

3 Multiplying Monomials

You will recall from your work with exponents that $x^2 = x \cdot x$, $x^3 = x \cdot x \cdot x$, and so on. Therefore, you can do the following.

$$\begin{aligned} x^2 \cdot x^3 &= (x \cdot x)(x \cdot x \cdot x) \\ &= x^5 \end{aligned}$$

You can use this idea to multiply any two monomials.

EXAMPLE 1 $x^4 \cdot x^3 = (x \cdot x \cdot x \cdot x)(x \cdot x \cdot x)$
 $= x^7$

EXAMPLE 2 $(3a^2)(a^3) = (3 \cdot a \cdot a)(a \cdot a \cdot a)$
 $= 3(a \cdot a \cdot a \cdot a \cdot a)$
 $= 3a^5$

Once you have done a few problems like these, you'll probably see a short cut.

When you multiply two powers of the same number, add the exponents.

$$x^a \cdot x^b = x^{a+b}$$

EXAMPLE 3 $x^5 \cdot x^7 = x^{5+7} = x^{12}$

EXAMPLE 4 $n^6 \cdot n = n^{6+1} = n^7$

Remember, $n = n^1$.

EXAMPLE 5 $(2x^2)(3x^3) = (2 \cdot 3)(x^2 \cdot x^3)$
 $= 6x^{2+3}$
 $= 6x^5$

EXAMPLE 6 $(4a^2b)(-3ab^2) = (4 \cdot -3)(a^2 \cdot a)(b \cdot b^2)$
 $= -12a^{2+1}b^{1+2}$
 $= -12a^3b^3$

Classroom Practice

Multiply.

- | | | |
|---------------------|-----------------------|-------------------------|
| 1. $x^4 \cdot x^2$ | 2. $n \cdot n^2$ | 3. $y^2 \cdot y^3$ |
| 4. $-2x \cdot x^5$ | 5. $(x^3)(4x^2)$ | 6. $(2a)(4a^3)$ |
| 7. $(-9y^2)(y^5)$ | 8. $(6n^4)(2n^4)$ | 9. $(5ab)(a^2b)$ |
| 10. $(2x^2)(4x^2y)$ | 11. $(-5x^2y)(-2y^2)$ | 12. $(8a^2b^5)(-4ab^2)$ |

Written Exercises

Multiply.

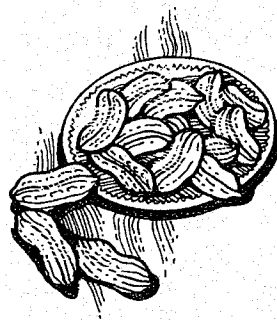
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|--------------------------|---------------------------|----------------------------|
| A 1. $a \cdot a^2$ | 2. $b^2 \cdot b^2$ | 3. $x^2 \cdot 3x$ |
| 4. $x^4 \cdot x$ | 5. $n^3 \cdot n^4$ | 6. $a^6 \cdot a$ |
| 7. $(2x)(2x^2)$ | 8. $(3x)(-2x^4)$ | 9. $(c^2)(-5c^3)$ |
| 10. $(ab)(a^2b)$ | 11. $(3x^2)(-2x^5)$ | 12. $(-y^2)(-y^7)$ |
| 13. $(3x^2)(4x^4)$ | 14. $(-x^2)(-4x)$ | 15. $(5a)(-ab^2)$ |
| 16. $(xy)(-2x)$ | 17. $(cd)(-3d^3)$ | 18. $(2mn)(-8m^2)$ |
| 19. $(5x^2y)(4xy^2)$ | 20. $(-5xy)(2xy^2)$ | 21. $(-r^2s)(-10r^2s^2)$ |
| 22. $(-6a^2)(4ab^5)$ | 23. $(-x^4)(-3xyz^2)$ | 24. $(-6a^2b^5)(abc^3)$ |
| 25. $(a^2b)(-5a^2b^2)$ | 26. $(-x^3)(-5x^2y)$ | 27. $(xy^3)(-2x^3y^2)$ |
| 28. $(-a^4b^3)(a^2bc^5)$ | 29. $(m^3n)(-4m^3n^2p^4)$ | 30. $(r^2s^3t^4)(r^5st^3)$ |

PUZZLE ♦ PROBLEMS

If you live in Parry Sound, then you live in Ontario.
If you live in Ontario, then you live in Canada.
Therefore, if you live in Parry Sound, you live in Canada.

Crackers are better than nothing.
Nothing is better than peanuts.
Therefore, crackers are better than peanuts.

What happened?



4 Powers of Monomials

Study the examples below and see if you can discover another important rule of exponents.

$$\begin{aligned}(x^3)^2 &= x^3 \cdot x^3 = x^6 \\ (a^4)^2 &= a^4 \cdot a^4 = a^8 \\ (y^5)^3 &= y^5 \cdot y^5 \cdot y^5 = y^{15}\end{aligned}$$

To find the power of a power of a number, multiply the exponents.

$$(x^a)^b = x^{ab}$$

EXAMPLE 1

$$(x^4)^3 = x^{4 \cdot 3} = x^{12}$$

EXAMPLE 2

$$(y^8)^2 = y^{8 \cdot 2} = y^{16}$$

Suppose you have an expression like $(xy)^2$. You can rewrite it in the following way.

$$(xy)^2 = (xy)(xy) = x^2y^2$$

Make a note of this rule of exponents.

$$(xy)^n = x^n y^n$$

EXAMPLE 3

$$(ab)^3 = a^3 b^3$$

EXAMPLE 4

$$(2a)^2 = 2^2 \cdot a^2 = 4a^2$$

WARNING! $2a^2$ and $(2a)^2$ are not the same.

$$\begin{array}{r|l} 2a^2 & \neq (2a)^2 \\ 2a^2 & 2^2 \cdot a^2 \\ & 4a^2 \end{array} \quad \boxed{2a^2 \neq (2a)^2}$$

EXAMPLE 5

$$(-4y)^2 = (-4)^2 \cdot y^2 = 16y^2$$

Classroom Practice

1. In 5^3 , 5 is used as a factor ? times.
2. In $(x^3)^2$, x^3 is used as a factor ? times.
3. Does $(3x)^2$ equal $3x^2$, or does it equal $9x^2$?
4. Does $(2x)^3$ equal $2x^3$, or does it equal $8x^3$?

