Simplify equation (2). $\dfrac{2x+y}{x-2y}=\dfrac{1}{3}$

Multiply both sides by $3(x-2y)$.

$$3(2x+y)=x-2y$$
$$6x+3y=x-2y$$
$$5x+5y=0$$
$$x+y=0 \qquad (4)$$

Equations (3) and (4), which are equivalent forms of equations (1) and (2), may now be solved by either of the standard methods.

Exercises [A-1]

Solve the following systems of equations:

1. $\begin{cases} x+4y=1 \\ x+7y=4 \end{cases}$
2. $\begin{cases} y=4x-7 \\ 16x-5y=25 \end{cases}$
3. $\begin{cases} 5x+3y=6 \\ 7x-y=11 \end{cases}$
4. $\begin{cases} 5x-3y=7 \\ 15x-9y=3 \end{cases}$
5. $\begin{cases} 5x+4y=3 \\ 2x+3y=4 \end{cases}$
6. $\begin{cases} x+y=14 \\ \dfrac{x}{y}=\dfrac{3}{4} \end{cases}$
7. $\begin{cases} \frac{1}{2}x-\frac{1}{3}y=2 \\ \frac{1}{3}x+\frac{1}{2}y=\frac{17}{3} \end{cases}$
8. $\begin{cases} .5x-y=1.7 \\ .3x+.4y=1.3 \end{cases}$
9. $\begin{cases} .1x+3y=2.6 \\ x-1.6y=10.2 \end{cases}$
10. $\begin{cases} 3x-2y+4=0 \\ \dfrac{x-y}{x+y}=3 \end{cases}$
11. $\begin{cases} y=3+.8x \\ 4y-2.2x=2 \end{cases}$
12. $\begin{cases} \dfrac{1}{u}+\dfrac{1}{t}=5 \\ \dfrac{1}{u}-\dfrac{1}{t}=1 \end{cases}$
13. $\begin{cases} \dfrac{6}{x}+\dfrac{4}{y}=9 \\ \dfrac{12}{x}-\dfrac{2}{y}=3 \end{cases}$
14. $\begin{cases} \dfrac{h}{x}=.35 \\ \dfrac{h}{x+10}=.25 \end{cases}$
15. $\begin{cases} \dfrac{10}{x}+3y=2 \\ \dfrac{8}{x}-2y=6 \end{cases}$
16. $\begin{cases} \frac{1}{2}(x-2)-\frac{1}{3}(y+3)=-2 \\ \dfrac{3(x-2)}{4}-\dfrac{5(y+1)}{2}=2 \end{cases}$
17. $\begin{cases} \dfrac{4}{3x}+\dfrac{1}{2y}=1 \\ \dfrac{5}{3x}-\dfrac{3}{4y}=4 \end{cases}$

18. How many pairs of values of x and y satisfy the equation $x+y=8$,
    (a) If there is no restriction on the values of the variables?
    (b) If both x and y must be positive, odd integers?
    (c) If the values of x and y must be equal?
19. If $y=7u+2$, and $u=4-3x$, eliminate u by substitution and express y in terms of x. Find the value of y for which $x=2$
20. In the equation $\frac{1}{2}x+\frac{1}{3}(y+8)=4$, find the value of y if (a) the value of x is $-\frac{5}{2}$, (b) the values of x and y are in the ratio 2 : 1.
21. If the equation $3x+ky=2$ is satisfied by the pair of values $x=2$, $y=-1$, find the value of k.

Exercises [A–2]

Solve the following systems of equations:

1. $\begin{cases} 3x - y = 5 \\ 5x - y = 7 \end{cases}$

2. $\begin{cases} y = 11 - 3x \\ 5x - 3y = 2 \end{cases}$

3. $\begin{cases} 3x + 5y = 7 \\ x - 3y = 0 \end{cases}$

4. $\begin{cases} 2x + y = 4 \\ 4x + 2y = 6 \end{cases}$

5. $\begin{cases} 3x + 4y = 1 \\ 5x + 3y = 9 \end{cases}$

6. $\begin{cases} x - y = 1 \\ \frac{y}{x} = \frac{1}{2} \end{cases}$

7. $\begin{cases} \frac{2x}{3} + \frac{y}{9} = 6 \\ \frac{1}{4}x + \frac{1}{2}y = 5 \end{cases}$

8. $\begin{cases} .5x + 1.2y = 1.4 \\ .6x - 7.0y = 5.9 \end{cases}$

9. $\begin{cases} .5x + .2y = 1.65 \\ .7x - .3y = 2.6 \end{cases}$

10. $\begin{cases} 2x - 3y - 3 = 0 \\ \frac{x + y}{x - y} = \frac{1}{2} \end{cases}$

