercise in mathematics. It did accomplish that. But, as we suspected, there is an easier way to get the general result we desired.

Suppose $|A| = |B|$. Now, we already know that $|A| = B \Longleftrightarrow A = \pm B$, providing B is not negative. Well $|B|$ certainly is not negative. So, $|A| = |B| \Longleftrightarrow A = \pm |B|$. But $\pm |B| = \pm B$. So $|A| = |B| \Longleftrightarrow A = \pm B$. And that's it!

■ Two problems compared

■ Solve for x. $|3x + 5| = |x + 4|$

Solution.

$|3x+5| = |x+4| \Longleftrightarrow 3x+5 = \pm(x+4)$

Case 1.
$3x+5 = x+4$
$\Longleftrightarrow 2x = -1$
$\Longleftrightarrow x = \frac{-1}{2}$

Case 2.
$3x+5 = -(x+4)$
$\Longleftrightarrow 3x+5 = -x-4$
$\Longleftrightarrow 4x = -9$
$\Longleftrightarrow x = \frac{-9}{4}$

$\therefore\ x = \frac{-1}{2}$ or $x = \frac{-9}{4}$.

▪ Solve for x. $|3x+5| = x+4$

This is actually the more difficult of the two, because you have to be careful to make $x+4$ nonnegative, otherwise you will have an positive quantity, $|3x+5|$ equal to a negative number. With that in mind, our work is as follows.

Solution.

Require that $x+4 \geq 0$. This is equivalent to $x \geq -4$. Under this condition,

$|3x+5| = x+4 \Longleftrightarrow 3x+5 = \pm(x+4)$.

Case 1.
$3x+5 = x+4$
$\Longleftrightarrow x = \frac{-1}{2}$

Case 2.
$3x+5 = -(x+4)$
$\Longleftrightarrow x = \frac{-9}{4}$