Simplify.

- | | | | |
|--------------|--------------|----------------|----------------|
| 5. $(a^4)^2$ | 6. $(c^2)^5$ | 7. $(ab)^4$ | 8. $(5a)^2$ |
| 9. $(x^2)^3$ | 10. $(2a)^2$ | 11. $(4n^2)^2$ | 12. $(-3xy)^2$ |

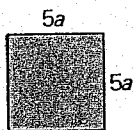
Written Exercises

Simplify.

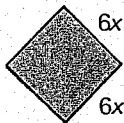
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|-----------------------|-----------------|---------------|----------------|
| A 1. $(x^2)^3$ | 2. $(a^3)^4$ | 3. $(b^6)^2$ | 4. $(x^2)^5$ |
| 5. $(c^3)^5$ | 6. $(n^4)^{10}$ | 7. $(2x)^2$ | 8. $(4a)^2$ |
| 9. $(ab)^4$ | 10. $(xy)^6$ | 11. $(6ax)^2$ | 12. $(-2xy)^3$ |

Find the area. Use $A = s^2$.

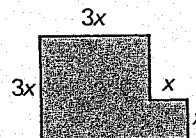
13.



14.



15.



Use both of the exponent rules discussed in the lesson. Simplify.

SAMPLE

$$(4x^3)^2 = 4^2(x^3)^2 = 16x^6$$

- | | | |
|----------------------------|----------------------|---------------------------|
| 16. $(5x^3)^2$ | 17. $(3a^2)^2$ | 18. $(2b^3)^2$ |
| 19. $(5x^5)^2$ | 20. $(-2a^5)^2$ | 21. $(2a^2b)^2$ |
| 22. $(3xy^3)^2$ | 23. $(-x^3y^3)^2$ | 24. $2(ab^4)^2$ |
| 25. $3(x^2y)^2$ | 26. $-(2n^2)^3$ | 27. $-(4n^2)^2$ |
| B 28. $(xy)^2(-xy)$ | 29. $(-xy^3)(-xy)^2$ | 30. $(3xy^4)^2(-4x^2)^2$ |
| 31. $(2mn)^3(3n)^2$ | 32. $-(x^2y^2)(x^2)$ | 33. $-(r^4s^3)^2(rs^2)^3$ |

5 Polynomials Times Monomials

Now that you can multiply monomials, you can put the distributive property to work.

You know that $\rightarrow 5(3 + 4) = (5 \cdot 3) + (5 \cdot 4)$

In the same way $\rightarrow 5(3x + 4y) = (5 \cdot 3x) + (5 \cdot 4y)$
 $= 15x + 20y$

EXAMPLE 1

$$3(a + 3b) = (3 \cdot a) + (3 \cdot 3b)$$
$$= 3a + 9b$$

EXAMPLE 2

$$-1(2x - 3y) = (-1 \cdot 2x) - (-1 \cdot 3y)$$
$$= -2x + 3y$$

EXAMPLE 3

$$a(a^2 + 2ab + b^2) = (a \cdot a^2) + (a \cdot 2ab) + (a \cdot b^2)$$
$$= a^3 + 2a^2b + ab^2$$

EXAMPLE 4

$$-2x(9x^3 + 3x^2 + x) = (-2x \cdot 9x^3) + (-2x \cdot 3x^2) + (-2x \cdot x)$$
$$= -18x^4 - 6x^3 - 2x^2$$

Classroom Practice

Multiply.

1. $2(a + 4)$

4. $x(x - 2)$

7. $-4(x^2 + 1)$

10. $-x(x^2 - 2x + 4)$

13. $xy(x^2 - 2y)$

2. $3(x + y)$

5. $a(a + 2b)$

8. $-5(a^2 - 2c)$

11. $x^2(x^2 - 3x + 1)$

14. $-4x(x + 2y + 3z)$

3. $-1(2x + 3y)$

6. $2c(a + b)$

9. $x(x^2 + 2x + 4)$

12. $a^2b^2(-a - b)$

15. $m(m^2 - 5m - 6)$

Written Exercises

Multiply.

- A**
- | | | |
|------------------------|--------------------------|-----------------------------|
| 1. $2(x + 4)$ | 2. $3(a - b)$ | 3. $4(x + y)$ |
| 4. $5(a^2 + b)$ | 5. $-6(n + 2m)$ | 6. $-1(5a + b^2)$ |
| 7. $a(a - b)$ | 8. $x(x + 3y)$ | 9. $-c(a + b)$ |
| 10. $-ab(2a - 4b)$ | 11. $-5x(3x + 2y)$ | 12. $2x(3x - 1)$ |
| 13. $4a(a + 2b + 3)$ | 14. $-4(1 + 5x + x^2)$ | 15. $-1(2x + y + z)$ |
| 16. $2x(x^2 - 2x - 4)$ | 17. $-4y(y^3 - 2y + 1)$ | 18. $ab(a^2 + 2ab - 1)$ |
| 19. $-x^2(x + 2x^2)$ | 20. $-3c(2c^2 + 4c - 5)$ | 21. $-y^2(y^3 - 2y^2 + 4y)$ |
22. A rectangle is $10n$ centimeters long by $(n + 6)$ centimeters wide.
Write its area as a polynomial.
- B** 23. You have collected $(3n + 1)$ dimes. What is their value in cents?

Solve.

- | | |
|----------------------------------|---------------------------------|
| 24. $4(2n + 3) - 3(n - 1) = 0$ | 25. $-(n + 3) + 2(n + 7) = 0$ |
| 26. $5x + 2 - 2(2x + 6) = 0$ | 27. $(2y - 3) - (y + 6) = 63$ |
| 28. $2(5x - 6) - 3(2x - 4) = 0$ | 29. $3(1 - 2a) - (6 - 2a) = -7$ |
| 30. $3(x - 4) + 2(2x + 1) = 4$ | 31. $2(n - 6) + 5(2n + 4) = 32$ |
| 32. $6(1 - 3x) - 2(2x + 5) = 40$ | 33. $4(2a - 3) - 2(a - 8) = 22$ |
| 34. $7(m + 3) - 5(2 - m) = -1$ | 35. $2(2x - 1) + 3(x + 4) = 52$ |

SELF-TEST

Simplify.

