REVIEW SET 15C (MAINLY 2-D)

1 If $\mathbf{p} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$, $\mathbf{q} = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$, and $\mathbf{r} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$ find: **a** $\mathbf{p} \bullet \mathbf{q}$ **b** $\mathbf{q} \bullet (\mathbf{p} - \mathbf{r})$

2 Using $\mathbf{p} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$, $\mathbf{q} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$ and $\mathbf{r} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$ verify that:

$\mathbf{p} \bullet (\mathbf{q} - \mathbf{r}) = \mathbf{p} \bullet \mathbf{q} - \mathbf{p} \bullet \mathbf{r}$.

3 Determine the value of t if $\begin{bmatrix} 3 \\ 3 - 2t \end{bmatrix}$ and $\begin{bmatrix} t^2 + t \\ -2 \end{bmatrix}$ are perpendicular.

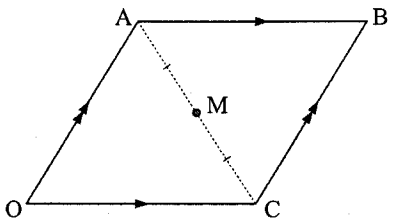
4 Given A(2, 3), B(−1, 4) and C(3, k), find k if $\angle$BAC is a right angle.

5 Find all vectors which are perpendicular to the vector $\begin{bmatrix} -4 \\ 5 \end{bmatrix}$.

6 Find the measure of all angles of triangle KLM for K(−2, 1), L(3, 2) and M(1, −3).

7 Find the angle between the two lines with equations $4x - 5y = 11$ and $2x + 3y = 7$.

8 **a** **Do not** assume any diagonal properties of parallelograms. OABC is a parallelogram with $\overrightarrow{OA} = \mathbf{p}$ and $\overrightarrow{OC} = \mathbf{q}$. M is the midpoint of AC.



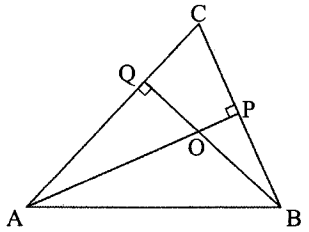
i Find in terms of $\mathbf{p}$ and $\mathbf{q}$:

(1) $\overrightarrow{OB}$ **(2)** $\overrightarrow{OM}$

ii Show using **i** only that O, M and B are collinear and M is the midpoint of OB.

b AP and BQ are altitudes of triangle ABC.

Let $\overrightarrow{OA} = \mathbf{p}$, $\overrightarrow{OB} = \mathbf{q}$ and $\overrightarrow{OC} = \mathbf{r}$.



i Find vector expressions for $\overrightarrow{AC}$ and $\overrightarrow{BC}$ in terms of $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$.

ii Deduce that $\mathbf{q} \bullet \mathbf{r} = \mathbf{p} \bullet \mathbf{q} = \mathbf{p} \bullet \mathbf{r}$.

iii Hence prove that OC is perpendicular to AB.

9 If $\mathbf{a} = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$, find:

a $2\mathbf{a} - 3\mathbf{b}$ **b** $\mathbf{x}$ if $\mathbf{a} - 3\mathbf{x} = \mathbf{b}$ **c** the projection vector of $\mathbf{a}$ on $\mathbf{b}$.

10 If $|\mathbf{a}| = 3$, $|\mathbf{b}| = \sqrt{7}$ and $\mathbf{a} \times \mathbf{b} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ find:

a $\mathbf{a} \bullet \mathbf{b}$ **b** the area of triangle OAB given that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

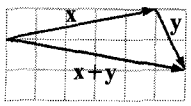
c the volume of tetrahedron OABC if C is the point (1, −1, 2).

