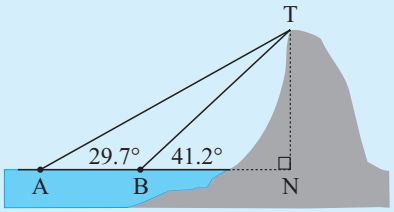
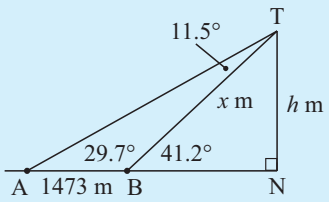
Example 6

The angles of elevation to the top of a mountain are measured from two beacons A and B, at sea.

These angles are as shown on the diagram.

If the beacons are 1473 m apart, how high is the mountain?





$\angle ATB = 41.2^o - 29.7^o$ {exterior angle of Δ}
$= 11.5^o$

We can now find x in ΔABT using the sine rule,

i.e., $\dfrac{x}{\sin 29.7} = \dfrac{1473}{\sin 11.5}$

$\therefore \quad x = \dfrac{1473}{\sin 11.5} \times \sin 29.7$

$\doteq 3660.62.....$

Now, in ΔBNT, $\sin 41.2^o = \dfrac{h}{x} = \dfrac{h}{3660.62....}$

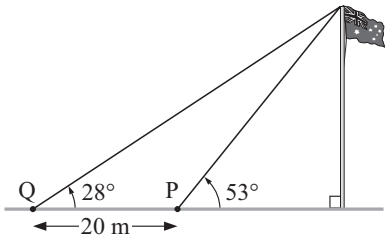
$\therefore \quad h = \sin 41.2^o \times 3660.62....$

$\therefore \quad h \doteq 2410$

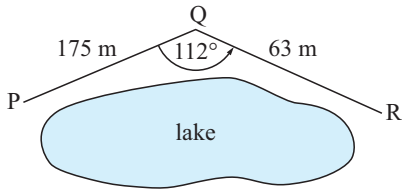
So, the mountain is about 2410 m high.

EXERCISE 12C

1 Manny wishes to determine the height of a flag pole. He takes a sighting of the top of the flagpole from point P. He then moves further away from the flagpole by 20 metres to point Q and takes a second sighting. The information is shown in the diagram alongside. How high is the flagpole?



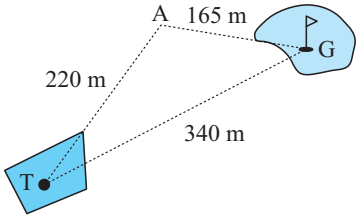
2



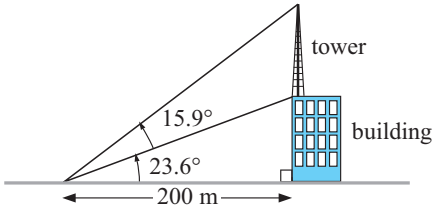
To get from P to R, a park ranger had to walk along a path to Q and then to R as shown.

What is the distance in a straight line from P to R?

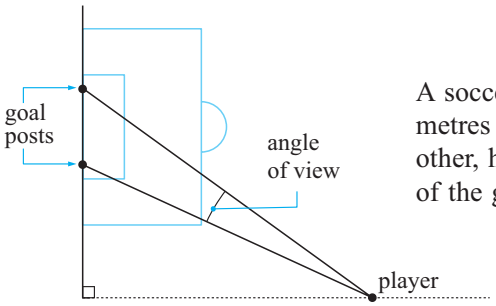
3 A golfer played his tee shot a distance of 220 m to a point A. He then played a 165 m six iron to the green. If the distance from tee to green is 340 m, determine the number of degrees the golfer was off line with his tee shot.



4 A Communications Tower is constructed on top of a building as shown. Find the height of the tower.



5



A soccer goal is 5 metres wide. When a player is 21 metres from one goal post and 19 metres from the other, he shoots for goal. What is the angle of view of the goals that the player sees?

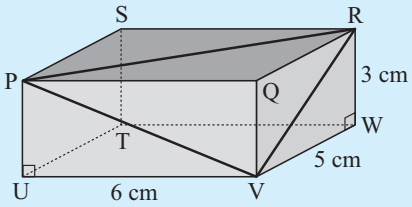
6 A tower 42 metres high, stands on top of a hill. From a point some distance from the base of the hill, the angle of elevation to the top of the tower is 13.2^o. From the same point the angle of elevation to the bottom of the tower is 8.3^o. Find the height of the hill.

7 From the foot of a building I have to look upwards at an angle of 22^o to sight the top of a tree. From the top of the building, 150 metres above ground level, I have to look down at an angle of 50^o below the horizontal to sight the tree top.

a How high is the tree? **b** How far from the building is this tree?

Example 7

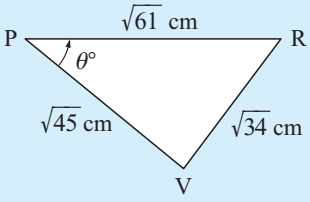
Find the measure of angle RPV.



