

A

SET BUILDER NOTATION

$A = \{x \mid x \in Z, -2 \leq x \leq 4\}$ reads “the set of all x such that x is an integer between -2 and 4 , including -2 and 4 .”

$A = \{x: x \in Z, -2 \leq x \leq 4\}$ reads the same as above.

Example 1

Given $A = \{x \mid x \in Z, 3 < x \leq 10\}$ write down:

- a** the meaning of the set builder notation.
b List the elements of set A . **c** Find $n(A)$.

- a** The set of all x such that x is an integer between 3 and 10, including 10.
b $A = \{4, 5, 6, 7, 8, 9, 10\}$ **c** $n(A) = 7$, as there are 7 elements

EXERCISE 3A

- Finite or infinite?
 - $\{x \mid x \in Z, -2 \leq x \leq 1\}$
 - $\{x \mid x \in R, -2 \leq x \leq 1\}$
 - $\{x \mid x \in Z, x \geq 5\}$
 - $\{x \mid x \in Q, 0 \leq x \leq 1\}$
- For the following sets:
 - Write down the meaning of the set builder notation.
 - If possible, list the elements of A .
 - Find $n(A)$.
 - Is A infinite?
 - $A = \{x \mid x \in Z, -1 \leq x < 7\}$
 - $A = \{x \mid x \in N, -2 < x < 8\}$
 - $A = \{x \mid x \in R, 0 \leq x \leq 1\}$
 - $A = \{x \mid x \in Q, 5 \leq x \leq 6\}$
- Write in set builder notation:
 - The set of all integers between -100 and 100 .
 - The set of all real numbers greater than 1000 .
 - The set of all rational numbers between 2 and 3 , inclusive.
- The subsets of $\{a\}$ are \emptyset and $\{a\}$, i.e., two of them. The subsets of $\{a, b\}$ are \emptyset , $\{a\}$, $\{b\}$ and $\{a, b\}$, i.e., four of them.
 - List the subsets of **i** $\{a, b, c\}$ **ii** $\{a, b, c, d\}$ and state the number of them in each case.
 - Copy and complete: “If a set has n elements it has subsets.”
- Is $A \subset B$ in these cases?
 - $A = \emptyset$ and $B = \{2, 5, 7, 9\}$
 - $A = \{2, 5, 8, 9\}$ and $B = \{8, 9\}$
 - $A = \{x \mid x \in R, 2 \leq x \leq 3\}$ and $B = \{x \mid x \in R\}$
 - $A = \{x \mid x \in Q, 3 \leq x \leq 9\}$ and $B = \{x \mid x \in R, 0 \leq x \leq 10\}$
 - $A = \{x \mid x \in Z, -10 \leq x \leq 10\}$ and $B = \{z \mid z \in Z, 0 \leq z \leq 5\}$
 - $A = \{x \mid x \in Q, 0 \leq x \leq 1\}$ and $B = \{y \mid y \in Q, 0 < y \leq 2\}$

B**COMPLEMENTS OF SETS****UNIVERSAL SETS**

If we are only interested in the natural numbers from 1 to 20 and we want to consider subsets of this set then $U = \{x \mid x \in N, 1 \leq x \leq 20\}$ is the universal set in this situation.

The symbol U is used to represent a universal set.

COMPLEMENTARY SETS

If the universal set is $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $A = \{1, 3, 5, 7, 8\}$ then the complement of A , denoted A' is $A' = \{2, 4, 6\}$.

The **complement** of A , denoted A' is the set of all elements of U which are **not in** A .

Three obvious relationships are observed connecting A and A' . These are:

- $A \cap A' = \emptyset$ as A' and A have no common members
- $A \cup A' = U$ as all elements of A and A' combined make up U
- $n(A') = n(U) - n(A)$ {see the example above where $n(A') = 3$ and $n(U) - n(A) = 8 - 5 = 3$ }

Example 2

Find C' given that:

a $U = \{\text{all integers}\}$ and $C = \{\text{all even integers}\}$

b $C = \{x \mid x \in Z, x \geq 2\}$ and $U = Z$

a $C' = \{\text{all odd integers}\}$

b $C' = \{x \mid x \in Z, x \leq 1\}$

EXERCISE 3B

1 Find C' , the complement of C given that:

a $U = \{\text{letters of the English alphabet}\}$ and $C = \{\text{vowels}\}$

b $U = \{\text{integers}\}$ and $C = \{\text{negative integers}\}$

c $U = Z$ and $C = \{x \mid x \in Z, x \leq -5\}$

d $U = Q$ and $C = \{x \mid x \in Q, x \leq 2 \text{ or } x \geq 8\}$

Example 3

If $U = \{x \mid x \in Z, -5 \leq x \leq 5\}$, $A = \{x \mid x \in Z, 1 \leq x \leq 4\}$ and $B = \{x \mid x \in Z, -3 \leq x < 2\}$ list the elements of these sets:

a A **b** B **c** A' **d** B' **e** $A \cap B$ **f** $A \cup B$ **g** $A' \cap B$

h $A' \cup B'$

- a** $A = \{1, 2, 3, 4\}$
b $B = \{-3, -2, -1, 0, 1\}$
c $A' = \{-5, -4, -3, -2, -1, 0, 5\}$
d $B' = \{-5, -4, 2, 3, 4, 5\}$
e $A \cap B = \{1\}$
f $A \cup B = \{-3, -2, -1, 0, 1, 2, 3, 4\}$
g $A' \cap B = \{-3, -2, -1, 0\}$
h $A' \cup B' = \{-5, -4, -3, -2, -1, 0, 2, 3, 4, 5\}$

- 2** If $U = \{x \mid x \in Z, 0 \leq x \leq 8\}$, $A = \{x \mid x \in Z, 2 \leq x \leq 7\}$ and $B = \{x \mid x \in Z, 5 \leq x \leq 8\}$ list the elements of:
a A **b** A' **c** B **d** B'
e $A \cap B$ **f** $A \cup B$ **g** $A \cap B'$
- 3** If $n(U) = 15$, $n(P) = 6$ and $n(Q') = 4$ where P and Q' are subsets of U , find:
a $n(P')$ **b** $n(Q)$.
- 4** True or false?
a If $n(U) = a$ and $n(A) = b$ then $n(A') = b - a$.
b If Q is a subset of U then $Q' = \{x \mid x \notin Q, x \in U\}$.
- 5** If $U = \{x \mid 0 < x \leq 12, x \in Z\}$, $A = \{x \mid x \in Z, 2 \leq x \leq 7\}$, $B = \{x \mid 3 \leq x \leq 9, x \in Z\}$ and $C = \{x \mid x \in Z, 5 \leq x \leq 11\}$ list the elements of:
a B' **b** C' **c** A' **d** $A \cap B$
e $(A \cap B)'$ **f** $A' \cap C$ **g** $B' \cup C$ **h** $(A \cup C) \cap B'$