11. $\begin{cases} x = 2 - .25y \\ 9x + 1.25y = 15 \end{cases}$

12. $\begin{cases} \frac{2}{u} + \frac{1}{t} = 7 \\ \frac{2}{u} - \frac{1}{t} = 1 \end{cases}$

14. $\begin{cases} \frac{h}{x} = .45 \\ \frac{h}{x + 50} = .30 \end{cases}$

16. $\begin{cases} \frac{1}{3}(x + 1) - \frac{1}{2}(3y - 1) = 1 \\ \frac{3(2x - 1)}{5} - \frac{2(y - 3)}{3} = 2 \end{cases}$

13. $\begin{cases} \frac{3}{x} + \frac{4}{y} = 8 \\ \frac{6}{x} - \frac{3}{y} = 5 \end{cases}$

15. $\begin{cases} \frac{8}{x} + 3y = 5 \\ \frac{6}{x} + 2y = 3 \end{cases}$

17. $\begin{cases} \frac{3}{2x} - \frac{1}{3y} = 2 \\ \frac{3}{4x} + \frac{5}{6y} = -2 \end{cases}$

18. How many pairs of values of x and y satisfy the equation $x + y = 10$,
   (a) If there is no restriction on the values of the variables?
   (b) If both x and y must be even integers, greater than zero?
   (c) If the value of x must be 25% of the value of y?

19. If $x = 4t + 3$, and $t = 3 - 2u$, eliminate t by substitution and express x in terms of u. Find the value of u for which $x = 0$.

20. In the equation $\frac{1}{3}(x - 2) + \frac{1}{4}y = \frac{1}{2}$, find the value of x if (a) the value of y is $-\frac{2}{3}$, (b) the values of x and y are equal.

21. If the equation $4x - 3y = c$ is satisfied by the pair of values $x = \frac{1}{2}$, $y = -1$, find the value of c.

22. Show that the equations $5x - y = 21$, $x + 3y = 1$, $13x + 7y = 45$ have a common solution.

Exercises [B]

1. Show that the equation $\frac{1}{2}x + \frac{1}{3}(y - 3) = \frac{1}{3}x + \frac{1}{2}(y - 2)$ is equivalent to the equation $x = y$.

2. State the number of solutions that may be found for each of the following pairs of equations:

   (a) $2x + 5y = 8$, $4x + 10y = 12$

   (b) $2x + 5y = 8$, $4x + 10y = 16$

   (c) $2x + 5y = 8$, $2x - y = 2$

3. In general, how many solutions may be found to a problem in two variables,
   (a) If only one equation in the two variables is to be satisfied?
   (b) If two independent equations in the variables are to be satisfied?
   (c) If three independent equations in the variables are to be satisfied?

4. If $7x + 3y = 18$, find the smallest integral value of y for which the corresponding value of x is less than 1.

5. Solve for x and y: $x + y = 28$, $\dfrac{7}{x+y} = \dfrac{4}{x-y}$.

6. Show that no pair of values of x and y satisfies the three equations $3x - 2y = 8$, $5x - 4y = -3$, $\dfrac{x}{y} = \dfrac{3}{4}$.

7. Find the value that a must have if the three equations $3x - 2y = 8$, $2x + 3y = 1$, $ax + 5y = -1$ are to have a common solution.

8. Solve for x and y: $x - \frac{1}{3}y = 3y - \frac{1}{2}x = 8\frac{1}{2}$.

9. State the number of solutions that may be found for each of the following pairs of equations:

   (a) $4x - 3y = 6$, $8x - 6y = 9$

   (b) $4x - 3y = 6$, $2x - \frac{3}{2}y = 3$

   (c) $4x - 3y = 6$, $x + 3y = 4$

10. How many pairs of numbers may be found to satisfy the following conditions?
    (a) The sum of the two numbers is 25.
    (b) The sum of the two numbers is 25, and the difference of the numbers is 12.
    (c) The sum of the two numbers is 25, their difference is 12, and the numbers are equal.

11. If $3x + 2y = 7$, find the smallest integral value of x for which the corresponding value of y is negative.

12. Solve for t and u: $\begin{cases} t - u = 5 \\ \dfrac{4}{u+t} = \dfrac{10}{u-t} \end{cases}$

13. Show that no values of x and y satisfy the three equations $x - y = 1$, $2x - 3y = 2$, $y = \frac{1}{2}x$.

14. Find the value that b must have if the three equations $2x - y = 5$, $4x + 3y = -5$, $3x + by = 9$ are to have a common solution.