In ΔRVW, $RV = \sqrt{5^2 + 3^2} = \sqrt{34}$ cm. {Pythagoras}

In ΔPUV, $PV = \sqrt{6^2 + 3^2} = \sqrt{45}$ cm. {Pythagoras}

Likewise in ΔPQR, $PR = \sqrt{6^2 + 5^2} = \sqrt{61}$ cm.



$$\cos\theta = \frac{(\sqrt{61})^2 + (\sqrt{45})^2 - (\sqrt{34})^2}{2\sqrt{61}\sqrt{45}}$$

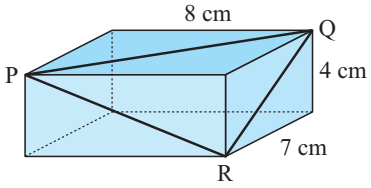
$$= \frac{61 + 45 - 34}{2\sqrt{61}\sqrt{45}}$$

$$= \frac{72}{2\sqrt{61}\sqrt{45}}$$

$$\therefore \quad \theta = \cos^{-1}\left(\frac{36}{\sqrt{61}\sqrt{45}}\right) \doteq 46.6$$

i.e., angle RPV measures 46.6^o.

8

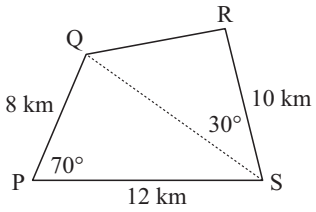


Find the measure of angle PQR in the rectangular box shown.

9 Two observation posts are 12 km apart at A and B. From A, a third observation post C is located such that angle CAB is 42^o while angle CBA is 67^o. Find the distance of C from both A and B.

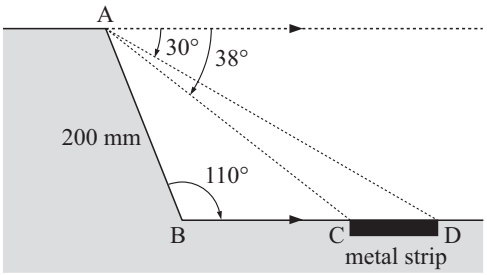
10 Stan and Olga are considering buying a sheep farm and the land agent supplies them with the given accurate sketch. Find the area of the property giving your answer in:

a km^2 **b** hectares.



11 Thabo and Palesa start at point A. They each walk in a straight line at an angle of 120^o to each other. Thabo walks at 6 kmph and Palesa walks at 8 kmph. How far apart are they after 45 minutes?

12 The design of the kerbing cross-section for a driverless-bus roadway is given. The metal strip is inlaid into the concrete and is used to control the direction of travel and speed of the bus. Find the width of the metal strip.



13 An orienteer runs for $4\frac{1}{2}$ km and then turns through an angle of 32^o and runs another 6 km. How far is she from her starting point?

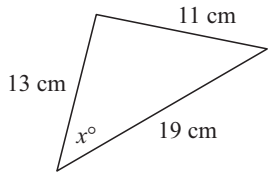
14 Sam and Markus are standing on level ground 100 metres apart. A large tree is due North of Markus and on a bearing of 065^o from Sam. The top of the tree appears at an angle of elevation of 25^o to Sam and 15^o to Markus. Find the height of the tree.

15 A helicopter A, flying at 4000 m, observes two ships B and C. B is 23.8 km from the helicopter and C is 31.9 km from it. The angle of view from the helicopter to B and C (angle BAC) is 83.6^o. How far are the ships apart?

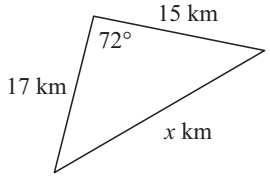
REVIEW SET 12

1 Determine the value of x:

a



b



2 **a** $a = 21.25$ cm **b** $b = 76.9$ cm **c** $c = 5.09$ cm

EXERCISE 12B.2

1 $\angle C = 62.1^o$ or $\angle C = 117.9^o$

2 **a** $\angle A = 49.5^o$ **b** $\angle B = 72.05^o$ or 107.95^o **c** $\angle C = 44.3^o$

3 No, $\frac{\sin 85^o}{11.4} \neq \frac{\sin 27^o}{9.8}$ **4** $\angle ABC = 66^o$, BD $= 4.55$ cm **5** $x = 17.7$, $y = 33.1$

6 **a** 88.7^o or 91.3^o **b** 91.3^o

c .... *cosine rule* as it avoids the *ambiguous case.*

EXERCISE 12C

1 17.7 m **2** 207 m **3** 23.9^o **4** 77.5 m **5** 13.2^o

6 69.1 m **7** **a** 38.0 m **b** 94.0 m **8** 55.1^o

9 AC $= 11.7$ km BC $= 8.49$ km

10 **a** 74.9 km^2 **b** 7490 hectares **11** 9.12 km

12 $\div$ 85 mm **13** 10.1 km **14** 29.2 m **15** 37.6 km

REVIEW SET 12

1 **a** $x = 34.1$ **b** $x = 18.9$ **2** **a** $x = 41.5$ **b** $x = 15.4$

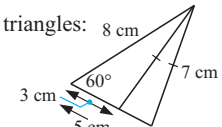
3 AC $= 12.55$ cm, $\angle A = 48.6^o$, $\angle C = 57.4^o$