Example 4

If $U = \{\text{positive integers}\}$, $P = \{\text{multiples of 4 less than 50}\}$
 $Q = \{\text{multiples of 6 less than 50}\}$:

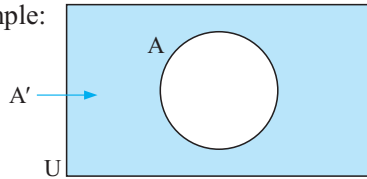
- a** list P and Q **b** find $P \cap Q$ **c** find $P \cup Q$.
d Verify that $n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$
- a** $P = \{4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48\}$
 $Q = \{6, 12, 18, 24, 30, 36, 42, 48\}$
b $P \cap Q = \{12, 24, 36, 48\}$
c $P \cup Q = \{4, 6, 8, 12, 16, 18, 20, 24, 28, 30, 32, 36, 40, 42, 44, 48\}$
d $n(P \cup Q) = 16$ and $n(P) + n(Q) - n(P \cap Q) = 12 + 8 - 4 = 16$
i.e., $n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$ is verified.

C VENN DIAGRAMS

Venn diagrams are diagrams used to represent sets of objects, numbers or things.

The universal set is usually represented by a rectangle whereas sets within it are usually represented by circles or ellipses.

For example:

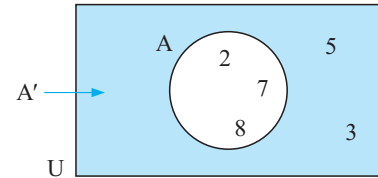


is a Venn diagram which shows set A within the universal set U.

A', the complement of A is shaded.

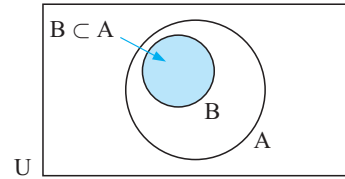
In this case U could be {2, 3, 5, 7, 8}, A = {2, 7, 8} and A' = {3, 5}.

These elements can be shown as:



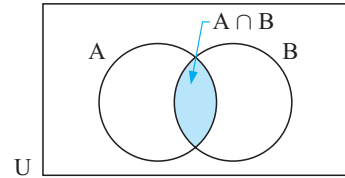
SUBSETS

As $B \subset A$ then B is contained within A as every element of B is also in A:



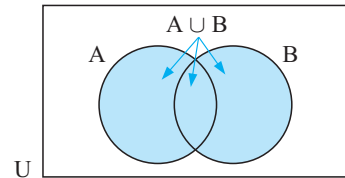
INTERSECTION

As $A \cap B$ consists of all elements common to both A and B, then $A \cap B$ is the shaded region:



UNION

As $A \cup B$ consists of all elements in A or B (or both A and B) then $A \cup B$ is the shaded region:

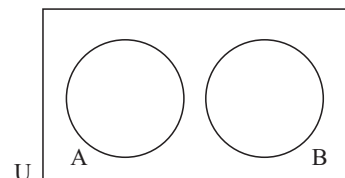


DISJOINT (OR MUTUALLY EXCLUSIVE) SETS

Disjoint sets do not have common elements.

For example, $A = \{2, 3, 8\}$ and $B = \{4, 5, 9\}$ i.e., $A \cap B = \emptyset$.

These are represented by non-overlapping circles.



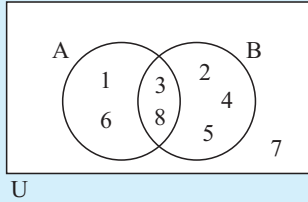
Example 5

Given that $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, illustrate on a Venn diagram the sets:

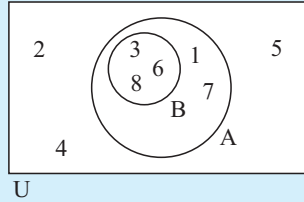
a $A = \{1, 3, 6, 8\}$ and $B = \{2, 3, 4, 5, 8\}$

b $A = \{1, 3, 6, 7, 8\}$ and $B = \{3, 6, 8\}$

a $A \cap B = \{3, 8\}$

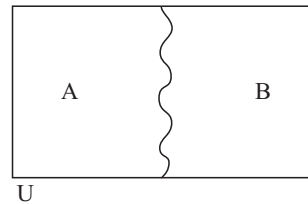


b $A \cap B = \{3, 6, 8\}$, $B \subset A$



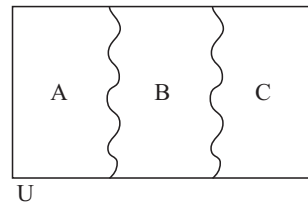
Note: • If two sets A and B are **disjoint** and **exhaustive** then $A \cap B = \emptyset$ and $U = A \cup B$.

We can represent this situation without using circles for the sets, as shown:



• Likewise if three sets A, B and C are **disjoint** and **mutually exclusive** then $A \cap B = \emptyset$, $B \cap C = \emptyset$, $A \cap C = \emptyset$ and $U = A \cup B \cup C$.

We can represent this situation:



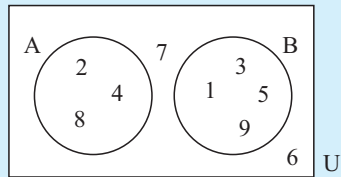
Example 6

Given $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, illustrate on a Venn diagram the sets:

a $A = \{2, 4, 8\}$ and $B = \{1, 3, 5, 9\}$

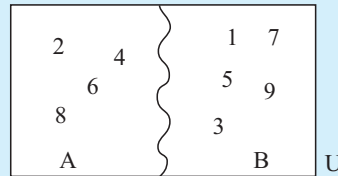
b $A = \{2, 4, 6, 8\}$ and $B = \{1, 3, 5, 7, 9\}$

a $A \cup B \neq U$



i.e., A and B are disjoint

b $A \cup B = U$



i.e., A and B are disjoint and mutually exclusive.