15. Solve for x and y:
$$\tfrac{1}{2}x - 7y - 5 = \tfrac{3}{2}x - y = 0.$$

3. $\{x \mid x = \pm 2\}$, or $\{x \mid x^2 = 4\}$ (other answers possible).
4. a. {B, D}. b. $\emptyset$.
5. a. {1, 2, 3, 4}. b. {2, -2}. c. None.
6. {0, 1, 2, 3, 4, 5, 6}.
7. $\{x \mid x \text{ is an integer and } 0 < x < 100\}$.
8. R is the set of rational numbers (or whatever universal set is in use).

Page 31

1. (-3, 1)
2. $(2\frac{1}{2}, 3)$
3. $(1\frac{1}{2}, -\frac{1}{2})$
4. No solution
5. (-1, 2)
6. (6, 8)
7. (8, 6)
8. (3.96, 0.28)
9. (11, 0.5)
10. $(-1, \frac{1}{2})$
11. (-10, -5)
12. $(u, t) = (\frac{1}{3}, \frac{1}{2})$
13. $(2, \frac{2}{3})$
14. (x, h) = (25, 8.75)
15. (2, -1)
16. (-2, -3)
17. $(\frac{2}{3}, -\frac{1}{2})$
18. a. An unlimited number
    b. Four: (1, 7), (3, 5), (5, 3), (7, 1)
    c. One: (4, 4)
19. $y = 30 - 21x$; $y = -12$
20. a. $y = 7\frac{3}{4}$
    b. $y = 1$
21. $k = 4$

Page 32

1. (1, -2)
2. $(2\frac{1}{2}, 3\frac{1}{2})$
3. $(\frac{3}{2}, \frac{1}{2})$
4. No solution
5. (3, -2)
6. (2, 1)
7. (8, 6)
8. $(4, -\frac{1}{2})$
9. (3.5, -0.5)
10. $(1, -\frac{1}{3})$
11. (1.25, 3)
12. $(u, t) = (\frac{1}{2}, \frac{1}{3})$
13. $(\frac{3}{4}, 1)$
14. (x, h) = (100, 45)
15. (-2, 3)
16. $(\frac{1}{2}, 0)$
17. $(\frac{3}{2}, -\frac{1}{3})$
18. a. An unlimited number
    b. Four: (2, 8), (4, 6), (6, 4), (8, 2)
    c. One: (2, 8)

19. $x = 15 - 8u$; $u = 1\frac{7}{8}$
20. a. $x = 4$ b. $x = 2$
21. $c = 5$
22. (4, -1) is common sol.

Pages 32-33

2. a. None
   b. Unlimited number
   c. One
3. a. Unlimited number
   b. One
   c. None
4. $y = 4$
5. (22, 6)
7. $a = 2$
8. $(10, 4\frac{1}{2})$
9. a. None
   b. Unlimited number
   c. One
10. a. Unlimited number
    b. One: $(18\frac{1}{2}, 6\frac{1}{2})$
    c. None
11. $x = 3$
12. $(t, u) = (1\frac{1}{2}, -3\frac{1}{2})$
14. $b = -2$
15. $(-\frac{1}{2}, -\frac{3}{4})$

Pages 35-36

1. (1, 5, 3)
2. (1, 1, 0)
3. (2, -1, 3)
4. $(\frac{1}{2}, -\frac{1}{4}, \frac{1}{3})$
5. (r, s, t) = (9, 11, 8)
6. $(-\frac{1}{3}, \frac{1}{5}, \frac{1}{2})$

Page 36 (middle)

1. (3, 2, 1)
2. (r, s, t) = (2, 0, 1)
3. (-1, 4, 8)
4. $(\frac{2}{3}, -\frac{1}{3}, \frac{1}{2})$
5. (s, t, u) = (6, 8, 5)
6. $(\frac{1}{4}, -\frac{1}{3}, \frac{1}{2})$

Page 36 (bottom)

2. (a,b,c) = (8,4,3)
4. (a,b,c) = (3k,2k,4k), k any constant.

Pages 40-42

1. a. $d = 47\frac{1}{2} - 6x$
   b. 43
   c. $1\frac{1}{2}$
2. a. hours
   b. $n = 5 + \frac{5}{24}x$
   c. $6\frac{1}{4}$
   d. 8
3. a. 35 m.p.h.
   b. $v = \frac{7x}{4}$
   c. 22.9
4. a. Width, (x + 4) in.
      Length, (x + 7) in.
   b. (8x + 28) sq.in.
   c. 18
5. a. 84 cents
   b. $(90 - \frac{3}{5}x)$ cents
   c. 35
6. a. $I = 275 - 0.03x$
   b. $230
   c. $2700