- | | | | |
|------------------|-----------------|-----------------------|-------|
| 1. $x \cdot x^3$ | 2. $(4x)(7x^3)$ | 3. $(-m^2n^2)(2mn^2)$ | (5-3) |
| 4. $(m^8)^4$ | 5. $(ab)^3$ | 6. $(-7x^2y)^2$ | (5-4) |

Multiply.

- | | | | |
|-------------------|--------------------|----------------------------|-------|
| 7. $x(y + 7)$ | 8. $m(m - n)$ | 9. $-a(a^2 - b)$ | (5-5) |
| 10. $-2(3x - 4y)$ | 11. $5x(4x + x^2)$ | 12. $m^2(3m^3 - m^2 + 2m)$ | |

6 Multiplying Polynomials

To multiply by a binomial you also use the distributive property. Let's use a vertical form in multiplying the binomials below. Your work is a lot like multiplying in arithmetic.

$$\begin{array}{r}
 (n+1)(n+3) \longrightarrow \begin{array}{r} n+1 \\ n+3 \\ \hline n^2+n \\ +3n+3 \\ \hline n^2+4n+3 \end{array} \\
 \begin{array}{l} \longleftarrow n(n+1) \\ \longleftarrow 3(n+1) \end{array} \\
 \begin{array}{c} \uparrow \\ \text{Be sure that like terms are lined up.} \end{array}
 \end{array}$$

Here are a few more examples to study.

$$\begin{array}{r}
 \text{EXAMPLE 1} \quad \begin{array}{r} x+7 \\ x-2 \\ \hline x^2+7x \\ -2x-14 \\ \hline x^2+5x-14 \end{array} \\
 \begin{array}{l} \longleftarrow x(x+7) \\ \longleftarrow -2(x+7) \end{array}
 \end{array}$$

$$\begin{array}{r}
 \text{EXAMPLE 2} \quad \begin{array}{r} 2y-5 \\ y-4 \\ \hline 2y^2-5y \\ -8y+20 \\ \hline 2y^2-13y+20 \end{array} \\
 \begin{array}{l} \longleftarrow y(2y-5) \\ \longleftarrow -4(2y-5) \end{array}
 \end{array}$$

$$\begin{array}{r}
 \text{EXAMPLE 3} \quad \begin{array}{r} a^2+2a-1 \\ a-3 \\ \hline a^3+2a^2-a \\ -3a^2-6a+3 \\ \hline a^3-a^2-7a+3 \end{array} \\
 \begin{array}{l} \longleftarrow a(a^2+2a-1) \\ \longleftarrow -3(a^2+2a-1) \end{array}
 \end{array}$$

Classroom Practice

Multiply.

- | | | |
|-----------------------|-----------------------|------------------------|
| 1. $(x + 2)(x + 4)$ | 2. $(n + 2)(n + 7)$ | 3. $(a + 4)(a - 1)$ |
| 4. $(x + 3)(x - 5)$ | 5. $(2x + 1)(x + 5)$ | 6. $(a + 6)(2a - 4)$ |
| 7. $(2a + b)(a + 3b)$ | 8. $(2x - 4)(5x + 1)$ | 9. $(3y - 6)(2y - 5)$ |
| 10. $(2x - 5)(x + 2)$ | 11. $(3n + 6)(n - 3)$ | 12. $(4x - 1)(2x + 3)$ |

Written Exercises

Multiply.

- | | | | |
|----------|---|------------------------------|------------------------|
| A | 1. $(x + 1)(x + 2)$ | 2. $(y + 3)(y + 4)$ | 3. $(n + 4)(n + 6)$ |
| | 4. $(a + 3)(a + 5)$ | 5. $(x + 1)(x + 7)$ | 6. $(y + 9)(y + 3)$ |
| | 7. $(x + y)(x + y)$ | 8. $(a + b)(a + c)$ | 9. $(x + y)(x + 4)$ |
| | 10. $(x - 1)(x - 2)$ | 11. $(y - 3)(y - 4)$ | 12. $(n - 3)(n - 1)$ |
| | 13. $(x - 1)(x + 2)$ | 14. $(y + 3)(y - 4)$ | 15. $(y - 5)(y + 4)$ |
| | 16. $(2x + y)(x + 2y)$ | 17. $(4x + 1)(4x + 1)$ | 18. $(4x + 1)(4x - 1)$ |
| | 19. $(a + b)(a - b)$ | 20. $(a + 2b)(a - 2b)$ | 21. $(2a - 1)(a + b)$ |
| | 22. $(6 - x)(6 - x)$ | 23. $(6x - 5)(6x - 5)$ | 24. $(3y - 2)(2y + 1)$ |
| B | 25. $(n + 1)(n^2 + 2n + 1)$ | 26. $(x + 2)(x^2 + 2x + 1)$ | |
| | 27. $(x - 2)(x^2 - 4x + 4)$ | 28. $(y + 3)(y^2 + 6y + 9)$ | |
| | 29. $(m - 2)(m^2 - 2m + 4)$ | 30. $(n - 4)(n^2 - 8n + 16)$ | |
| | 31. $(a + b)(a^2 + 2ab + b^2)$ | 32. $(a - b)(a^2 - b^2)$ | |
| C | 33. $(n^2 + 2n + 1)(n^2 + 2n + 1)$ | | |
| | 34. $(x^2 - 7x + 12)(x^2 - 3x - 1)$ | | |
| | 35. $(2a^2 - ab + 4b^2)(5a^2 + ab + b^2)$ | | |
| | 36. $(x - 4)(2x^4 - 5x^3 - 7x^2 + 10)$ | | |
| | 37. $(n - 1)(n^4 - 3n^3 + n^2 + 1)$ | | |
| | 38. $(a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$ | | |

7 Multiplying at Sight

When you multiplied binomials by using the vertical form, you may have noticed a pattern. The pattern is pointed out at the right.

$$\begin{array}{r}
 x + 2 \\
 x + 3 \\
 \hline
 x^2 + 2x \\
 + 3x + 6 \\
 \hline
 x^2 + 5x + 6
 \end{array}$$

Diagram illustrating the vertical multiplication of $(x + 2)(x + 3)$. The result is $x^2 + 5x + 6$. The terms are labeled as follows:

- x^2 : product of the first terms
- $5x$: sum of $2 \cdot x$ and $3 \cdot x$
- 6 : product of the last terms

You don't have to write out each step every time you multiply two binomials. You can multiply at sight if you follow three steps.