4 113 cm^2 **5** 7.32 m **6** 204 m

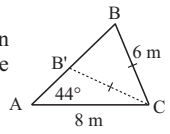
7 530 m, bearing 077.2^o **8** 179 km, bearing 352^o

9 If the unknown is an angle, use the cosine rule to avoid the ambiguous case.

10 **a** $x = 3$ or 5 **b** Kady can draw 2 triangles:



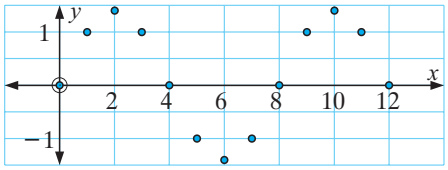
11 **a** The information given could give two triangles:



b $\div 2.23$ m^3

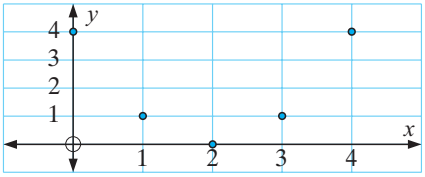
EXERCISE 13A

1 **a**



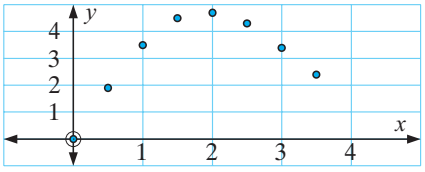
Data exhibits periodic behaviour.

b



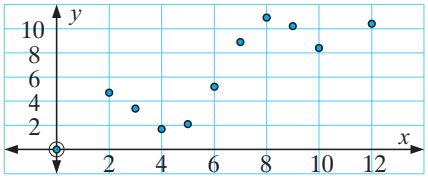
Not enough information to say data is periodic. It may in fact be quadratic.

c



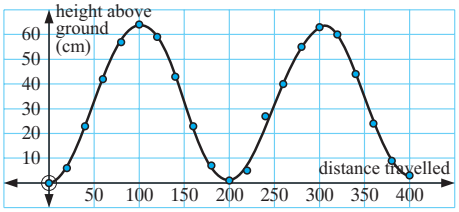
Not enough information to say data is periodic. It may in fact be quadratic.

d



Not enough information to say data is periodic.

2 **a**



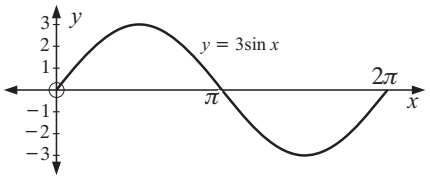
b The data is periodic. **i** $y = 32$ (approx.) **ii** $\div 64$ cm **iii** $\div 200$ cm **iv** $\div 32$ cm

c A curve can be fitted to the data.

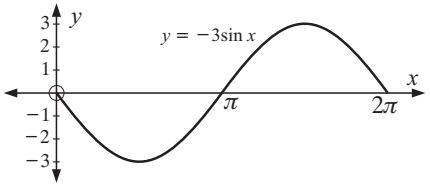
3 **a** periodic **b** periodic **c** periodic **d** not periodic **e** periodic **f** periodic

EXERCISE 13B.1

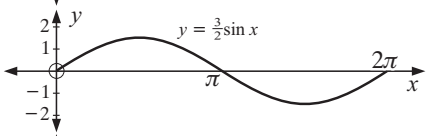
1 **a**



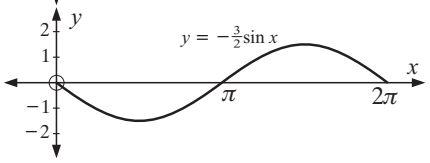
b



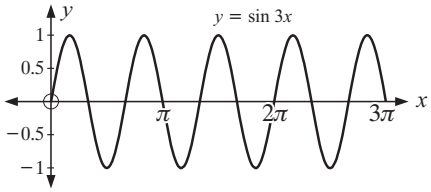
c



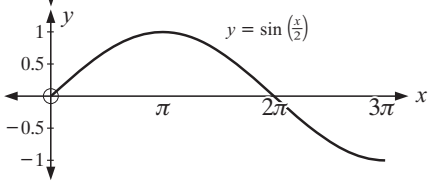
d



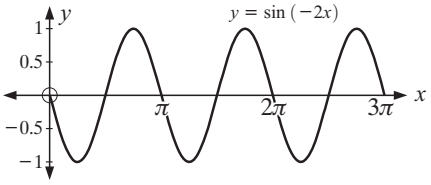
2 **a**



b



c



3 **a** $\frac{\pi}{2}$ **b** $\frac{\pi}{2}$ **c** 6π **d** $\frac{10\pi}{3}$

4 **a** $B = \frac{2}{5}$ **b** $B = 3$ **c** $B = \frac{1}{6}$ **d** $B = \frac{\pi}{2}$ **e** $B = \frac{\pi}{50}$