5 Factoring Trinomials—Two Differences

You know that when you want to factor a positive number like 15, you might also consider the negative factors as well as the positive ones.

$15 = 3 \times 5$ but also $15 = (-3) \times (-5)$

In this section you will use this idea to factor some trinomials.

EXAMPLE 1 Factor $x^2 - 8x + 15$.

Step 1: $x^2 - 8x + 15 = (\quad)(\quad)$

Step 2: $x^2 - 8x + 15 = (x \quad)(x \quad)$

Step 3: $x^2 - 8x + 15 = (x\ ?\ )(x\ ?\ )$

What two negative numbers have a product of 15?

−1 and −15 −3 and −5

Which of the pairs of factors of 15 have a sum of −8?

−3 and −5

Step 4: $x^2 - 8x + 15 = (x - 3)(x - 5)$

Check: $(x - 3)(x - 5) = x^2 - 8x + 15$ ✓

EXAMPLE 2 Factor $n^2 - 10n + 16$.

Step 1: $n^2 - 10n + 16 = (\quad)(\quad)$

Step 2: $n^2 - 10n + 16 = (n \quad)(n \quad)$

Step 3: $n^2 - 10n + 16 = (n\ ?\ )(n\ ?\ )$

You have to find two numbers whose product is 16 and whose sum is −10.

−8 and −2

Step 4: $n^2 - 10n + 16 = (n - 8)(n - 2)$

Check: $(n - 8)(n - 2) = n^2 - 10n + 16$ ✓

Classroom Practice

Tell whether the factors will be sums or differences. Factor.

1. $x^2 - 6x + 9$
2. $x^2 + 2x + 1$
3. $y^2 + 3y + 2$
4. $n^2 - 7n + 10$
5. $n^2 - 10n + 24$
6. $n^2 + 5n + 6$
7. $c^2 + 7c + 6$
8. $x^2 - 4x + 4$
9. $a^2 - 9a + 20$

Written Exercises

Complete.

A

	If the product of two numbers is:	→ and their sum is: →	then the two numbers are:
1.	6	-5	?
2.	10	-7	?
3.	18	-9	?
4.	12	-7	?
5.	24	-10	?

Factor.

6. $n^2 - 3n + 2$
7. $x^2 - 7x + 10$
8. $y^2 - 8y + 12$
9. $x^2 - 5x + 6$
10. $n^2 - 10n + 21$
11. $y^2 - 6y + 5$
12. $b^2 - 2b + 1$
13. $x^2 - 11x + 24$
14. $x^2 - 13x + 30$
15. $n^2 + 9n + 18$
16. $n^2 - 11n + 18$
17. $n^2 - 5n + 4$
18. $a^2 - 9a + 14$
19. $x^2 + 6x + 8$
20. $x^2 - 7x + 12$
21. $y^2 - 11y + 28$
22. $x^2 - 12x + 27$
23. $n^2 + 10n + 25$
24. $n^2 - 11n + 24$
25. $y^2 - 11y + 30$
26. $r^2 - 14r + 33$
27. $x^2 - 14x + 49$
28. $a^2 - 12a + 36$
29. $m^2 - 13m + 40$
30. $a^2 - 4a + 4$
31. $z^2 - 18z + 32$
32. $x^2 - 11x + 30$
33. $y^2 - 15y + 36$
34. $b^2 - 12b + 36$
35. $d^2 - 14d + 45$
36. $c^2 - 24c + 80$
37. $x^2 - 52x + 100$
38. $n^2 - 30n + 200$

6 Factoring Trinomial Squares

When the two factors of a trinomial are equal, the trinomial is a *trinomial square.*

This is a trinomial square because it is the product of two equal factors.

$$x^2 + 6x + 9 = (x + 3)(x + 3) = (x + 3)^2$$

Take a look at trinomial squares. Learn how to recognize them.

square of x · square of 3 → $x^2 + 6x + 9$ ← twice $3 \cdot x$

square of a · square of b → $a^2 + 2ab + b^2$ ← twice $a \cdot b$

So, $x^2 + 6x + 9 = (x + 3)^2$. So, $a^2 + 2ab + b^2 = (a + b)^2$.

