BRWN-B

## 2 Common Monomial Factors

Now you can find common factors of variable expressions.

$2a = 2 \cdot a$
$6a^2 = 2 \cdot 3 \cdot a \cdot a$

$2 \cdot a$, or $2a$, is the greatest common factor.

Use this idea to find a common monomial factor of the terms of a polynomial. When you do this, you **factor** the polynomial.

$$2a + 6a^2 = (2a \cdot 1) + (2a \cdot 3a)$$
$$= 2a(1 + 3a)$$

This is a common monomial factor of $2a$ and $6a^2$.

*Check:* Use the distributive property. → $2a(1 + 3a) = 2a + 6a^2$ ✓

**EXAMPLE 1** Factor $3x^2 - ax^2$.

$$3x^2 - ax^2 = (3 \cdot x^2) - (a \cdot x^2)$$
$$= x^2(3 - a)$$

With practice you will be able to recognize the common monomial factors at sight.

**EXAMPLE 2**

$$12mx - 6m^2x^2 = 6mx(2 - mx)$$

$6mx \cdot 2$ $\quad$ $6mx \cdot mx$

**EXAMPLE 3**

$$x^3 + x^2 + 2x = x(x^2 + x + 2)$$

$x \cdot x^2$ $\quad$ $x \cdot x$ $\quad$ $2 \cdot x$

**EXAMPLE 4**

$$y^4 + y^3 + 4y^2 = y^2(y^2 + y + 4)$$

$y^2 \cdot y^2$ $\quad$ $y^2 \cdot y$ $\quad$ $4 \cdot y^2$

# Classroom Practice

**Complete.**

1. $4xy = 4x(\_\_?\_\_)$
2. $25x^2 = 5x(\_\_?\_\_)$
3. $3x^2y^2 = 3xy(\_\_?\_\_)$
4. $36a^3 = 12a^2(\_\_?\_\_)$
5. $72b^2 = 8b(\_\_?\_\_)$
6. $40a^4b^4 = 4a^2b^2(\_\_?\_\_)$

**Find the greatest common factor of the monomial pair.**

7. $2a, 6a$
8. $15n, 5n$
9. $n^2, 3n$
10. $3x, 6x^2$

**Factor.**

11. $3x + 2x^2$
12. $2a^2 + 4a$
13. $7x^3 + 7x^2$
14. $3x^2 + 6xy$

# Written Exercises

**Factor.**

A

1. $9 + 3x$
2. $5 - 15n$
3. $2x - 10$
4. $4y - 16$
5. $3x^2 - x$
6. $5n^2 - 2n$
7. $2x^2 - 6x$
8. $y^2 - 2y$
9. $28n^2 - 7n$
10. $2xy - y^2$
11. $4x^2 - 8x$
12. $21y^2 - 7xy$
13. $25mn - 5m^2n^2$
14. $8a^2b - 24ab^2$
15. $9x^2 - 27x^2y$
16. $12a^2 + 36a^2b$
17. $3x^2 - 6x + 21$
18. $5n^3 + 15n^2 + 25n$
19. $55y^2 + 22y + 44$
20. $2n^2 + 4mn + 80m^2$
21. $4a^2 + 12ab - 16b^2$
22. $3x^2 - 12xy + 9y^2$

B

23. $6x^2 + 6x + 24xy + 42$
24. $-13x + 26x^2 + 39x^3$
25. $-50a^2 + 25b^2 + 75ab$
26. $48mn + 72m^2n^2 + 60m^3n^3$
27. $56x^3y^3 - 72x^2y^2 - 64xy$
28. $32a^2b^4 - 16ab^3 + 48a^3b^5$

C

29. $x(a - b) + 2(a - b)$
30. $y(a + d) - 4(a + d)$
31. $mn - nx + my - xy$
32. $ab - 3ad + b^2 - 3bd$
33. $2a - a^2 + 2b - ab$
34. $3x^2 - 2xy - 3x + 2y$
35. $2x^2 - 8 + x^2y - 4y$
36. $a^3 - ab^2 - a^2b + b^3$

# 4 Factoring Trinomials—Two Sums

Up to now the only trinomials you have factored are those having a common monomial factor.

$$3x^3 + 2x^2 + x = x(3x^2 + 2x + 1)$$

Common monomial factor

Not all trinomials can be factored, but in the next few lessons you will see that many are the product of two binomials. For example, if $(x + 2)(x + 3) = x^2 + 5x + 6$, then we know that $x^2 + 5x + 6$ has two factors, $x + 2$ and $x + 3$.