Step 1: $(x + 3)(x + 2) = x^2 \dots$ ← Multiply the *first* terms.

Step 2: $(x + 3)(x + 2) = x^2 + 5x \dots$ ← Multiply the *outer* terms. Multiply the *inner* terms. Add the two products.

Step 3: $(x + 3)(x + 2) = x^2 + 5x + 6$ ← Multiply the *last* terms.

An easy way to remember this is to think of it as the **FOIL method**.

FOIL stands for First, Outer, Inner, Last!

Here's how the FOIL method works.

EXAMPLE 1

Multiply $(x + 4)(x + 5)$.

Step 1: $(x + 4)(x + 5) = x^2 \dots$ ← F

Step 2: $(x + 4)(x + 5) = x^2 + 9x \dots$ ← O + I

Step 3: $(x + 4)(x + 5) = x^2 + 9x + 20$ ← L

Be sure to watch the signs!

EXAMPLE 2

Multiply $(x - 4)(x - 2)$.

Step 1: $(x - 4)(x - 2) = x^2 \dots \leftarrow$ F

Step 2: $(x - 4)(x - 2) = x^2 - 6x \dots \leftarrow$ O + I

Step 3: $(x - 4)(x - 2) = x^2 - 6x + 8 \leftarrow$ L

EXAMPLE 3

Multiply $(x - 4)(x + 2)$.

$$(x - 4)(x + 2) = x^2 - 2x - 8$$

F O + I L

EXAMPLE 4

Multiply $(2x + 3y)(x + y)$.

$$(2x + 3y)(x + y) = 2x^2 + 5xy + 3y^2$$

F O + I L

Classroom Practice

Complete.

1. $(x + 1)(x + 4) = \underline{\quad?} + 5x + 4$

2. $(a + 2)(a + 5) = a^2 + 7a + \underline{\quad?}$

3. $(y + 3)(y + 2) = y^2 + 5y + \underline{\quad?}$

4. $(x + 4)(x + 5) = x^2 + \underline{\quad?} + 20$

5. $(n + 1)(n + 7) = n^2 + \underline{\quad?} + 7$

6. $(x + 5)(x - 2) = x^2 + 3x - \underline{\quad?}$

7. $(a + 8)(a - 2) = a^2 + 6a - \underline{\quad?}$

8. $(c - 6)(c + 1) = c^2 - 5c - \underline{\quad?}$

9. $(x - 3)(x - 4) = x^2 - 7x + \underline{\quad?}$

10. $(a - 5)(a - 9) = a^2 - \underline{\quad?} + 45$

Multiply.

11. $(x + 2)(x + 3)$

12. $(y + 5)(y + 1)$

13. $(a + 6)(a + 2)$

14. $(x - 4)(x - 3)$

15. $(y - 6)(y - 1)$

16. $(x + 3)(x - 7)$

17. $(c - 9)(c + 6)$

18. $(2x + 1)(x + 3)$

19. $(4y + 1)(2y - 6)$

20. $(5x + 2)(3x - 1)$

21. $(2a - b)(3a + 4b)$

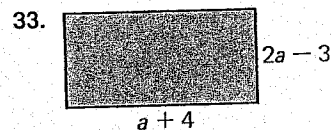
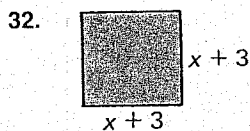
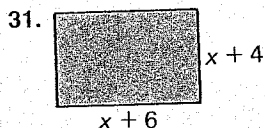
22. $(7r - s)(r + 4s)$

Written Exercises

Multiply.

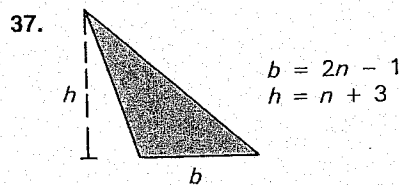
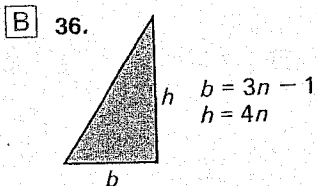
- | | | |
|-------------------------|-------------------------|--------------------------|
| 1. $(x + 4)(x + 3)$ | 2. $(x + 3)(x + 5)$ | 3. $(x + 6)(x + 2)$ |
| 4. $(x + 2)(x + 1)$ | 5. $(x - 1)(x - 2)$ | 6. $(x - 6)(x - 3)$ |
| 7. $(x - 4)(x - 3)$ | 8. $(a - 3)(a - 5)$ | 9. $(y - 4)(y + 1)$ |
| 10. $(a - 1)(a + 3)$ | 11. $(n + 2)(n - 1)$ | 12. $(x + 5)(x - 2)$ |
| 13. $(y + 6)(y - 2)$ | 14. $(a + 5)(a - 3)$ | 15. $(2x + 1)(x + 4)$ |
| 16. $(x + 2)(x - 2)$ | 17. $(a + b)(a - b)$ | 18. $(2x + 5)(2x - 5)$ |
| 19. $(6a + 7)(6a - 7)$ | 20. $(2y - 3)(2y + 3)$ | 21. $(3x + 4)(3x - 4)$ |
| 22. $(5n - 1)(2n + 3)$ | 23. $(3x - 1)(2x + 5)$ | 24. $(3x + 2y)(2x + 3y)$ |
| 25. $(3x + y)(7x + 2y)$ | 26. $(3a - b)(a - 3b)$ | 27. $(4x - 3y)(x - 5y)$ |
| 28. $(3a - b)(7a + 4b)$ | 29. $(6x - y)(4x + 2y)$ | 30. $(3n + 2p)(7n - p)$ |

Find the area. Use $A = lw$.