EXERCISE 3C

1 Represent sets A and B on a Venn diagram, given:

- a $U = \{2, 3, 4, 5, 6, 7\}, \quad A = \{2, 4, 6\}, \quad B = \{5, 7\}$
- b $U = \{2, 3, 4, 5, 6, 7\}, \quad A = \{2, 4, 6\}, \quad B = \{3, 5, 7\}$
- c $U = \{1, 2, 3, 4, 5, 6, 7\}, \quad A = \{2, 4, 5, 6\} \quad B = \{1, 4, 6, 7\}$
- d $U = \{3, 4, 5, 7\}, \quad A = \{3, 4, 5, 7\} \quad B = \{3, 5\}$

2 Given that $U = \{x \mid x \in Z, 1 \leq x \leq 10\}$,

$A = \{\text{odd numbers} < 10\}, \quad B = \{\text{primes} < 10\}:$

- a List sets A and B. b Find $A \cap B$ and $A \cup B$.
- c Represent the sets A and B on a Venn diagram.

3 Given that $U = \{\text{even numbers between 0 and 30}\}$,

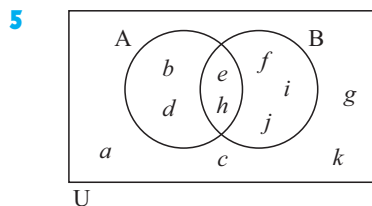
$A = \{\text{multiples of 4 less than 30}\}, \quad B = \{\text{multiples of 6 less than 30}\}:$

- a List sets A and B. b Find $A \cap B$ and $A \cup B$.
- c Represent the sets A and B on a Venn diagram.

4 Given that $U = \{x \mid x \in Z^+, x \leq 30\}$,

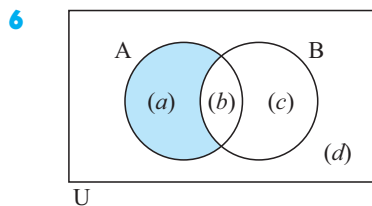
$R = \{\text{primes less than 30}\}, \quad S = \{\text{composites less than 30}\}:$

- a List sets R and S. b Find $R \cap S$ and $R \cup S$.
- c Represent the sets R and S on a Venn diagram.



List the letters in set:

- a A b B c A' d B'
- e $A \cap B$ f $A \cup B$ g $(A \cup B)'$
- h $A' \cup B'$

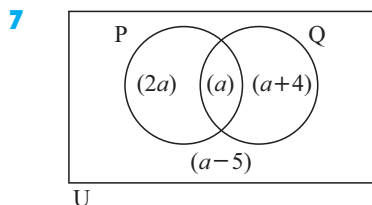


(a) means that there are a elements in the shaded region.

Notice that $n(A) = a + b$.

Find:

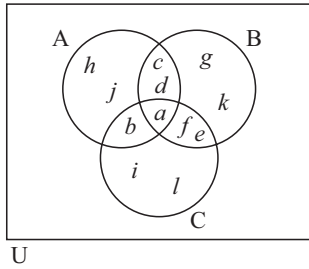
- a $n(B)$ b $n(A')$ c $n(A \cap B)$
- d $n(A \cup B)$ e $n((A \cap B)')$ f $n((A \cup B)')$



The Venn diagram shows us that $n(P \cap Q) = a$ and $n(P) = 3a$.

- a Find:
 - i $n(Q)$ ii $n(P \cup Q)$
 - iii $n(Q')$ iv $n(U)$
- b Find a if:
 - i $n(U) = 29$ ii $n(U) = 31$ Comment!

8



This Venn diagram consists of three overlapping circles A, B and C.

a List the letters in set:

- | | | |
|------------------------------|-------------------------------|----------------------|
| i A | ii B | iii C |
| iv $A \cap B$ | v $A \cup B$ | vi $B \cap C$ |
| vii $A \cap B \cap C$ | viii $A \cup B \cup C$ | |

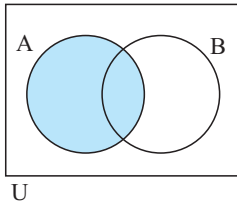
b Find:

- i** $n(A \cup B \cup C)$
ii $n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

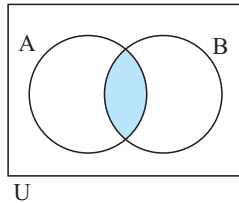
D

VENN DIAGRAM REGIONS

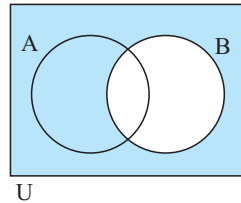
We use shading to show various sets being considered. For example, for two intersecting sets:



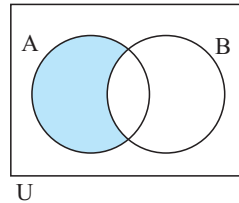
A is shaded



$A \cap B$ is shaded



B' is shaded

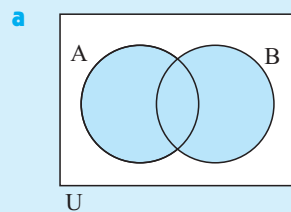


$A \cap B'$ is shaded

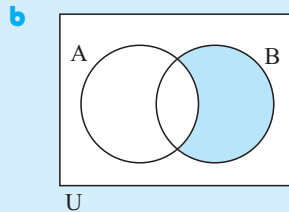
Example 7

On separate Venn diagrams shade these regions for two overlapping sets A and B:

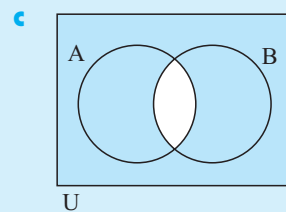
- a** $A \cup B$ **b** $A' \cap B$ **c** $(A \cap B)'$



(in A, B or both)



(outside A intersected with B)



(outside $A \cap B$)

Click on the icon to **find the shaded region** representing various subsets for two intersecting sets.



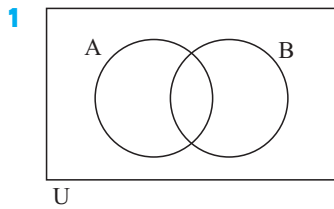
Click on the icon to **practice shading** regions representing various subsets. If you are correct you will be informed of this.



This demo has two parts:

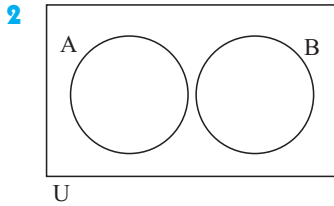
- two intersecting sets
- three intersecting sets.

