[11-03-16-S]

## 6 REVIEW

### EXERCISES

**1.** Two regions $\mathcal{R}_1$ and $\mathcal{R}_2$ are shown in the figure. Express each of the following quantities as an integral:

(a) The area of $\mathcal{R}_1$

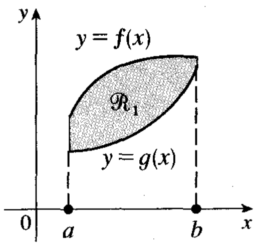
(b) The area of $\mathcal{R}_2$

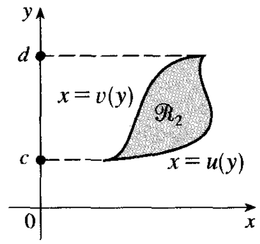
(c) The volume of the solid obtained by rotating $\mathcal{R}_1$ about the $x$-axis

(d) The volume of the solid obtained by rotating $\mathcal{R}_1$ about the $y$-axis

(e) The volume of the solid obtained by rotating $\mathcal{R}_2$ about the $x$-axis

(f) The volume of the solid obtained by rotating $\mathcal{R}_2$ about the $y$-axis




**2–8** ▪ Find the area of the region bounded by the given curves.

**2.** $y = 4 + 3x - x^2, \quad y = 0$

**3.** $y = x^2 - 6x, \quad y = 12x - 2x^2$

**4.** $y = x^2 - 6, \quad y = 12 - x^2, \quad x = -5, \quad x = 5$

**5.** $y = x^3, \quad x = y^3$

**6.** $x - 2y + 7 = 0, \quad y^2 - 6y - x = 0$

**7.** $y = \sin x, \quad y = -\cos x, \quad x = 0, \quad x = \pi$

**8.** $y = x^3, \quad y = x^2 - 4x + 4, \quad x = 0, \quad x = 2$

**9–13** ▪ Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.

**9.** $y = \sqrt{x - 1}, y = 0, x = 3$; about the $x$-axis

**10.** $y = x^3, y = x^2$; about the $x$-axis

**11.** $x + 3 = 4y - y^2, x = 0$; about the $x$-axis

**12.** $y = x^3, y = 8, x = 0$; about the $y$-axis

13. $x^2 - y^2 = a^2$, $x = a + h$ (where $a > 0$, $h > 0$); about the $y$-axis

**14–16** ■ Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.

14. $y = \cos x$, $y = 0$, $x = 3\pi/2$, $x = 5\pi/2$; about the $y$-axis

15. $y = x^3$, $y = x^2$; about $y = 1$

16. $y = x^3$, $y = 8$, $x = 0$; about $x = 2$

17. Find the volumes of the solids obtained by rotating the region bounded by the curves $y = x$ and $y = x^2$ about the following lines:
(a) The $x$-axis (b) The $y$-axis (c) $y = 2$

18. Let $\mathfrak{R}$ be the region in the first quadrant bounded by the curves $y = x^3$ and $y = 2x - x^2$. Calculate the following quantities:
(a) The area of $\mathfrak{R}$
(b) The volume obtained by rotating $\mathfrak{R}$ about the $x$-axis
(c) The volume obtained by rotating $\mathfrak{R}$ about the $y$-axis

19. Let $\mathfrak{R}$ be the region bounded by the curves $y = \tan(x^2)$, $x = 1$, and $y = 0$. Use the Midpoint Rule with $n = 4$ to estimate the following:
(a) The area of $\mathfrak{R}$
(b) The volume obtained by rotating $\mathfrak{R}$ about the $x$-axis

20. Let $\mathfrak{R}$ be the region bounded by the curves $y = 1 - x^2$ and $y = x^6 - x + 1$. Estimate the following:
(a) The $x$-coordinates of the points of intersection of the curves
(b) The area of $\mathfrak{R}$
(c) The volume generated when $\mathfrak{R}$ is rotated about the $x$-axis
(d) The volume generated when $\mathfrak{R}$ is rotated about the $y$-axis

**21–24** ■ Each integral represents the volume of a solid. Describe the solid.

21. $\int_0^{\pi} \pi \sin^2 x\, dx$

22. $\int_0^{\pi} 2\pi x \sin x\, dx$

23. $\int_0^{2} 2\pi y(4 - y^2)\, dy$

24. $\int_0^{1} \pi[(2 - x^2)^2 - (2 - \sqrt{x})^2]\, dx$

25. The base of a solid is a circular disk with radius 3. Find the volume of the solid if parallel cross-sections perpendicular to the base are isosceles right triangles with hypotenuse lying along the base.