EXAMPLE 1 Is $x^2 + 8x + 16$ a trinomial square? See if the terms meet the test.

square of x · square of 4 → $x^2 + 8x + 16$ ← twice $4 \cdot x$

Yes, $x^2 + 8x + 16 = (x + 4)^2$.

EXAMPLE 2 Is $a^2 + 5a + 6$ a trinomial square?

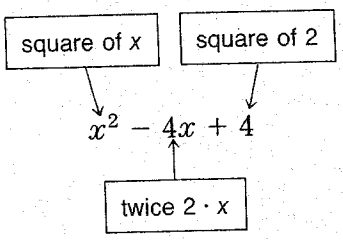
square of a · Square of an integer? No! → $a^2 + 5a + 6$

No, $a^2 + 5a + 6$ is not a trinomial square.

Of course, if you forget how to recognize a trinomial square, you can factor using the method you learned in the last two sections.

Sometimes a trinomial is the square of a difference. In that case, the middle term will have a minus sign.

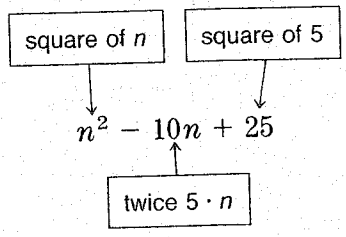
EXAMPLE 3 Is $x^2 - 4x + 4$ a trinomial square?



Yes, $x^2 - 4x + 4 = (x - 2)^2$.

Use − because the middle term in the trinomial is negative.

EXAMPLE 4 Is $n^2 - 10n + 25$ a trinomial square?



Yes, $n^2 - 10n + 25 = (n - 5)^2$.

Written Exercises

Which of the following are trinomial squares?

A

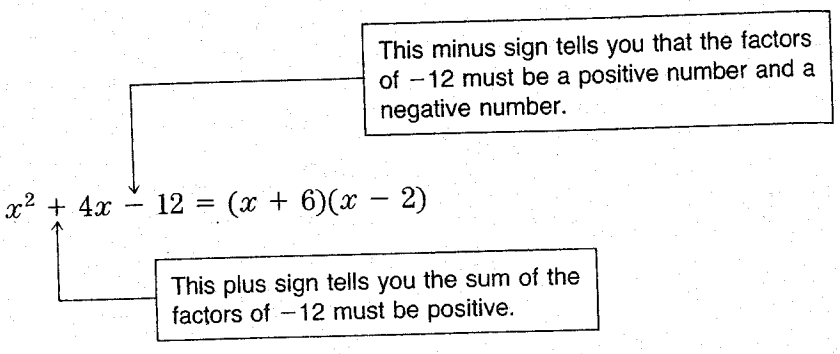
1. $x^2 + 2x + 1$
2. $y^2 + 4y + 4$
3. $a^2 + 6a + 9$
4. $y^2 + y + 4$
5. $y^2 + 4y + 1$
6. $y^2 + 2y + 4$

Factor. Note that four exercises are not trinomial squares. Factor those as you did in the last sections.

7. $a^2 + 10a + 25$
8. $n^2 + 4n + 4$
9. $x^2 + 8x + 16$
10. $n^2 + 12n + 36$
11. $x^2 - 4x + 4$
12. $a^2 - 9a + 20$
13. $y^2 + 14y + 49$
14. $x^2 - 20x + 100$
15. $n^2 + 18n + 81$
16. $x^2 + 15x + 56$
17. $x^2 - 19x + 90$
18. $b^2 - 15b + 36$
19. $a^2 - 2ab + b^2$
20. $m^2n^2 + 18mn + 81$
21. $1 - 20x + 100x^2$

7 Factoring Trinomials

Here is a trinomial having one factor a sum and one factor a difference. Notice how the signs help you discover the factors.