EXERCISE 3D



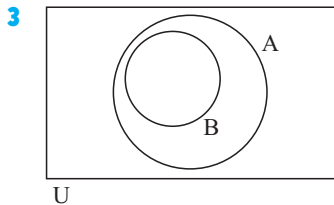
Draw separate Venn diagrams with shading of:

- a** $A \cap B$ **b** $A \cap B'$ **c** $A' \cup B$
d $A \cup B'$ **e** $(A \cap B)'$ **f** $(A \cup B)'$



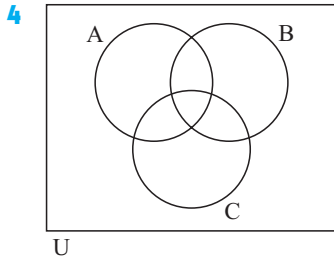
A and B are two disjoint sets. Shade on separate Venn diagrams:

- a** A **b** B **c** A'
d B' **e** $A \cap B$ **f** $A \cup B$
g $A' \cap B$ **h** $A \cup B'$ **i** $(A \cap B)'$



$B \subset A$ in the given Venn diagram. Shade on separate Venn diagrams:

- a** A **b** B **c** A'
d B' **e** $A \cap B$ **f** $A \cup B$
g $A' \cap B$ **h** $A \cup B'$ **i** $(A \cap B)'$

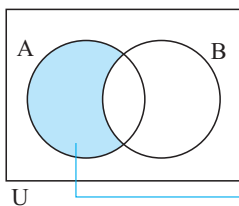


This Venn diagram consists of three overlapping circles (sets). On separate Venn diagrams shade:

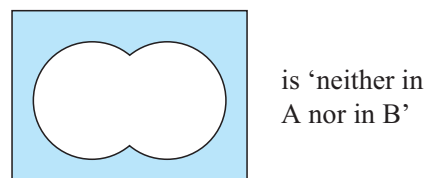
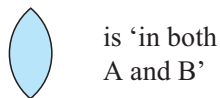
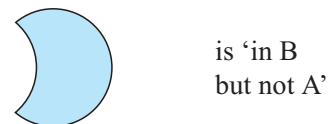
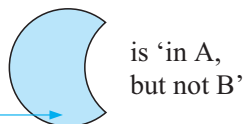
- a** A **b** B' **c** $B \cap C$
d $A \cup B$ **e** $A \cap B \cap C$ **f** $A \cup B \cup C$
g $(A \cap B \cap C)'$ **h** $(A \cup B) \cap C$ **i** $(B \cap C) \cup A$



E NUMBERS IN REGIONS



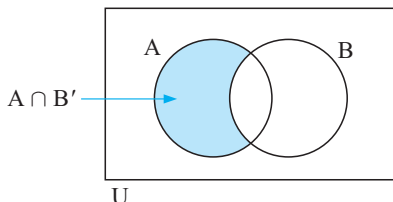
There are four regions on a Venn diagram which contains two overlapping sets A and B.



5 Given $n(U) = 50$, $n(S) = 30$, $n(R) = 25$ and $n(R \cup S) = 48$, find:

- a $n(R \cap S)$ b $n(S, \text{ but not } R)$

Note:



$$n(A \cap B') = n(A) - n(A \cap B)$$

$$n(A' \cap B) = n(B) - n(A \cap B)$$

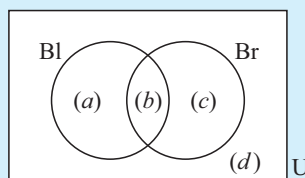
are useful identities which could be used in questions like 3, 4 and 5 above.

Example 10

A squash club has 27 members. 19 have black hair, 14 have brown eyes and 11 have both black hair and brown eyes.

- a Place this information on a Venn diagram.
 b Hence find the number of members with:
 i black hair or brown eyes ii black hair, but not brown eyes.

a Let Bl represent the black hair set and Br represent the brown eyes set.



$$a + b + c + d = 27$$

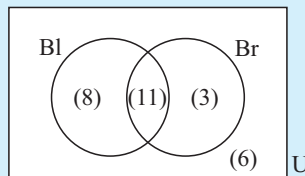
$$a + b = 19$$

$$b + c = 14$$

$$b = 11$$

$$\therefore a = 8, c = 3, d = 6$$

b i.e.,



- i $n(\text{Bl} \cup \text{Br}) = 8 + 11 + 3 = 22$
 ii $n(\text{Bl, but not Br}) = 8$

6 Pele has 14 cavies as pets. Five have long hair and 8 are brown. Two are both brown and have long hair.

- a Place this information on a Venn diagram.
 b Hence find the number of cavies that:
 i are short haired ii have short hair and are brown
 iii have short hair and are not brown.

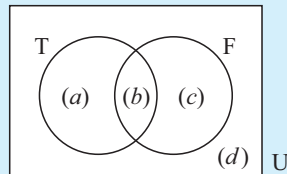
7 During a 2 week period, Murielle took her umbrella with her on 8 days, it rained on 9 days, and Murielle took her umbrella on five of the days when it rained.

- a Display this information on a Venn diagram.
 b Hence find the number of days that:
 i Murielle did not take her umbrella and it rained
 ii Murielle did not take her umbrella and it did not rain.

Example 11

A platform diving squad of 25 has 18 members who dive from 10 m and 17 who dive from 4 m. How many dive from both platforms?

Let T represent those who dive from 10 m and
 F represent those who dive from 4 m.



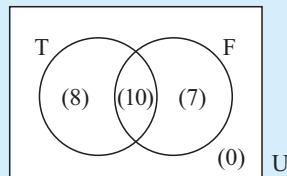
$d = 0$ {as all divers in the squad dive from 10 m or 4 m}

$$a + b = 18$$

$$b + c = 17 \quad \therefore c = 7, a = 8, b = 10$$

$$a + b + c = 25$$

i.e.,



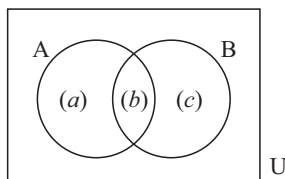
$$\begin{aligned} n(\text{both } T \text{ and } F) \\ &= n(T \cap F) \\ &= 10 \end{aligned}$$

- 8** A badminton club has 31 playing members. 28 play singles and 16 play doubles. How many play both singles and doubles?
- 9** In a factory, 56 people work on the assembly line. 47 work day shifts and 29 work night shifts. How many work both day shifts and night shifts?

