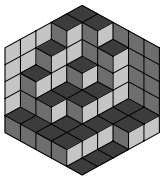

What is Problem Solving?

by Richard Rusczyk

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I was invited to the Math Olympiad Summer Program (MOP) in the 10th grade. I went to MOP certain that I must really be good at math. In my five weeks at MOP, I encountered over sixty problems on various tests. I didn't solve a single one. That's right - I was 0-for-60+. I came away no longer confident that I was good at math. I assumed that most of the other kids did better at MOP because they knew more tricks than I did. My formula sheets were pretty thorough, but perhaps they were missing something. By the end of MOP, I had learned a somewhat unsettling truth. The others knew fewer tricks than I did, not more. They didn't even have formula sheets!

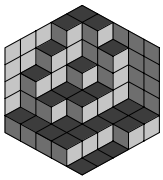
At another contest later that summer, a younger student, Alex, from another school asked me for my formula sheets. In my local and state circles, students' formula sheets were the source of knowledge, the source of power that fueled the top students and the top schools. They were studied, memorized, revered. But most of all, they were not shared. But when Alex asked for my formula sheets I remembered my experience at MOP and I realized that formula sheets are not really math. Memorizing formulas is no more mathematics than memorizing dates is history or memorizing spelling words is literature. I gave him the formula sheets. (Alex must later have learned also that the formula sheets were fool's gold - he became a Rhodes scholar.)

The difference between MOP and many of these state and local contests I participated in was the difference between problem solving and what many people call mathematics. For these people, math is a series of tricks to use on a series of specific problems. Trick A is for Problem A, Trick B for Problem B, and so on. In this vein, school can become a routine of 'learn tricks for a week - use tricks on a test - forget most tricks quickly.' The tricks get forgotten quickly primarily because there are so many of them, and also because the students don't see how these 'tricks' are just extensions of a few basic principles.

I had painfully learned at MOP that true mathematics is not a process of memorizing formulas and applying them to problems tailor-made for those formulas. Instead, the successful mathematician possesses fewer tools, but knows how to apply them to a much broader range of problems. We use the term "problem solving" to distinguish this approach to mathematics from the 'memorize-use-forget' approach.

After MOP I relearned math throughout high school. I was unaware that I was learning much more. When I got to Princeton I enrolled in organic chemistry. There were over 200 students in the course, and we quickly separated into two groups. One group understood that all we would be taught could largely be derived from a very small number of basic principles. We loved the class - it was a year long exploration of where these fundamental concepts could take us. The other, much larger, group saw each new destination not as the result of a path from the building blocks, but as yet another place whose coordinates had to be memorized if ever they were to visit again. Almost to a student, the difference between those in the happy group and those in the struggling group was how they learned mathematics. The class seemingly involved no math at all, but those who took a memorization approach to math were doomed to do it again in chemistry. The skills the problem solvers developed in math transferred, and these students flourished.

We use math to teach problem solving because it is the most fundamental logical discipline. Not only is it the foundation upon which sciences are built, it is the clearest way to learn and understand how to develop a rigorous logical argument. There are no loopholes, there are no half-truths. The



language of mathematics is precise, as is 'right' and 'wrong' (or 'proven' and 'unproven'). Success and failure are immediate and indisputable; there isn't room for subjectivity. This is not to say that those who cannot do math cannot solve problems. There are many paths to strong problem solving skills. Mathematics is the shortest.

Problem solving is crucial in mathematics education because it transcends mathematics. By developing problem solving skills, we learn not only how to tackle math problems, but also how to logically work our way through any problems we may face. The memorizer can only solve problems he has encountered already, but the problem solver can solve problems she's never seen before. The problem solver is flexible; she can diversify. Above all, she can create.