EXAMPLE 1 Factor $n^2 + 4n - 21$.

Step 1: $n^2 + 4n - 21 = (\quad)(\quad)$

Step 2: $n^2 + 4n - 21 = (n \quad)(n \quad)$

Step 3: $n^2 + 4n - 21 = (n\ ?\ )(n\ ?\ )$

Look for factors of -21 whose sum is $+4$.

$3 \times -7 = -21$ and $3 + (-7) = -4$

$-3 \times 7 = -21$ and $-3 + 7 = 4$ ✓

Step 4: $n^2 + 4n - 21 = (n - 3)(n + 7)$

EXAMPLE 2 Factor $x^2 - 2x - 15$.

Step 1: $x^2 - 2x - 15 = (\quad)(\quad)$

Step 2: $x^2 - 2x - 15 = (x \quad)(x \quad)$

Step 3: $x^2 - 2x - 15 = (x\ ?\ )(x\ ?\ )$

Look for factors of -15 whose sum is -2.

$-3 \times 5 = -15$ and $-3 + 5 = 2$

$3 \times -5 = -15$ and $3 + (-5) = -2$ ✓

Step 4: $x^2 - 2x - 15 = (x + 3)(x - 5)$

In the next example, notice that -24 has many different factors. You may have to check quite a few of these before you find the right ones.

EXAMPLE 3 Factor $x^2 - 2x - 24$.

Step 1: $x^2 - 2x - 24 = (\quad)(\quad)$

Step 2: $x^2 - 2x - 24 = (x \quad)(x \quad)$

Step 3: $x^2 - 2x - 24 = (x\ ?\ )(x\ ?\ )$

Look for factors of -24 whose sum is -2.

$2 \times -12 = -24$ and $2 + (-12) = -10$
$3 \times -8 = -24$ and $3 + (-8) = -5$
$4 \times -6 = -24$ and $4 + (-6) = -2$ ✓

Step 4: $x^2 - 2x - 24 = (x + 4)(x - 6)$

It is always wise to check your answer by multiplying.

Classroom Practice

Name the possible pairs of factors.

1. -6
2. -5
3. -10
4. -8
5. -7
6. -12
7. -16
8. -20
9. -36
10. -24

Factor.

11. $a^2 + 5a - 14$
12. $x^2 + x - 6$
13. $y^2 + 2y - 8$
14. $n^2 - 3n - 10$
15. $x^2 - 6x - 16$
16. $n^2 - 4n - 5$
17. $x^2 + 4x - 12$
18. $a^2 - 3a - 40$
19. $x^2 + 5x - 24$
20. $y^2 + 6y - 16$
21. $b^2 - 5b - 24$
22. $m^2 + 8m - 33$
23. $x^2 + x - 30$
24. $y^2 + y - 56$
25. $n^2 - 2n - 63$
26. $x^2 - 2x - 8$
27. $x^2 + 2x - 15$
28. $a^2 + a - 20$
29. $y^2 - 5y - 36$
30. $n^2 + 2n - 35$
31. $x^2 - 4x - 32$
32. $x^2 - 3x - 18$
33. $w^2 + 4w - 12$
34. $z^2 + 10z - 39$
35. $y^2 - 2y - 48$
36. $r^2 + 6r - 55$
37. $m^2 - 4m - 60$

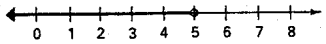
Written Exercises

Complete.

	If the product of two numbers is:	→ and their sum is:	→ then the two numbers are:
A 1.	−10	−3	?
2.	−18	7	?
3.	−4	−3	?
4.	−21	−4	?
5.	−2	−1	?
6.	−6	1	?
7.	−30	−7	?
8.	−30	−1	?
9.	−28	3	?
10.	−15	2	?
11.	−14	13	?
12.	−24	−2	?

Factor.