In an earlier exercise we verified that:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

10



Use the figure given to show that:

$$n(A) + n(B) - n(A \cap B) = n(A \cup B).$$

Example 12 (The Opening Problem)

A city has three newspapers A, B and C. Of the adult population, 1% read none of these newspapers, 36% read A, 40% read B, 52% read C, 8% read A and B, 11% read B and C, 13% read A and C and 3% read all three papers. What percentage of the adult population read:

- a** newspaper A only **b** newspaper B or newspaper C
c newspaper A or B but not C.

From the given information, working in %'s

$$a = 3$$

$$a + d = 8 \quad \{8\% \text{ read A and B}\}$$

$$a + b = 11 \quad \{11\% \text{ read B and C}\}$$

$$a + c = 13 \quad \{13\% \text{ read A and C}\}$$

$$\therefore d = 5, \quad b = 8 \quad \text{and} \quad c = 10$$

$$g + 3 + 5 + 10 = 36 \quad \therefore g = 18$$

$$e + 3 + 5 + 8 = 40 \quad \therefore e = 24$$

$$f + 3 + 8 + 10 = 52 \quad \therefore f = 31$$

and $h = 1$

a $P(\text{reads A only}) = 18\%$ {shaded}

b $n(\text{B or C}) = n(\text{B} \cup \text{C})$
 $= 100\% - 1\% - 18\%$
 $= 81\%$

c $n(\text{A or B, but not C}) = 18\% + 5\% + 24\%$
 $= 47\%$

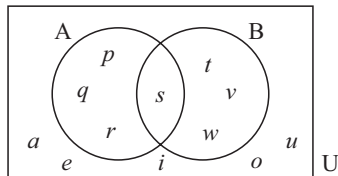
- 11** In a year group of 63 students, 22 study Biology, 26 study Chemistry and 25 study Physics. 18 study both Physics and Chemistry, four study both Biology and Chemistry and three study both Physics and Biology. One studies all three subjects. How many students study:
- a** Biology only
 - b** Physics or Chemistry
 - c** none of Biology, Physics or Chemistry
 - d** Physics but not Chemistry?
- 12** 36 students participated in the mid-year adventure trip. 19 students went paragliding, 21 went abseiling and 16 went white water rafting. 7 did abseiling and rafting, 8 did paragliding and rafting and 11 did paragliding and abseiling. 5 students did all three activities. Find the number of students who:
- a** went paragliding or abseiling
 - b** only went white water rafting
 - c** did not participate in any of the activities mentioned
 - d** did exactly two of the activities mentioned.
- 13** There were 32 students available for the woodwind section of the school orchestra. 11 students could play the flute, 15 could play the clarinet and 12 could play the saxophone. Two could play the flute and the saxophone, two could play the flute and the clarinet and 6 could play the clarinet and the saxophone. One student could play all three instruments. Find the number of students who could play:
- a** woodwind instruments other than the flute, clarinet and saxophone
 - b** only the saxophone
 - c** the saxophone and clarinet but not the flute
 - d** only one of the clarinet, saxophone or flute.

- 14** In a particular region, most farms had livestock and crops. A survey of 21 farms showed that 15 grew crops, 9 had cattle and 11 had sheep. Four had sheep and cattle, 7 had cattle and crops and 8 had sheep and crops. Three had cattle, sheep and crops. Two specialised in goats and did not grow crops. Find the number of farms with:
- a** only crops **b** only animals **c** one type of animal and crops.

REVIEW SET 3A

- 1** If $P = \{x \mid x \in Z, 2 \leq x < 9\}$:
- a** list the elements of P **b** find $n(P)$
- 2** **a** Write in set builder notation:
- i** the set of all integers greater than 3
ii the set of rational numbers between -5 and 5
- b** **i** Is the set in **a i** finite or infinite?
ii Is the set in **a ii** finite or infinite?
- 3** If $P = \{x \mid x \in R, 0 \leq x \leq 10\}$ and $Q = \{2, 5, 7\}$:
- a** Is $Q \subset P$? **b** Is $\{ \} \subset Q$?
- 4** If $U = \{x: x \in Z, 0 \leq x \leq 15\}$, $A = \{\text{multiples of 4 less than 16}\}$ and $B = \{\text{prime numbers less than 15}\}$, list the elements of:
- a** A **b** B **c** B' **d** $A \cup B$ **e** $A \cap B'$

5



- a** List the letters in:
- i** A **ii** B **iii** $A \cap B$
iv $(A \cup B)'$ **v** $A \cup B'$ **vi** $A' \cap B$
- b** Find: **i** $n(A \cup B)$ **ii** $n(A \cap B')$
- 6** Of the 13 people in an elevator, eight were company executives, 10 of the people wore suits, and 7 of the people wearing suits were company executives.
- a** Display this information on a Venn diagram.
b Hence find the number of people who:
- i** were not company executives and wore suits
ii were not company executives and did not wear suits.
- 7** All members of a class of 27 students participated in at least one event on sports day. 20 students participated in individual events and 12 students took part in team events. How many students took part in both individual and team events?
- 8** In a class of 24 students, 13 study economics, 13 study history and 12 study information technology (IT). Five study both economics and IT, six study both history and IT and six study both economics and history. Two study all three subjects. Find how many students study:
- a** economics or history **b** economics and history but not IT
c exactly one of economics, history or IT.

- 3 a $\frac{1}{5}$ b $\frac{3}{5}$ c $\frac{9}{10}$ d $\frac{3}{10}$
 4 a 70 km/h b 25 km/h c 76 km/h
 5 a 4500 rpm b 2750 rpm c 2500 rpm d 4250 rpm
 6 a 38.5°C b 36.8°C

EXERCISE 2H

- 1 a $\pm\frac{1}{2}$ mm b $\pm\frac{1}{2}$ kg c $\pm\frac{1}{2}$ cm d ± 50 mL
 2 a 154.5 cm to 155.5 cm b 209.5 cm to 210.5 cm
 c 188.5 cm to 189.5 cm
 3 a 52.5 kg to 53.5 kg b 94.5 kg to 95.5 kg
 c 78.5 kg to 79.5 kg
 4 a 26.5 mm to 27.5 mm b 38.25 cm to 38.35 cm
 c 4.75 m to 4.85 m d 1.45 kg to 1.55 kg
 e 24.5 g to 25.5 g f 3.745 kg to 3.755 kg
 5 a 6.4 m b 6.0 m c 10 cm graduations
 6 246 cm and 250 cm 7 between 788 cm and 792 cm
 8 a 55.25 cm^2 b 41.25 cm^2
 9 between 1058.25 cm^2 and 1126.25 cm^2
 10 between 31.875 cm^2 and 40.375 cm^2
 11 between 144.4 cm^3 and 248.6 cm^3
 12 between 1502.1 cm^3 and 1546.7 cm^3
 13 between 922.5 cm^3 and 1473.0 cm^3
 14 between 332.2 cm^3 and 347.7 cm^3