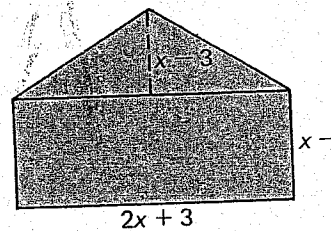


34. A rectangle measures $(17 - x)$ centimeters by $(x - 4)$ centimeters. Find its area.
35. A rectangle measures $(2y + 3)$ centimeters by $(3y + 1)$ centimeters. Find its area.

Find the area. Use $A = \frac{1}{2}bh$.

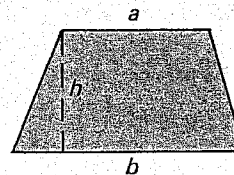


38. Find the area of the side of the house shown at the right.

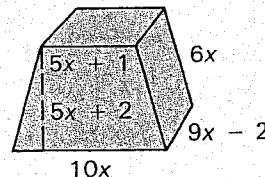


- C 39. A trapezoid is shown at the right. Its area may be found by using the formula $A = \frac{1}{2}h(a + b)$.

Find the area of the trapezoid if $h = 2x$, $a = 3x - 1$, and $b = 3x + 1$.



40. The box shown at the right is designed to hold a cup and saucer. The top, the bottom, and two sides are rectangles. Two sides are trapezoids. Find the area of the whole surface of the box.



CONSUMER APPLICATIONS

USING A CHECK REGISTER

When you have a checking account you have to be careful to keep track of how much money you have spent and how much money you have in your account. You can use a check register to keep track of your transactions and your account balances. See the sample below.

CHECK NO.	DATE	DESCRIPTION OF TRANSACTION	AMOUNT OF CHECK	AMOUNT OF DEPOSIT	BALANCE
BALANCE BROUGHT FORWARD →					406 19
161	9/21	Jones Cleaners	12 15		394 04
162	9/30	Statewide Electric Co.	34 83		359 21
	10/6	Deposit		125 62	484 83
163	10/7	Southern Savings + Loan	152 30		332 53

Be sure to record all checks when you write them. It's just too easy to forget about the check in only a few hours.

When you deposit money into your account, remember to save the deposit slips. Record your deposit in the register as well.

After writing a check or making a deposit, be sure to record the amount of money you have in your account in the Balance column.

8 Square of a Binomial

You know that when you square a number you multiply it by itself.

$$6^2 = 6 \cdot 6 \quad x^2 = x \cdot x$$

The same is true when you square a binomial. All you do is to write the two factors, then multiply as you did in the last section.

$$\begin{aligned}(a + b)^2 &= (a + b)(a + b) \\ &= a^2 + 2ab + b^2\end{aligned}$$

The square of a binomial is called a **trinomial square**.

Study some more examples.

EXAMPLE 1

$$\begin{aligned}\text{a. } (x + 3)^2 &= (x + 3)(x + 3) \\ &= x^2 + 6x + 9\end{aligned} \quad \begin{aligned}\text{b. } (x - 4)^2 &= (x - 4)(x - 4) \\ &= x^2 - 8x + 16\end{aligned}$$

Once you have done a few multiplications like those above, you might notice a pattern in squaring a binomial.

SQUARING A BINOMIAL

1. Square the first term.
2. Add or subtract twice the product of the two terms.
3. Add the square of the second term.

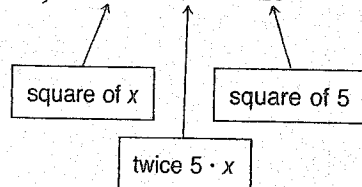
$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

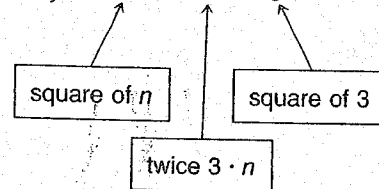
Here are a couple of examples. Be sure to watch the signs too.

EXAMPLE 2

$$\text{a. } (x + 5)^2 = x^2 + 10x + 25$$



$$\text{b. } (n - 3)^2 = n^2 - 6n + 9$$



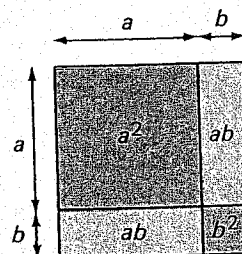
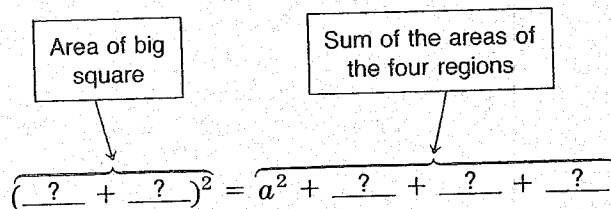
Of course, if you find this short cut difficult to remember, you can multiply as in Example 1.

Written Exercises

Express as a trinomial.

- A**
- | | | | |
|-------------------|--------------------|-------------------|--------------------|
| 1. $(x + 4)^2$ | 2. $(m + 1)^2$ | 3. $(r + 2)^2$ | 4. $(y - 3)^2$ |
| 5. $(x + 3)^2$ | 6. $(n + 6)^2$ | 7. $(x - 2)^2$ | 8. $(m - 6)^2$ |
| 9. $(x - 4)^2$ | 10. $(m + 5)^2$ | 11. $(y - 7)^2$ | 12. $(a + 10)^2$ |
| 13. $(x + 9)^2$ | 14. $(y - 8)^2$ | 15. $(m + 7)^2$ | 16. $(n + 8)^2$ |
| 17. $(n - 5)^2$ | 18. $(y - 1)^2$ | 19. $(3a + 2)^2$ | 20. $(2x + 3)^2$ |
| 21. $(4x - 2)^2$ | 22. $(2a + b)^2$ | 23. $(3x + y)^2$ | 24. $(6n - 1)^2$ |
| 25. $(10a - b)^2$ | 26. $(2m + 3n)^2$ | 27. $(4x - 3y)^2$ | 28. $(3x - 4y)^2$ |
| 29. $2(a + 1)^2$ | 30. $2(a + 5)^2$ | 31. $3(x - 2)^2$ | 32. $-1(x + 3)^2$ |
| 33. $-2(x - 2)^2$ | 34. $-1(2n + 3)^2$ | 35. $2(2x + y)^2$ | 36. $-1(a - 2b)^2$ |

- B** 37. Study the figure. Then complete the equation below.