Recall how to multiply two binomials at sight.

Step 1: $(x + 2)(x + 3) = x^2 \ldots$

Step 2: $(x + 2)(x + 3) = x^2 + 5x \ldots$

Step 3: $(x + 2)(x + 3) = x^2 + 5x + 6$

Now reverse this process to find the factors.

Step 1: $x^2 + 5x + 6 = (\quad)(\quad)$ ← Draw your parentheses.

Step 2: $x^2 + 5x + 6 = (x \quad)(x \quad)$ ← Your first terms must give you $x^2$.

Step 3: $x^2 + 5x + 6 = (x\ ?\ )(x\ ?\ )$

What two numbers have a product of 6?

1 and 6 2 and 3

Which of the pairs of factors of 6 have a sum of 5?

2 and 3

Step 4: $x^2 + 5x + 6 = (x + 2)(x + 3)$

Of course, you should check to see if your answer is correct.

$$(x + 2)(x + 3) = x^2 + 5x + 6 \quad \checkmark$$

Study these two examples.

**EXAMPLE 1** Factor $x^2 + 7x + 10$.

Step 1: $x^2 + 7x + 10 = (\quad)(\quad)$

Step 2: $x^2 + 7x + 10 = (x \quad)(x \quad)$

Step 3: $x^2 + 7x + 10 = (x\ ?\ )(x\ ?\ )$

What two numbers have a product of 10?

1 and 10 2 and 5

Which of the pairs of factors of 10 have a sum of 7?

2 and 5

Step 4: $x^2 + 7x + 10 = (x + 2)(x + 5)$

**EXAMPLE 2** Factor $a^2 + 10a + 16$.

Step 1: $a^2 + 10a + 16 = (\quad)(\quad)$

Step 2: $a^2 + 10a + 16 = (a \quad)(a \quad)$

Step 3: $a^2 + 10a + 16 = (a\ ?\ )(a\ ?\ )$

What two numbers have a product of 16?

1 and 16 2 and 8

4 and 4

Which of the pairs of factors of 16 have a sum of 10?

2 and 8

Step 4: $a^2 + 10a + 16 = (a + 2)(a + 8)$

## Classroom Practice

**Name the possible pairs of factors.**

| | | | | |
|---|---|---|---|---|
| **1.** 10 | **2.** 7 | **3.** 9 | **4.** 6 | **5.** 11 |
| **6.** 12 | **7.** 15 | **8.** 17 | **9.** 16 | **10.** 20 |
| **11.** 21 | **12.** 24 | **13.** 25 | **14.** 30 | **15.** 36 |

## Written Exercises

**Complete.**

| | If the product of two numbers is: | → and their sum is: → | then the two numbers are: |
|---|---|---|---|
| Sample | 18 | 11 | 2 and 9 |
| A 1. | 5 | 6 | ? |
| 2. | 6 | 5 | ? |
| 3. | 8 | 6 | ? |
| 4. | 12 | 8 | ? |
| 5. | 12 | 7 | ? |
| 6. | 9 | 6 | ? |
| 7. | 15 | 8 | ? |
| 8. | 14 | 9 | ? |
| 9. | 18 | 9 | ? |
| 10. | 20 | 9 | ? |
| 11. | 21 | 10 | ? |
| 12. | 24 | 14 | ? |
| 13. | 24 | 11 | ? |
| 14. | 36 | 13 | ? |
| 15. | 36 | 12 | ? |

16. $x^2 + 8x + 7 = (x \_\_?\_\_)(x \_\_?\_\_)$

17. $a^2 + 7a + 6 = (a \_\_?\_\_)(a \_\_?\_\_)$

18. $n^2 + 9n + 20 = (n \_\_?\_\_)(n \_\_?\_\_)$

19. $x^2 + 9x + 18 = (x \_\_?\_\_)(x \_\_?\_\_)$

20. $y^2 + 9y + 14 = (y \_\_?\_\_)(y \_\_?\_\_)$

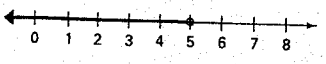
21. $r^2 + 10r + 21 = (r \_\_?\_\_)(r \_\_?\_\_)$