EXERCISE 2I

- 1 a i \$2460 ii 0.180% b i -467 people ii 1.48%
 c i \$1890 ii 0.413% d i 189 cars ii 6.72%
 2 a i 1.238 kg ii 19.8% b i 2.4 m ii 2.46%
 c i 3.8 L ii 16.0% d i 22 hours ii 30.6%
 3 a 100 m^2 b 99.91 m^2 c 0.09 m^2 d 0.090%
 4 a 3254.224 cm^3 b 3240 cm^3 c -14.224 cm^3 d 0.437%
 5 a 65.25 km/h b 4.75 km/h c 7.28%

REVIEW SET 2A

- 1 6 hours 17 minutes 2 a 3.35 am next day b 9.50 am
 3 9 pm 4 1188 pounds 5 95°F
 6 a 1392000 km b 0.0000084 m
 7 a $8.64 \times 10^5\text{ km}$ b $3.16 \times 10^8\text{ km}$
 8 a i 6.4 ii 6.38 b i 0.05 ii 0.047
 9 a 25.28 km/h b 5.583 hours (or 5 hours 35 minutes)
 10 a $\pm 0.5\text{ cm}$ b 35.5 cm to 36.5 cm
 c between 1260.25 cm^2 and 1332.25 cm^2
 11 a i $-\$590$ ii 22.8% b i -0.109 cm ii 0.417%

REVIEW SET 2B

- 1 165 minutes 2 11 hours 4 minutes
 3 2 am the next day 4 28.9°C 5 180.34 cm
 6 a 46000000000 b 1.9 c 0.0032
 7 a $1.276 \times 10^7\text{ m}$ b $4.2 \times 10^{-7}\text{ cm}$ 8 313 sheets
 9 a i 59.4 ii 59.40 b i 0.01 ii 0.0084
 10 a \$508.75 b \$14.54 per hour 11 \$230.80
 12 a -2.3 m b 6.71% 13 between 48 cm and 52 cm

EXERCISE 3A

- 1 a finite b infinite c infinite d infinite
 2 a i The set of all x such that x is an integer between -1 and 7 , including -1 .
 ii $A = \{-1, 0, 1, 2, 3, 4, 5, 6\}$ iii $n(A) = 8$
 iv no, A is finite

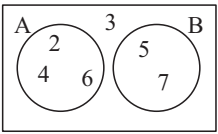
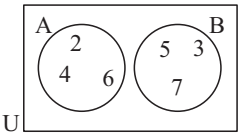
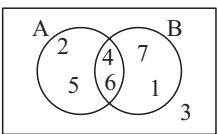
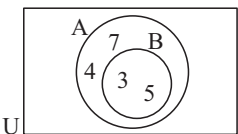
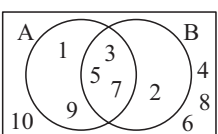
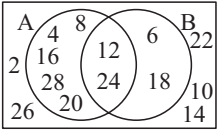
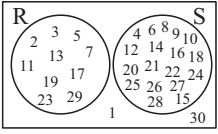
- b i The set of all x such that x is a natural number between -2 and 8 .
 ii $A = \{0, 1, 2, 3, 4, 5, 6, 7\}$ iii $n(A) = 8$
 iv no, A is finite
 c i The set of all x such that x is a real number between 0 and 1 , including both 0 and 1 .
 ii $A = \{\text{all real numbers between } 0 \text{ and } 1\}$
 iii $n(A) = \infty$ iv yes, A is infinite
 d i The set of all x such that x is a rational number between 5 and 6 , including both 5 and 6 .
 ii not possible iii $n(A)$ is infinite iv yes, A is infinite
 3 a $\{x \mid x \in \mathbb{Z}, -100 < x < 100\}$
 b $\{x \mid x \in \mathbb{R}, x > 1000\}$ c $\{x \mid x \in \mathbb{Q}, 2 \leq x \leq 3\}$
 4 a i $\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}$;
 8 of them
 ii $\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\},$
 $\{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\},$
 $\{b, c, d\}, \{a, b, c, d\}$; 16 of them
 b If a set has n elements it has 2^n subsets.
 5 a yes b no c yes d yes e no f no

EXERCISE 3B

- 1 a $C' = \{\text{all consonants}\}$ b $C' = \{\text{all positive integers}\}$
 c $C' = \{x \mid x \in \mathbb{Z}, x \geq -4\}$
 d $C' = \{x \mid x \in \mathbb{Q}, 2 < x < 8\}$
 2 a $A = \{2, 3, 4, 5, 6, 7\}$ b $A' = \{0, 1, 8\}$
 c $B = \{5, 6, 7, 8\}$ d $B' = \{0, 1, 2, 3, 4\}$
 e $A \cap B = \{5, 6, 7\}$ f $A \cup B = \{2, 3, 4, 5, 6, 7, 8\}$
 g $A \cap B' = \{2, 3, 4\}$
 3 a $n(P') = 9$ b $n(Q) = 11$ 4 a false b true
 5 a $B' = \{1, 2, 10, 11, 12\}$ b $C' = \{1, 2, 3, 4, 12\}$
 c $A' = \{1, 8, 9, 10, 11, 12\}$ d $A \cap B = \{3, 4, 5, 6, 7\}$
 e $(A \cap B)' = \{1, 2, 8, 9, 10, 11, 12\}$
 f $A' \cap C = \{8, 9, 10, 11\}$
 g $B' \cap C = \{1, 2, 5, 6, 7, 8, 9, 10, 11, 12\}$
 h $(A \cup C) \cap B' = \{2, 10, 11\}$
 6 a $P = \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$
 b $P \cap Q = \{2, 5, 11\}$
 c $P \cup Q = \{2, 3, 4, 5, 7, 11, 12, 13, 15, 17, 19, 23\}$
 d $n(P \cup Q) = 12$; $n(P) + n(Q) - n(P \cap Q) = 9 + 6 - 3 = 12$
 7 a $P = \{3, 6, 9, 12, 15, 18, 21, 24, 27\}$
 b $P \cap Q = \{12, 24\}$
 c $P \cup Q = \{3, 4, 6, 8, 9, 12, 15, 16, 18, 20, 21, 24, 27, 28\}$
 d $n(P \cup Q) = 14$; $n(P) + n(Q) - n(P \cap Q) = 9 + 7 - 2 = 14$
 8 a $M = \{32, 36, 40, 44, 48, 52, 56\}$, $N = \{36, 42, 48, 54\}$
 b $M \cup N = \{32, 36, 40, 42, 44, 48, 52, 54, 56\}$
 c $M \cap N = \{36, 48\}$
 d $n(M \cup N) = 11$; $n(M) + n(N) - n(M \cap N) = 7 + 4 - 2 = 9$
 9 a $R = \{-2, -1, 0, 1, 2, 3, 4\}$; $S = \{0, 1, 2, 3, 4, 5, 6\}$
 b $R \cap S = \{0, 1, 2, 3, 4\}$
 c $R \cup S = \{-2, -1, 0, 1, 2, 3, 4, 5, 6\}$
 d $n(R \cup S) = 9$, $n(R) + n(S) - n(R \cap S) = 7 + 7 - 5 = 9$
 10 a $C = \{-4, -3, -2, -1\}$
 $D = \{-7, -6, -5, -4, -3, -2, -1, 0\}$
 b $C \cap D = \{-4, -3, -2, -1\}$ (= C)
 c $C \cup D = \{-7, -6, -5, -4, -3, -2, -1, 0\}$ (= D)
 d $n(C \cup D) = 8$; $n(C) + n(D) - n(C \cap D) = 4 + 8 - 4 = 8$
 11 a $A = \{4, 8, 12, 16, 20, 24, 28, 32, 36\}$