SELF-TEST

Vocabulary

FOIL method (p. 164) trinomial square (p. 168)

Multiply using the vertical form.

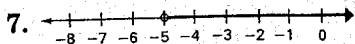
1. $(x + 2)(x + 4)$ 2. $(a - 1)(a - 5)$ 3. $(a - 1)(a^2 - 2a + 1)$ (5-6)

Multiply using the FOIL method.

4. $(m - 5)(m + 1)$ 5. $(2a - 7b)(2a + 3b)$ 6. $(3m + 1)(2m + 5)$ (5-7)

Express as a trinomial.

7. $(a + 4)^2$ 8. $(n - 3)^2$ 9. $(6x + y)^2$ 10. $(4a - 7b)^2$ (5-8)



9. 0 11. -2 13. 4 15. 12 17. 140
 19. 45 21. $-21ay$ 23. $-2t + 5$ 25. -3
 27. 6 29. -6 31. -2 33. 16 cm^2
 35. $2w + t$ 37. 68 pies

CHAPTER 5

- Written Exercises, page 153** 1. $3y + 11$
 3. $2m^2$ 5. $5ab + 2b^2$ 7. $2x^2 + xy + 2y^2$
 9. $-5y^2 + 1$ 11. $12n + 2$
 13. $10m + 12n + 15$ 15. $-4m + 8$
 17. $10x^2 - 20x + 2$ 19. $5x^2 + 2x - 1$
 21. $k^3 + k^2 - 2k + 6$ 23. $x^3 + 6x^2 - 3$
 25. $a^2 - ab - b^2$

- Written Exercises, page 155** 1. $x + 4y$
 3. $-2x^2 + 2y^2$ 5. $2x^2 + 4$ 7. $-4xy$
 9. $x^2y + 4xy^2 + 2$ 11. $b + 2$
 13. $4m - 18$ 15. $4y + 14$ 17. $-3x^2 + 5x - 6$
 19. $-a^2 + 5ab - 10c$ 21. $2y^2 + 5y - 18$ 23. $n = 10$ 25. $n = 80$

- Written Exercises, page 157** 1. a^3
 3. $3x^3$ 5. n^7 7. $4x^3$ 9. $-5c^5$
 11. $-6x^7$ 13. $12x^6$ 15. $-5a^2b^2$
 17. $-3cd^4$ 19. $20x^3y^3$ 21. $10r^4s^3$
 23. $3x^5yz^2$ 25. $-5a^4b^3$ 27. $-2x^4y^5$
 29. $-4m^6n^3p^4$

Puzzle Problems, page 157 The word "nothing" has two different meanings here: in the first sentence, the absence of something; in the second, no thing. Thus, logic cannot be used to conclude the third sentence.

- Written Exercises, page 159** 1. x^6
 3. b^{12} 5. c^{15} 7. $4x^2$ 9. a^4b^4
 11. $36a^2x^2$ 13. $25a^2$ 15. $10x^2$ 17. $9a^4$
 19. $25x^{10}$ 21. $4a^4b^2$ 23. x^6y^6
 25. $3x^4y^2$ 27. $-16n^4$ 29. $-x^3y^5$
 31. $72m^3n^5$ 33. $-r^{11}s^{12}$

- Written Exercises, page 161** 1. $2x + 8$
 3. $4x + 4y$ 5. $-6n - 12m$ 7. $a^2 - ab$
 9. $-ca - cb$ 11. $-15x^2 - 10xy$
 13. $4a^2 + 8ab + 12a$ 15. $-2x - y - z$

17. $-4y^4 + 8y^2 - 4y$ 19. $-x^3 - 2x^4$
 21. $-y^5 + 2y^4 - 4y^3$ 23. $(30n + 10)$ cents
 25. $n = -11$ 27. $y = 72$
 29. $a = 1$ 31. $n = 2$ 33. $a = 3$
 35. $x = 6$

Written Exercises, page 163

1. $x^2 + 3x + 2$ 3. $n^2 + 10n + 24$
 5. $x^2 + 8x + 7$ 7. $x^2 + 2xy + y^2$
 9. $x^2 + xy + 4x + 4y$ 11. $y^2 - 7y + 12$
 13. $x^2 + x - 2$ 15. $y^2 - y - 20$
 17. $16x^2 + 8x + 1$ 19. $a^2 - b^2$
 21. $2a^2 - a + 2ab - b$
 23. $36x^2 - 60x + 25$
 25. $n^3 + 3n^2 + 3n + 1$
 27. $x^3 - 6x^2 + 12x - 8$
 29. $m^3 - 4m^2 + 8m - 8$
 31. $a^3 + 3a^2b + 3ab^2 + b^3$
 33. $n^4 + 4n^3 + 6n^2 + 4n + 1$
 35. $10a^4 - 3a^3b + 21a^2b^2 + 3ab^3 + 4b^4$
 37. $n^5 - 4n^4 + 4n^3 - n^2 + n - 1$

Written Exercises, pages 166-167

1. $x^2 + 7x + 12$ 3. $x^2 + 8x + 12$
 5. $x^2 - 3x + 2$ 7. $x^2 - 7x + 12$
 9. $y^2 - 3y - 4$ 11. $n^2 + n - 2$
 13. $y^2 + 4y - 12$ 15. $2x^2 + 9x + 4$
 17. $a^2 - b^2$ 19. $36a^2 - 49$
 21. $9x^2 - 16$ 23. $6x^2 + 13x - 5$
 25. $21x^2 + 13xy + 2y^2$
 27. $4x^2 - 23xy + 15y^2$
 29. $24x^2 + 8xy - 2y^2$
 31. $x^2 + 10x + 24$ 33. $2a^2 + 5a - 12$
 35. $(6y^2 + 11y + 3) \text{ cm}^2$
 37. $\frac{1}{2}(2n^2 + 5n - 3)$ 39. $6x^2$

