Table 1-1 Laws of the Algebra of Sets

Idempotent laws	
(1*a*) $A \cup A = A$	(1*b*) $A \cap A = A$
Associative laws	
(2*a*) $(A \cup B) \cup C = A \cup (B \cup C)$	(2*b*) $(A \cap B) \cap C = A \cap (B \cap C)$
Commutative laws	
(3*a*) $A \cup B = B \cup A$	(3*b*) $A \cap B = B \cap A$
Distributive laws	
(4*a*) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	(4*b*) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Identity laws	
(5*a*) $A \cup \varnothing = A$	(5*b*) $A \cap \mathbf{U} = A$
(6*a*) $A \cup \mathbf{U} = \mathbf{U}$	(6*b*) $A \cap \varnothing = \varnothing$
Involution law	
(7) $(A^c)^c = A$	
Complement laws	
(8*a*) $A \cup A^c = \mathbf{U}$	(8*b*) $A \cap A^c = \varnothing$
(9*a*) $\mathbf{U}^c = \varnothing$	(9*b*) $\varnothing^c = \mathbf{U}$
DeMorgan's laws	
(10*a*) $(A \cup B)^c = A^c \cap B^c$	(10*b*) $(A \cap B)^c = A^c \cup B^c$

Duality

The identities in Table 1-1 are arranged in pairs, as, for example, (2*a*) and (2*b*). We now consider the principle behind this arrangement. Let E be an equation of set algebra. The *dual* E^* of E is the equation obtained by replacing each occurrence of $\cup, \cap, \mathbf{U}, \varnothing$ in E by $\cap, \cup, \varnothing, \mathbf{U}$, respectively. For example, the dual of

$$(\mathbf{U} \cap A) \cup (B \cap A) = A \quad \text{is} \quad (\varnothing \cup A) \cap (B \cup A) = A$$

Observe that the pairs of laws in Table 1-1 are duals of each other. It is a fact of set algebra, called the *principle of duality*, that, if any equation E is an identity, then its dual E^* is also an identity.

1.8 FINITE SETS, COUNTING PRINCIPLES

A set is said to be *finite* if it contains exactly m distinct elements where m denotes some nonnegative integer. Otherwise a set is said to be infinite. For example, the empty set $\varnothing$ and the set of letters of the English alphabet are finite sets, whereas the set of even positive integers $\{2, 4, 6, \ldots\}$ is infinite. [Infinite sets will be studied in detail in Chapter 6.]

The notation $n(A)$ or $|A|$ will denote the number of elements in a finite set A.

First we begin with a special case.

Lemma 1.5: Suppose A and B are finite disjoint sets. Then $A \cup B$ is finite and

$$n(A \cup B) = n(A) + n(B)$$