**Factor. Check by multiplication.**

22. $y^2 + 3y + 2$
23. $n^2 + 17n + 16$
24. $y^2 + 16y + 15$
25. $a^2 + 5a + 4$
26. $m^2 + 7m + 6$
27. $x^2 + 11x + 10$
28. $x^2 + 11x + 18$
29. $n^2 + 6n + 8$
30. $y^2 + 6y + 9$

**Factor.**

31. $x^2 + 9x + 14$
32. $a^2 + 9a + 18$
33. $y^2 + 12y + 11$
34. $r^2 + 11r + 30$
35. $b^2 + 12b + 27$
36. $m^2 + 11m + 28$
37. $x^2 + 12x + 32$
38. $y^2 + 24y + 23$
39. $x^2 + 13x + 30$
40. $a^2 + 9a + 20$
41. $a^2 + 12a + 20$
42. $n^2 + 12n + 35$
B 43. $x^2 + 27x + 50$
44. $x^2 + 17x + 72$
45. $m^2 + 28m + 75$
46. $x^2 + 20x + 64$
47. $a^2 + 24a + 63$
48. $y^2 + 24y + 44$

**15.** 


**17.** $25a^2 - 10a + 1$ **19.** $a = 4$
**21.** $-12x^4y^6$ **23.** $x^3 - x^2 - 10x + 12$
**25.** $-7 < 0 < 2$ **27.** $(x^3 + 2x^2)$ cubic units **29.** $45m^7n^6$ **31.** 15 cm
**33.** 0, 1, 2, 3 **35.** $>$ **37.** $a = 2$
**39.** $h = 9$

## CHAPTER 6

### *Written Exercises, page 187*

**1.** $2 \cdot 3 \cdot 7$ **3.** $2 \cdot 2 \cdot 3 \cdot 3$ **5.** $2 \cdot 5 \cdot 31$
**7.** $2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$ **9.** $2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 13$
**11.** 3 **13.** 7 **15.** 1 **17.** 11 **19.** 16
**21.** 12 **23.** 14 **25.** 25 **27.** 4 **29.** 8
**31.** 4 **33.** 20 **35.** 6 **37.** 100

### *Written Exercises, page 189*

**1.** $3(3 + x)$ **3.** $2(x - 5)$ **5.** $x(3x - 1)$
**7.** $2x(x - 3)$ **9.** $7n(4n - 1)$
**11.** $4x(x - 2)$ **13.** $5mn(5 - mn)$
**15.** $9x^2(1 - 3y)$ **17.** $3(x^2 - 2x + 7)$
**19.** $11(5y^2 + 2y + 4)$
**21.** $4(a^2 + 3ab - 4b^2)$
**23.** $6(x^2 + x + 4xy + 7)$
**25.** $25(-2a^2 + b^2 + 3ab)$
**27.** $8xy(7x^2y^2 - 9xy - 8)$
**29.** $(x + 2)(a - b)$ **31.** $(n + y)(m - x)$
**33.** $(2 - a)(a + b)$ **35.** $(x^2 - 4)(y + 2)$

### *Written Exercises, page 191*

**1.** $(4 - \pi)r^2$ **3.** $(a^2 - b^2)\pi$ **5.** 72
**7.** $x^2(1 - \pi) + 5(x + 3)$

### *Written Exercises, pages 194–195*

**1.** 1, 5 **3.** 2, 4 **5.** 3, 4 **7.** 3, 5 **9.** 3, 6
**11.** 3, 7 **13.** 3, 8 **15.** 6, 6
**17.** $(a + 6)(a + 1)$ **19.** $(x + 6)(x + 3)$
**21.** $(r + 3)(r + 7)$ **23.** $(n + 1)(n + 16)$
**25.** $(a + 1)(a + 4)$ **27.** $(x + 1)(x + 10)$
**29.** $(n + 4)(n + 2)$ **31.** $(x + 7)(x + 2)$
**33.** $(y + 1)(y + 11)$ **35.** $(b + 3)(b + 9)$
**37.** $(x + 4)(x + 8)$ **39.** $(x + 3)(x + 10)$
**41.** $(a + 2)(a + 10)$ **43.** $(x + 25)(x + 2)$
**45.** $(m + 25)(m + 3)$ **47.** $(a + 21)(a + 3)$