REVIEW SET 15D (MAINLY 3-D)

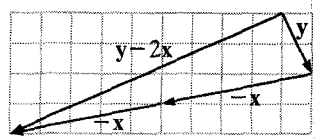
Click on the icon to obtain printable review sets and answers

REVIEW SET 15D

2 **a**



b



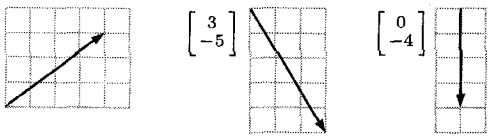
3 **a** $\overrightarrow{PQ}$ **b** $\overrightarrow{PR}$ **4** 4.845 km, 208° **5** **a** $\overrightarrow{AC}$ **b** $\overrightarrow{AD}$

6 **a** $AB = \frac{1}{2}CD$, $AB \parallel CD$ **b** C is midpoint AB

7 **a** $\mathbf{p} + \mathbf{r} = \mathbf{q}$ **b** $\mathbf{l} + \mathbf{m} = \mathbf{k} - \mathbf{j} + \mathbf{n}$

8 **a** $\mathbf{r} + \mathbf{q}$ **b** $-\mathbf{p} + \mathbf{r} + \mathbf{q}$ **c** $\mathbf{r} + \frac{1}{2}\mathbf{q}$ **d** $-\frac{1}{2}\mathbf{p} + \frac{1}{2}\mathbf{r}$

9 **a** $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$ **b** $\begin{bmatrix} 3 \\ -5 \end{bmatrix}$ **c** $\begin{bmatrix} 0 \\ -4 \end{bmatrix}$



10 **a** $\begin{bmatrix} -4 \\ -2 \end{bmatrix}$ **b** $\begin{bmatrix} -1 \\ -13 \end{bmatrix}$ **c** $\begin{bmatrix} -4 \\ 8 \end{bmatrix}$ **11** $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$

12 **a** $\sqrt{17}$ units **b** $\sqrt{13}$ units **c** $\sqrt{10}$ units **d** $\sqrt{109}$ units

13 **a** $\mathbf{p} + \mathbf{q}$ **b** $\frac{3}{2}\mathbf{p} + \frac{1}{2}\mathbf{q}$

14 **a** $\mathbf{x} = \begin{bmatrix} -1 \\ \frac{1}{3} \end{bmatrix}$ **b** $\mathbf{x} = \begin{bmatrix} 1 \\ -10 \end{bmatrix}$ **16** $r = 4,\ s = 7$

17 **a** $\mathbf{q} + \mathbf{r}$ **b** $\mathbf{r} + \mathbf{q}$, $DB = AC$, $DB \parallel AC$

REVIEW SET 15B

1 **a** $\overrightarrow{PQ} = \begin{bmatrix} -3 \\ 12 \\ 3 \end{bmatrix}$ **b** $\sqrt{162}$ units **c** $\sqrt{61}$ units

2 **a** $\begin{bmatrix} 3 \\ -3 \\ 11 \end{bmatrix}$ **b** $\begin{bmatrix} 7 \\ -3 \\ -26 \end{bmatrix}$ **c** $\sqrt{74}$ units **3** $\begin{bmatrix} 8 \\ -8 \\ 7 \end{bmatrix}$

4 $m = 5,\ n = -\frac{1}{2}$ **5** 2 : 3 **6** $t = 2 \pm \sqrt{2}$ **7** 80.3°

8 40.7° **9** **a** $\begin{bmatrix} -6 \\ 1 \\ 3 \end{bmatrix}$ **b** $\sqrt{46}$ units **c** $(-1, 3\frac{1}{2}, \frac{1}{2})$

10 **a** -1 **b** $\begin{bmatrix} 4 \\ -1 \\ 7 \end{bmatrix}$ **c** 60°

11 $\angle K \doteq 123.7°$, $\angle L \doteq 11.3°$, $\angle M \doteq 45.0°$

12 63.95° **13** $c = \frac{50}{3}$

14 **a** $\mathbf{a} \bullet \mathbf{b}$ is a scalar, so $\mathbf{a} \bullet \mathbf{b} \bullet \mathbf{c}$ is a scalar dotted with a vector, which is meaningless.

b $\mathbf{b} \times \mathbf{c}$ must be done first otherwise we have a scalar crossed with a vector which is meaningless.