Written Exercises, page 169

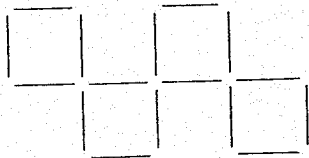
1. $x^2 + 8x + 16$ 3. $r^2 + 4r + 4$
 5. $x^2 + 6x + 9$ 7. $x^2 - 4x + 4$
 9. $x^2 - 8x + 16$ 11. $y^2 - 14y + 49$
 13. $x^2 + 18x + 81$ 15. $m^2 + 14m + 49$
 17. $n^2 - 10n + 25$ 19. $9a^2 + 12a + 4$
 21. $16x^2 - 16x + 4$ 23. $9x^2 + 6xy + y^2$
 25. $100a^2 - 20ab + b^2$
 27. $16x^2 - 24xy + 9y^2$

29. $2a^2 + 4a + 2$ 31. $3x^2 - 12x + 12$
 33. $-2x^2 + 8x - 8$ 35. $8x^2 + 8xy + 2y^2$
 37. $(a + b)^2 = a^2 + ab + ab + b^2$

Written Exercises, page 171

1. 1 3. a 5. $-5x$ 7. c^5 9. $7n$
 11. -6 13. $-4rs$ 15. $3x^2y$ 17. $-7x^2$
 19. $-7b$ 21. $2cd^2$ 23. $-7ab$ 25. $25mn$
 27. $5a^4b$ 29. $12xy^2$ 31. $2x$
 33. $4a$ 35. $4ab$

Puzzle Problems, page 171



Written Exercises, page 173

1. a^3 3. $\frac{1}{x^4}$ 5. $9x^3$ 7. $\frac{-2}{n^3}$ 9. $-3a$
 11. $-4x^3y^2$ 13. $2x^3$ 15. $\frac{6m^2}{n}$
 17. $-4x^5y$ 19. $3x^8y^3z$ 21. $\frac{-7a^4}{c^2}$
 23. $3r^3t^2$ 25. $-6x$ 27. $4abc^2$
 29. $-7a^2b$ 31. $27a^3c^4$ 33. $\frac{8b}{a}$ 35. $\frac{-z}{x^2}$

Puzzle Problems, page 173 Since the number doubles every minute, and the basket is full after 1 hour, the basket was half full in 1 hour - 1 minute, or 59 minutes.

Written Exercises, page 175

1. $a + 3b$ 3. $2x - y$ 5. $4x - 2y$
 7. $8m + 9n$ 9. $a + 9$ 11. $b^2 - ab$
 13. $a^2 + ab + b^2$ 15. $3mn^2 - 1 + 2m$
 17. $x - 3x^2y + 2$ 19. $2rs + 3s^2 - 6r^2$
 21. $-4a^2b + a^2b^2 - 2a^3$ 23. $-2 - 6n$
 25. $3x^2 - 8xy + 9y^2$
 27. $-3xb + 4x^3 - 6b^2$

Written Exercises, page 177

1. $x + 2$ 3. $y + 4$ 5. $x + 2$ 7. $x - 4$
 9. $a - 5$ 11. $x - 6$ 13. $x - 2$

Calculator Activities, page 177 1. 256

3. -4608 5. 221,184

Skills Review, page 178 1. Let $n =$ the number. $2n + 7 = 11$

3. Let $x =$ the number of tickets Clarisse sold. $x + x + 82 = 286$
 5. Let $n =$ the larger number. $2n = 5(n - 8) + 4$
 7. Let $r =$ André's rate. $\frac{1}{2}r +$

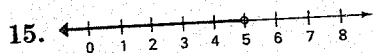
$$\frac{1}{2}(r + 12) = 13$$

Chapter Review Exercises,

- pages 179-180** 1. $2x - 2$ 2. $3x^2 - 6$
 3. $7n + 2$ 4. $5a^2 - 1$ 5. $3n + 8$
 6. $11x - 8y$ 7. $4n^2$ 8. $4x + 7$
 9. $6a^2 + 8b^2$ 10. $6 + 8x^2$ 11. 5
 12. $y - 2$ 13. $2x - 12y$ 14. $4m^2 + 3n$
 15. $4m - 5n$ 16. $-a - 2b$ 17. $7x - 2y$
 18. x 19. $-7x$ 20. $7a + 6b$ 21. a^6
 22. $2a^3$ 23. a^4 24. $-30a^3$ 25. $28x^4$
 26. $24x^3$ 27. $-56y^5$ 28. $18n^5$ 29. a^6
 30. $-a^6$ 31. $-a^6$ 32. $-27x^6$ 33. m^4n^8
 34. $27m^3n^6$ 35. $-x^6y^9$ 36. $9x^2y^8$
 37. $30 + 5n$ 38. $42x + 21$ 39. $a^2 + 4a$
 40. $n^2 - 5n + 6$ 41. $12n^2 - 7n + 1$
 42. $6x^2 - 5xy - y^2$ 43. $2r^2 + 16r + 32$
 44. $10a^2 - 31a - 14$ 45. $3x^2 - 2xy - y^2$
 46. $n^2 - 9n + 18$ 47. $14x^2 - 3x - 2$
 48. $30n^2 + 16n + 2$ 49. $1 + 2n - 8n^2$
 50. $3y^2 + 13y + 4$ 51. $x^2 - 12x + 35$
 52. $n^2 + 8n + 16$ 53. $n^2 - 8n + 16$
 54. $9m^2 - 12mn + 4n^2$
 55. $16a^2 + 24ab + 9b^2$ 56. $3n$ 57. $10x^2$
 58. $-10x$ 59. $-3y$ 60. $-6ab^3$ 61. $5a^3b$
 62. $11n^2$ 63. $-12y$ 64. $6n + 1$
 65. $m + 3$ 66. $2x - 1$ 67. $4ab - 3$
 68. $2a^2 - a + 1$ 69. $3x + 4y + 5$ 70. $\frac{1}{y^4}$
 71. $\frac{3x}{y}$ 72. $\frac{2m^3}{n^5}$ 73. $\frac{3a^2}{c^2}$ 74. $x - 5$
 75. $y + 7$ 76. $m + 3$

Mixed Review, pages 182-183

1. $-x^2 + 2x + 13$ 3. $x = -6$ 5. $n = 1$
 7. -12 9. 17 and 18 11. $y = -7 + 5x$
 13. $-8a^2b^3 + 6ab^2 - 7b$