13. $x^2 + 4x - 21$
14. $x^2 - 3x - 4$
15. $x^2 + 7x - 18$
16. $x^2 - 6x - 16$
17. $b^2 + b - 12$
18. $b^2 - 4b - 12$
19. $n^2 - 3n - 18$
20. $y^2 - 9y - 10$
21. $x^2 - x - 20$
22. $a^2 + 3a - 10$
23. $y^2 + 14y - 15$
24. $n^2 - 3n - 28$
25. $b^2 + 2b - 24$
26. $x^2 - x - 30$
27. $b^2 - 7b - 30$
28. $a^2 - 9a - 22$
29. $x^2 + 12x - 28$
30. $a^2 + 11a - 26$
31. $y^2 + 4y - 32$
32. $x^2 - 5x - 36$
33. $y^2 + 6y - 27$
34. $x^2 + 9x - 36$
35. $m^2 + 12m - 64$
36. $x^2 - x - 72$
37. $n^2 - 2n - 63$
38. $b^2 + 10b - 24$
39. $y^2 + y - 42$
40. $x^2 - 4x - 45$
41. $y^2 - y - 56$
42. $z^2 + 8z - 65$
43. $c^2 + 16c - 80$
44. $m^2 - 24m - 81$
45. $a^2 + 21a - 10$

15. 

17. $25a^2 - 10a + 1$ **19.** $a = 4$
21. $-12x^4y^6$ **23.** $x^3 - x^2 - 10x + 12$
25. $-7 < 0 < 2$ **27.** $(x^3 + 2x^2)$ cubic units **29.** $45m^7n^6$ **31.** 15 cm
33. 0, 1, 2, 3 **35.** $>$ **37.** $a = 2$
39. $h = 9$

CHAPTER 6

Written Exercises, page 187
1. $2 \cdot 3 \cdot 7$ **3.** $2 \cdot 2 \cdot 3 \cdot 3$ **5.** $2 \cdot 5 \cdot 31$
7. $2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$ **9.** $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 13$
11. 3 **13.** 7 **15.** 1 **17.** 11 **19.** 16
21. 12 **23.** 14 **25.** 25 **27.** 4 **29.** 8
31. 4 **33.** 20 **35.** 6 **37.** 100

Written Exercises, page 189
1. $3(3 + x)$ **3.** $2(x - 5)$ **5.** $x(3x - 1)$
7. $2x(x - 3)$ **9.** $7n(4n - 1)$
11. $4x(x - 2)$ **13.** $5mn(5 - mn)$
15. $9x^2(1 - 3y)$ **17.** $3(x^2 - 2x + 7)$
19. $11(5y^2 + 2y + 4)$
21. $4(a^2 + 3ab - 4b^2)$
23. $6(x^2 + x + 4xy + 7)$
25. $25(-2a^2 + b^2 + 3ab)$
27. $8xy(7x^2y^2 - 9xy - 8)$
29. $(x + 2)(a - b)$ **31.** $(n + y)(m - x)$
33. $(2 - a)(a + b)$ **35.** $(x^2 - 4)(y + 2)$

Written Exercises, page 191
1. $(4 - \pi)r^2$ **3.** $(a^2 - b^2)\pi$ **5.** 72
7. $x^2(1 - \pi) + 5(x + 3)$

Written Exercises, pages 194–195
1. 1, 5 **3.** 2, 4 **5.** 3, 4 **7.** 3, 5 **9.** 3, 6
11. 3, 7 **13.** 3, 8 **15.** 6, 6
17. $(a + 6)(a + 1)$ **19.** $(x + 6)(x + 3)$
21. $(r + 3)(r + 7)$ **23.** $(n + 1)(n + 16)$
25. $(a + 1)(a + 4)$ **27.** $(x + 1)(x + 10)$
29. $(n + 4)(n + 2)$ **31.** $(x + 7)(x + 2)$
33. $(y + 1)(y + 11)$ **35.** $(b + 3)(b + 9)$
37. $(x + 4)(x + 8)$ **39.** $(x + 3)(x + 10)$
41. $(a + 2)(a + 10)$ **43.** $(x + 25)(x + 2)$
45. $(m + 25)(m + 3)$ **47.** $(a + 21)(a + 3)$