15 **a** $k = \pm\frac{7}{\sqrt{33}}$ **b** $k = \pm\frac{1}{\sqrt{2}}$

REVIEW SET 15C

1 **a** -13 **b** -36 **3** $t = \frac{2}{3}$ or -3 **4** $k = 6$

5 $k\begin{bmatrix} 5 \\ 4 \end{bmatrix},\ k \neq 0$ **6** $\angle K = 64.44°$, $\angle L = 56.89°$, $\angle M = 58.67°$

7 72.35° or 107.65°

8 **a** **i** **(1)** $\mathbf{p} + \mathbf{q}$ **(2)** $\frac{1}{2}\mathbf{p} + \frac{1}{2}\mathbf{q}$

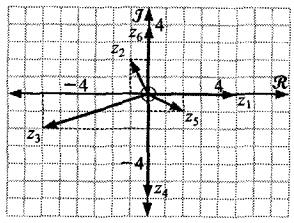
b **i** $\overrightarrow{AC} = -\mathbf{p} + \mathbf{r}$, $\overrightarrow{BC} = -\mathbf{q} + \mathbf{r}$

9 **a** $\begin{bmatrix} 7 \\ -12 \\ -7 \end{bmatrix}$ **b** $\begin{bmatrix} 1 \\ -\frac{5}{3} \\ -\frac{2}{3} \end{bmatrix}$ **c** $\begin{bmatrix} \frac{5}{14} \\ -\frac{5}{7} \\ -\frac{15}{14} \end{bmatrix}$

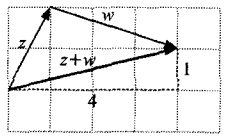
10 **a** ± 7 **b** $\frac{\sqrt{14}}{2}$ units^2 **c** $\frac{7}{6}$ units^3

EXERCISE 16A.1

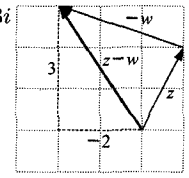
1



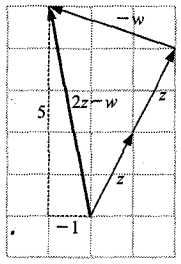
2 **a** $4 + i$



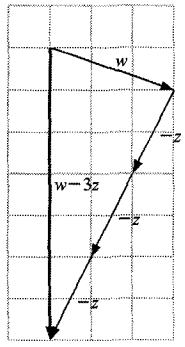
b $-2 + 3i$



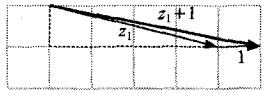
c $-1 + 5i$



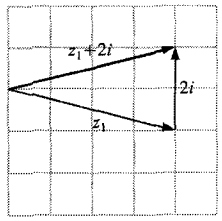
d $-7i$



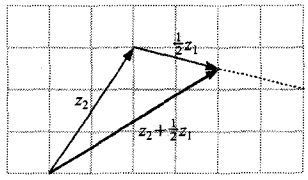
3 **a** $5 - i$



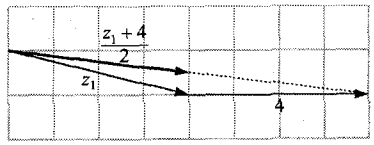
b $4 + i$



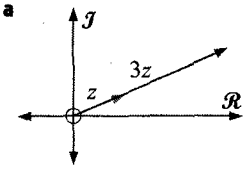
c $4 + \frac{5}{2}i$



d $4 - \frac{1}{2}i$



4 **a**



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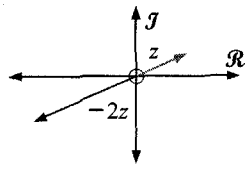
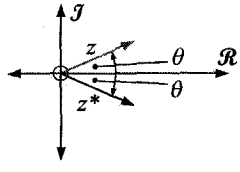


c



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