17. $25a^2 - 10a + 1$ 19. $a = 4$
 21. $-12x^4y^6$ 23. $x^3 - x^2 - 10x + 12$
 25. $-7 < 0 < 2$ 27. $(x^3 + 2x^2)$ cubic
 units 29. $45m^7n^6$ 31. 15 cm
 33. 0, 1, 2, 3 35. $>$ 37. $a = 2$
 39. $h = 9$

CHAPTER 6

Written Exercises, page 187

1. 2 · 3 · 7 3. 2 · 2 · 3 · 3 5. 2 · 5 · 31
 7. 2 · 2 · 2 · 3 · 5 9. 2 · 2 · 2 · 2 · 3 · 13
 11. 3 13. 7 15. 1 17. 11 19. 16
 21. 12 23. 14 25. 25 27. 4 29. 8
 31. 4 33. 20 35. 6 37. 100

Written Exercises, page 189

1. $3(3 + x)$ 3. $2(x - 5)$ 5. $x(3x - 1)$
 7. $2x(x - 3)$ 9. $7n(4n - 1)$
 11. $4x(x - 2)$ 13. $5mn(5 - mn)$
 15. $9x^2(1 - 3y)$ 17. $3(x^2 - 2x + 7)$
 19. $11(5y^2 + 2y + 4)$
 21. $4(a^2 + 3ab - 4b^2)$
 23. $6(x^2 + x + 4xy + 7)$
 25. $25(-2a^2 + b^2 + 3ab)$
 27. $8xy(7x^2y^2 - 9xy - 8)$
 29. $(x + 2)(a - b)$ 31. $(n + y)(m - x)$
 33. $(2 - a)(a + b)$ 35. $(x^2 - 4)(y + 2)$

Written Exercises, page 191

1. $(4 - \pi)r^2$ 3. $(a^2 - b^2)\pi$ 5. 72
 7. $x^2(1 - \pi) + 5(x + 3)$

Written Exercises, pages 194-195

1. 1, 5 3. 2, 4 5. 3, 4 7. 3, 5 9. 3, 6
 11. 3, 7 13. 3, 8 15. 6, 6
 17. $(a + 6)(a + 1)$ 19. $(x + 6)(x + 3)$
 21. $(r + 3)(r + 7)$ 23. $(n + 1)(n + 16)$
 25. $(a + 1)(a + 4)$ 27. $(x + 1)(x + 10)$
 29. $(n + 4)(n + 2)$ 31. $(x + 7)(x + 2)$
 33. $(y + 1)(y + 11)$ 35. $(b + 3)(b + 9)$
 37. $(x + 4)(x + 8)$ 39. $(x + 3)(x + 10)$
 41. $(a + 2)(a + 10)$ 43. $(x + 25)(x + 2)$
 45. $(m + 25)(m + 3)$ 47. $(a + 21)(a + 3)$

Written Exercises, page 197

1. -3, -2 3. -6, -3 5. -6, -4
 7. $(x - 5)(x - 2)$ 9. $(x - 3)(x - 2)$
 11. $(y - 5)(y - 1)$ 13. $(x - 3)(x - 8)$
 15. $(n + 6)(n + 3)$ 17. $(n - 4)(n - 8)$
 19. $(x + 4)(x + 2)$ 21. $(y - 7)(y - 8)$
 23. $(n + 5)(n + 5)$ 25. $(y - 6)(y - 8)$
 27. $(x - 7)(x - 7)$ 29. $(m - 5)(m - 8)$
 31. $(z - 2)(z - 16)$ 33. $(y - 3)(y - 12)$
 35. $(d - 5)(d - 9)$ 37. $(x - 2)(x - 50)$

Written Exercises, page 199

1. yes 3. yes 5. no 7. $(a + 5)^2$
 9. $(x + 4)^2$ 11. $(x - 2)^2$ 13. $(y + 7)^2$
 15. $(n + 9)^2$ 17. $(x - 9)(x - 10)$
 19. $(a - b)^2$ 21. $(1 - 10x)^2$

Written Exercises, page 202

1. -5, 2 3. -4, 1 5. -2, 1 7. -10, 3
 9. 7, -4 11. 14, -1 13. $(x + 7)(x - 3)$
 15. $(x + 9)(x - 2)$ 17. $(b - 3)(b + 4)$
 19. $(n - 6)(n + 3)$ 21. $(x - 5)(x + 4)$
 23. $(y - 1)(y + 15)$ 25. $(b + 6)(b - 4)$
 27. $(b - 10)(b + 3)$ 29. $(x + 14)(x - 2)$
 31. $(y + 8)(y - 4)$ 33. $(y + 9)(y - 3)$
 35. $(m - 4)(m + 16)$ 37. $(n - 9)(n + 7)$
 39. $(y - 6)(y + 7)$ 41. $(y + 7)(y - 8)$
 43. $(c - 4)(c + 20)$ 45. $(a - 4)(a + 25)$

Mixed Practice Exercises, page 203

1. $(x + 9)(x + 1)$ 3. $(n - 6)(n - 2)$
 5. $(y - 3)(y + 4)$ 7. $(y - 7)(y + 3)$
 9. $(x + 3)(x + 7)$ 11. $(b - 1)(b + 5)$
 13. $(b - 7)(b - 1)$ 15. $(y - 4)(y + 5)$
 17. $(x + 7)(x + 5)$ 19. $(n - 7)(n - 8)$
 21. $(x + 9)(x + 6)$ 23. $(n + 9)(n + 5)$
 25. $(n - 10)(n + 5)$ 27. $(y + 26)(y - 2)$
 29. $(a + 4)(a - 11)$ 31. $(c - 6)(c + 10)$
 33. $(m - 4)(m - 10)$ 35. $4x - 4$
 37. $4y - 4$

Written Exercises, page 205

1. $n^2 - 49$ 3. $a^2 - 100$ 5. $m^2 - n^2$
 7. $x^2 - y^2$ 9. $1 - 4x^2$ 11. $9x^2 - 4$
 13. $25y^2 - 4$ 15. $81m^2 - n^2$
 17. $25x^2 - 36$ 19. $25x^2 - 9y^2$
 21. $100x^2 - 25y^2$ 23. $a^4 - 4$ 25. 399
 27. 2496 29. 896 31. 8091 33. 864
 35. 3575 37. 6384 39. 2496