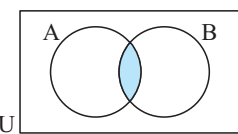
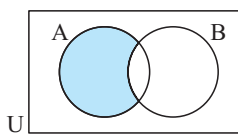
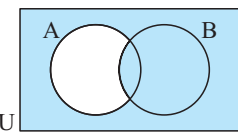
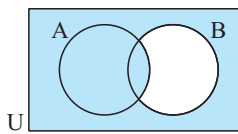
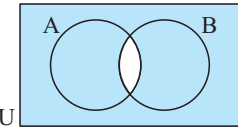
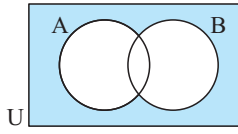
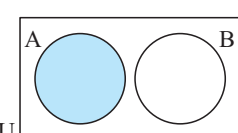
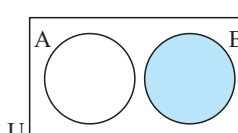
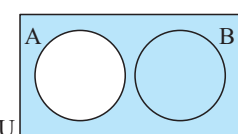
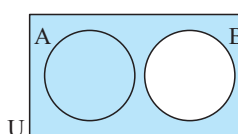
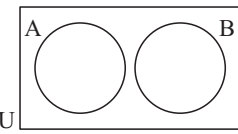
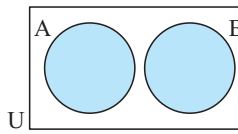
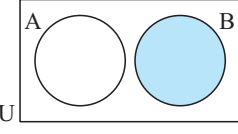
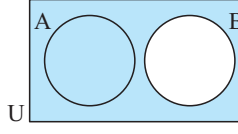
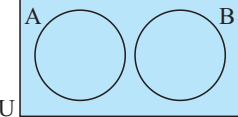
- $B = \{6, 12, 18, 24, 30, 36\}$; $C = \{12, 24, 36\}$
- b** **i** $A \cap B = \{12, 24, 36\}$ **ii** $B \cap C = \{12, 24, 36\}$
iii $A \cap C = \{12, 24, 36\}$ **iv** $A \cap B \cap C = \{12, 24, 36\}$
- c** $A \cup B \cup C = \{4, 6, 8, 12, 16, 18, 20, 24, 28, 30, 32, 36\}$
- d** $n(A \cup B \cup C) = 12$;
 $n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C) = 9 + 6 + 3 - 3 - 3 - 3 + 3 = 12$
- 12** **a** $A = \{6, 12, 18, 24, 30\}$; $B = \{1, 2, 3, 5, 6, 10, 15, 30\}$;
 $C = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$
- b** **i** $A \cap B = \{6, 30\}$ **ii** $B \cap C = \{2, 3, 5\}$
iii $A \cap C = \emptyset$ **iv** $A \cap B \cap C = \emptyset$
- c** $A \cup B \cup C = \{1, 2, 3, 5, 6, 7, 10, 11, 12, 13, 15, 17, 18, 19, 23, 24, 29, 30\}$
- d** $n(A \cup B \cup C) = 18$;
 $n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C) = 5 + 8 + 10 - 2 - 3 - 0 + 0 = 18$