### *Written Exercises, page 197*

**1.** $-3, -2$ **3.** $-6, -3$ **5.** $-6, -4$
**7.** $(x - 5)(x - 2)$ **9.** $(x - 3)(x - 2)$
**11.** $(y - 5)(y - 1)$ **13.** $(x - 3)(x - 8$
**15.** $(n + 6)(n + 3)$ **17.** $(n - 4)(n -$
**19.** $(x + 4)(x + 2)$ **21.** $(y - 7)(y -$
**23.** $(n + 5)(n + 5)$ **25.** $(y - 6)(y$
**27.** $(x - 7)(x - 7)$ **29.** $(m - 5)(m - 8)$
**31.** $(z - 2)(z - 16)$ **33.** $(y - 3)(y - 12)$
**35.** $(d - 5)(d - 9)$ **37.** $(x - 2)(x - 50)$

### *Written Exercises, page 199*

**1.** yes **3.** yes **5.** no **7.** $(a + 5)^2$
**9.** $(x + 4)^2$ **11.** $(x - 2)^2$ **13.** $(y + 7)^2$
**15.** $(n + 9)^2$ **17.** $(x - 9)(x - 10)$
**19.** $(a - b)^2$ **21.** $(1 - 10x)^2$

### *Written Exercises, page 202*

**1.** $-5, 2$ **3.** $-4, 1$ **5.** $-2, 1$ **7.** $-10, 3$
**9.** $7, -4$ **11.** $14, -1$ **13.** $(x + 7)(x - 3)$
**15.** $(x + 9)(x - 2)$ **17.** $(b - 3)(b + 4)$
**19.** $(n - 6)(n + 3)$ **21.** $(x - 5)(x + 4)$
**23.** $(y - 1)(y + 15)$ **25.** $(b + 6)(b - 4)$
**27.** $(b - 10)(b + 3)$ **29.** $(x + 14)(x - 2)$
**31.** $(y + 8)(y - 4)$ **33.** $(y + 9)(y - 3)$
**35.** $(m - 4)(m + 16)$ **37.** $(n - 9)(n + 7)$
**39.** $(y - 6)(y + 7)$ **41.** $(y + 7)(y - 8)$
**43.** $(c - 4)(c + 20)$ **45.** $(a - 4)(a + 25)$

### *Mixed Practice Exercises, page 203*

**1.** $(x + 9)(x + 1)$ **3.** $(n - 6)(n - 2)$
**5.** $(y - 3)(y + 4)$ **7.** $(y - 7)(y + 3)$
**9.** $(x + 3)(x + 7)$ **11.** $(b - 1)(b + 5)$
**13.** $(b - 7)(b - 1)$ **15.** $(y - 4)(y + 5)$
**17.** $(x + 7)(x + 5)$ **19.** $(n - 7)(n - 8)$
**21.** $(x + 9)(x + 6)$ **23.** $(n + 9)(n + 5)$
**25.** $(n - 10)(n + 5)$ **27.** $(y + 26)(y - 2)$
**29.** $(a + 4)(a - 11)$ **31.** $(c - 6)(c + 10)$
**33.** $(m - 4)(m - 10)$ **35.** $4x - 4$
**37.** $4y - 4$

### *Written Exercises, page 205*

**1.** $n^2 - 49$ **3.** $a^2 - 100$ **5.** $m^2 - n^2$
**7.** $x^2 - y^2$ **9.** $1 - 4x^2$ **11.** $9x^2 - 4$
**13.** $25y^2 - 4$ **15.** $81m^2 - n^2$
**17.** $25x^2 - 36$ **19.** $25x^2 - 9y^2$
**21.** $100x^2 - 25y^2$ **23.** $a^4 - 4$ **25.** 399
**27.** 2496 **29.** 896 **31.** 8091 **33.** 864
**35.** 3575 **37.** 6384 **39.** 2496