Written Exercises, page 197
1. $-3, -2$ **3.** $-6, -3$ **5.** $-6, -4$
7. $(x - 5)(x - 2)$ **9.** $(x - 3)(x - 2)$
11. $(y - 5)(y - 1)$ **13.** $(x - 3)(x - 8$
15. $(n + 6)(n + 3)$ **17.** $(n - 4)(n -$
19. $(x + 4)(x + 2)$ **21.** $(y - 7)(y -$
23. $(n + 5)(n + 5)$ **25.** $(y - 6)(y$
27. $(x - 7)(x - 7)$ **29.** $(m - 5)(m - 8)$
31. $(z - 2)(z - 16)$ **33.** $(y - 3)(y - 12)$
35. $(d - 5)(d - 9)$ **37.** $(x - 2)(x - 50)$

Written Exercises, page 199
1. yes **3.** yes **5.** no **7.** $(a + 5)^2$
9. $(x + 4)^2$ **11.** $(x - 2)^2$ **13.** $(y + 7)^2$
15. $(n + 9)^2$ **17.** $(x - 9)(x - 10)$
19. $(a - b)^2$ **21.** $(1 - 10x)^2$

Written Exercises, page 202
1. $-5, 2$ **3.** $-4, 1$ **5.** $-2, 1$ **7.** $-10, 3$
9. $7, -4$ **11.** $14, -1$ **13.** $(x + 7)(x - 3)$
15. $(x + 9)(x - 2)$ **17.** $(b - 3)(b + 4)$
19. $(n - 6)(n + 3)$ **21.** $(x - 5)(x + 4)$
23. $(y - 1)(y + 15)$ **25.** $(b + 6)(b - 4)$
27. $(b - 10)(b + 3)$ **29.** $(x + 14)(x - 2)$
31. $(y + 8)(y - 4)$ **33.** $(y + 9)(y - 3)$
35. $(m - 4)(m + 16)$ **37.** $(n - 9)(n + 7)$
39. $(y - 6)(y + 7)$ **41.** $(y + 7)(y - 8)$
43. $(c - 4)(c + 20)$ **45.** $(a - 4)(a + 25)$

Mixed Practice Exercises, page 203
1. $(x + 9)(x + 1)$ **3.** $(n - 6)(n - 2)$
5. $(y - 3)(y + 4)$ **7.** $(y - 7)(y + 3)$
9. $(x + 3)(x + 7)$ **11.** $(b - 1)(b + 5)$
13. $(b - 7)(b - 1)$ **15.** $(y - 4)(y + 5)$
17. $(x + 7)(x + 5)$ **19.** $(n - 7)(n - 8)$
21. $(x + 9)(x + 6)$ **23.** $(n + 9)(n + 5)$
25. $(n - 10)(n + 5)$ **27.** $(y + 26)(y - 2)$
29. $(a + 4)(a - 11)$ **31.** $(c - 6)(c + 10)$
33. $(m - 4)(m - 10)$ **35.** $4x - 4$
37. $4y - 4$

Written Exercises, page 205
1. $n^2 - 49$ **3.** $a^2 - 100$ **5.** $m^2 - n^2$
7. $x^2 - y^2$ **9.** $1 - 4x^2$ **11.** $9x^2 - 4$
13. $25y^2 - 4$ **15.** $81m^2 - n^2$
17. $25x^2 - 36$ **19.** $25x^2 - 9y^2$
21. $100x^2 - 25y^2$ **23.** $a^4 - 4$ **25.** 399
27. 2496 **29.** 896 **31.** 8091 **33.** 864
35. 3575 **37.** 6384 **39.** 2496