26. The base of a solid is the region bounded by the parabolas $y = x^2$ and $y = 2 - x^2$. Find the volume of the solid if the cross-sections perpendicular to the $x$-axis are squares with one side lying along the base.

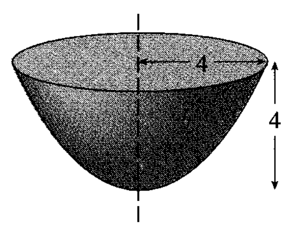
27. The height of a monument is 20 m. A horizontal cross-section at a distance $x$ meters from the top is an equilateral triangle with side $x/4$ meters. Find the volume of the monument.

28. (a) The base of a solid is a square with vertices at $(1, 0)$, $(0, 1)$, $(-1, 0)$, and $(0, -1)$. Each cross-section perpendicular to the $x$-axis is a semicircle. Find the volume of the solid.
(b) Show that by cutting the solid of part (a), we can rearrange it to form a cone. Thus compute its volume more simply.

29. A force of 30 N is required to maintain a spring stretched from its natural length of 12 cm to a length of 15 cm. How much work is done in stretching the spring from 12 cm to 20 cm?

30. A 1600-lb elevator is suspended by a 200-ft cable that weighs 10 lb/ft. How much work is required to raise the elevator from the basement to the third floor, a distance of 30 ft?

31. A tank full of water has the shape of a paraboloid of revolution as in the figure; that is, its shape is obtained by rotating a parabola about a vertical axis.
(a) If its height is 4 ft and the radius at the top is 4 ft, find the work required to pump the water out of the tank.
(b) After 4000 ft-lb of work has been done, what is the depth of the water remaining in the tank?




32. Find the average value of the function $f(x) = x^3$ on the interval $[2, 4]$.

33. If $f$ is a continuous function, what is the limit as $h \to 0$ of the average value of $f$ over the interval $[x, x + h]$?

34. Let $\mathfrak{R}_1$ be the region bounded by $y = x^2$, $y = 0$, and $x = b$, where $b > 0$. Let $\mathfrak{R}_2$ be the region bounded by $y = x^2$, $x = 0$, and $y = b^2$.
(a) Is there a value of $b$ such that $\mathfrak{R}_1$ and $\mathfrak{R}_2$ have the same area?
(b) Is there a value of $b$ such that $\mathfrak{R}_1$ sweeps out the same volume when rotated about the $x$-axis and the $y$-axis?
(c) Is there a value of $b$ such that $\mathfrak{R}_1$ and $\mathfrak{R}_2$ sweep out the same volume when rotated about the $x$-axis?
(d) Is there a value of $b$ such that $\mathfrak{R}_1$ and $\mathfrak{R}_2$ sweep out the same volume when rotated about the $y$-axis?

**Review Exercises for Chapter 6 ■ page 410**

**1.** (a) $A = \int_a^b [f(x) - g(x)]\,dx$ (b) $A = \int_c^d [u(y) - v(y)]\,dy$
(c) $V = \pi \int_a^b \{[f(x)]^2 - [g(x)]^2\}\,dx$
(d) $V = 2\pi \int_a^b x[f(x) - g(x)]\,dx$
(e) $V = 2\pi \int_c^d y[u(y) - v(y)]\,dy$
(f) $V = \pi \int_c^d \{[u(y)]^2 - [v(y)]^2\}\,dy$

**3.** 108 **5.** 1 **7.** $2\sqrt{2}$ **9.** $2\pi$

**11.** $16\pi/3$ **13.** $4\pi(2ah + h^2)^{3/2}/3$
**15.** $\int_0^1 \pi[(1 - x^3)^2 - (1 - x^2)^2]\,dx$
**17.** (a) $2\pi/15$ (b) $\pi/6$ (c) $8\pi/15$
**19.** (a) 0.38 (b) 0.87
**21.** Solid obtained by rotating the region under $y = \sin x$ from 0 to $\pi$ about the $x$-axis
**23.** Solid obtained by rotating the region in the first quadrant bounded by $x = 4 - y^2$ and the axes about the $x$-axis
**25.** 36 **27.** $125\sqrt{3}/3\ \text{m}^3$ **29.** 3.2 J
**31.** (a) $8000\pi/3$ ft-lb (b) 2.1 ft **33.** $f(x)$