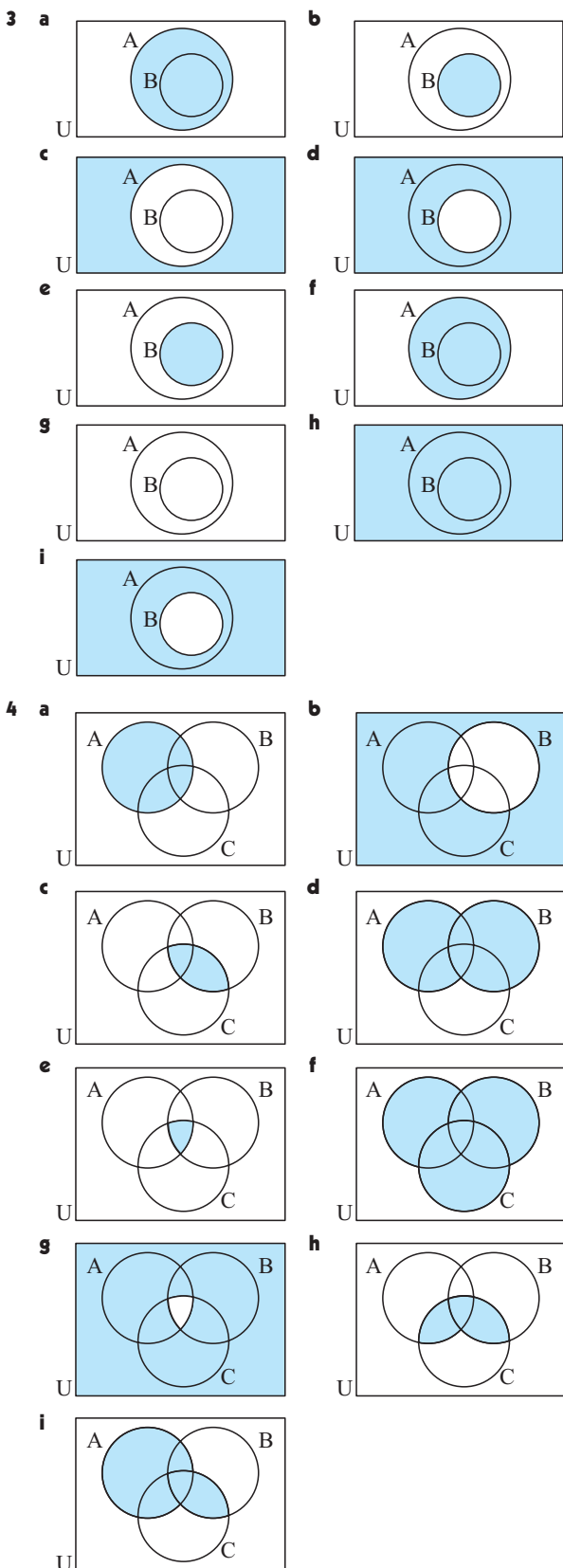
EXERCISE 3C

- 1** **a**  **b** 
- c**  **d** 
- 2** **a** $A = \{1, 3, 5, 7, 9\}$; $B = \{2, 3, 5, 7\}$
b $A \cap B = \{3, 5, 7\}$; $A \cup B = \{1, 2, 3, 5, 7, 9\}$
- c** 
- 3** **a** $A = \{4, 8, 12, 16, 20, 24, 28\}$; $B = \{6, 12, 18, 24\}$
b $A \cap B = \{12, 24\}$
 $A \cup B = \{4, 6, 8, 12, 16, 18, 20, 24, 28\}$
- c** 
- 4** **a** $R = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$
 $S = \{4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28\}$
- b** $R \cap S = \emptyset$
 $R \cup S = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29\}$
 $= \{x \mid x \in \mathbb{Z}^+, 1 < x < 30\}$
- c** 
- 5** **a** $A = \{b, d, e, h\}$ **b** $B = \{e, f, h, i, j\}$
c $A' = \{a, c, f, g, i, j, k\}$ **d** $B' = \{a, b, c, d, g, k\}$

- e** $A \cap B = \{e, h\}$ **f** $A \cup B = \{b, d, e, f, h, i, j\}$
g $(A \cup B)' = \{a, c, g, k\}$
h $A' \cup B' = \{a, b, c, d, f, g, i, j, k\}$
- 6** **a** $n(B) = b + c$ **b** $n(A') = c + d$ **c** $n(A \cap B) = b$
d $n(A \cup B) = a + b + c$ **e** $n((A \cap B)') = a + c + d$
f $n((A \cup B)') = d$
- 7** **a** **i** $n(Q) = 2a + 4$ **ii** $n(P \cup Q) = 4a + 4$
iii $n(Q') = 3a - 5$ **iv** $n(U) = 5a - 1$
- b** **i** $a = 6$
ii $n(U) = 31$ gives $a = \frac{32}{5}$ but $a \in \mathbb{Z}^+ \therefore$ no solution
- 8** **a** **i** $A = \{a, b, c, d, h, j\}$ **ii** $B = \{a, c, d, e, f, g, k\}$
iii $C = \{a, b, e, f, i, l\}$ **iv** $A \cap B = \{a, c, d\}$
v $A \cup B = \{a, b, c, d, e, f, g, h, j, k\}$
vi $B \cap C = \{a, e, f\}$ **vii** $A \cap B \cap C = \{a\}$
viii $A \cup B \cup C = \{a, b, c, d, e, f, g, h, i, j, k, l\}$
- b** **i** 12 **ii** 12

EXERCISE 3D

- 1** **a**  **b** 
- c**  **d** 
- e**  **f** 
- 2** **a**  **b** 
- c**  **d** 
- e**  **f** 
- g**  **h** 
- i** 



EXERCISE 3E

- 1** a 7 b 14 c 14 d 7 e 5 f 9
2 a 5 b 6 c 17 d 8 e 3 f 2
3 a 15 b 4 c 4 a 18 b 6 c 5 a 7 b 23
6 a where LH represents long hair and Br represents brown
b i 9 ii 6 iii 3
7 a where Um represents taking umbrella and R represents rain
b i 4 days ii 2 days
8 13 members **9** 20 people
10 $n(A) + n(B) + n(A \cap B) = a + b + b + c - b$
 $= a + b + c$
 $= n(A \cup B)$
11 a 16 b 33 c 14 d 7
12 a 29 b 6 c 1 d 11
13 a 3 b 5 c 5 d 21 **14** a 3 b 6 c 9

REVIEW SET 3A

- 1** a $P = \{2, 3, 4, 5, 6, 7, 8\}$ b $n(P) = 7$
2 a i $\{x \mid x \in \mathbb{Z}, x > 3\}$ ii $\{x \mid x \in \mathbb{Q}, -5 \leq x \leq 5\}$
b i infinite ii infinite
3 a yes, $Q \subset P$ b yes, $\{ \} \subset Q$
4 a $A = \{4, 8, 12\}$ b $B = \{2, 3, 5, 7, 11, 13\}$
c $B' = \{0, 1, 4, 6, 8, 9, 10, 12, 14, 15\}$
d $A \cup B = \{2, 3, 4, 5, 7, 8, 11, 12, 13\}$
e $A \cap B' = \{4, 8, 12\}$
5 a i $A = \{p, q, r, s\}$ ii $B = \{s, t, v, w\}$
iii $A \cap B = \{s\}$ iv $(A \cup B)' = \{a, e, i, o, u\}$
v $A \cup B' = \{a, e, i, o, p, q, r, s, u\}$
vi $A' \cap B = \{t, v, w\}$
b i $n(A \cup B) = 7$ ii $n(A \cap B) = 3$
6 a where CE represents company executive and S represents wearing suit
b i 3 ii 2
7 5 students **8** a 20 b 4 c 10

REVIEW SET 3B

- 1** a The set of all x such that x is a negative number between -4 and 0 .
b $A = \{-3, -2, -1\}$ c A is finite
2 yes, $T \subset S$
3 a $P = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$; $Q = \{4, 8, 12, 16\}$
b $P \cap Q = \{4, 8, 12, 16\}$
c $P \cup Q = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$ d yes, $Q \subset P$
e $n(P \cup Q) = 9$; $n(P) + n(Q) - n(P \cap Q) = 9 + 4 - 4 = 9$
4 a