EXERCISE 7.4

A *In Problems 1–6, find the value of each expression using De Moivre's theorem. Leave your answer in polar form.*

1. $(3e^{(15°)i})^3$ **2.** $(2e^{(30°)i})^8$

3. $(\sqrt{2}e^{(45°)i})^{10}$ **4.** $(\sqrt{2}e^{(60°)i})^8$

5. $(\sqrt{3}+i)^6$ **6.** $(1+\sqrt{3}\,i)^3$

B *Find the value of each expression using De Moivre's theorem, and write the result in exact rectangular form.*

7. $(-1+i)^4$ **8.** $(-\sqrt{3}-i)^4$

9. $(-\sqrt{3}+i)^5$ **10.** $(1-i)^8$

11. $\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)^3$ **12.** $\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^3$

In Problems 13–18, find all nth roots of z for n and z as given. Leave answers in polar form.

13. $z = 4e^{(30°)i}$; $n = 2$ **14.** $z = 16e^{(60°)i}$; $n = 2$

15. $z = 8e^{(90°)i}$; $n = 3$ **16.** $z = 27e^{(120°)i}$; $n = 3$

17. $z = -1+i$; $n = 5$ **18.** $z = 1-i$; $n = 5$

In Problems 19–24, find all nth roots of z for n and z as given. Write answers in polar form and plot in a complex plane.

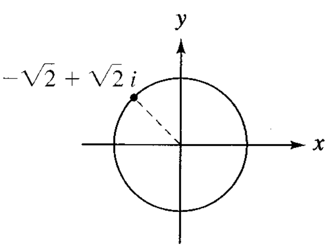
19. $z = -8$; $n = 3$ **20.** $z = -16$; $n = 4$

21. $z = 1$; $n = 4$ **22.** $z = 8$; $n = 3$

23. $z = -i$; $n = 5$ **24.** $z = i$; $n = 6$

25. (A) Show that $-\sqrt{2}+\sqrt{2}\,i$ is a root of $x^4+16=0$. How many other roots does the equation have?

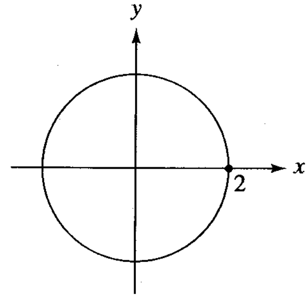
(B) The root $-\sqrt{2}+\sqrt{2}\,i$ is located on a circle of radius 2 in the complex plane, as shown in the figure. Without using the *n*th-root theorem, locate all other roots on the figure, and explain geometrically how you found their location.



(C) Verify that each complex number found in part (B) is a root of $x^4+16=0$. Show your steps.

26. (A) Show that 2 is a root of $x^3-8=0$. How many other roots does the equation have?

(B) The root 2 is located on a circle of radius 2 in the complex plane, as shown in the figure. Without using the *n*th-root theorem, locate all other roots on the figure, and explain geometrically how you found their location.



(C) Verify that each complex number found in part (B) is a root of $x^3-8=0$. Show your steps.

In Problems 27–30, solve each equation for all roots. Write the final answers in exact rectangular form.

27. $x^3+27=0$ **28.** $x^3-27=0$

29. $x^3-64=0$ **30.** $x^3+64=0$

C **31.** Show that

$$r^{1/n}e^{(\theta/n + k360°/n)i}$$

is the same number for $k = 0$ and $k = n$.

32. Show that

$$(r^{1/n}e^{(\theta/n + k360°/n)i})^n = re^{i\theta}$$

for any natural number *n* and any integer *k*.

Solve each equation for all roots. Write final answers in rectangular form, a + bi, where a and b are computed to three decimal places.

33. $x^5-1=0$ **34.** $x^4+1=0$

35. $x^3+5=0$ **36.** $x^5-6=0$

Exercise 7.4

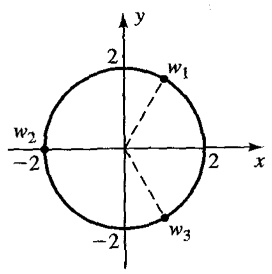
1. $27e^{(45°)i}$ **3.** $32e^{(450°)i} = 32e^{(90°)i}$ **5.** $64e^{(180°)i}$

7. -4 **9.** $16\sqrt{3} + 16i$ **11.** 1 **13.** $2e^{(15°)i}, 2e^{(195°)i}$

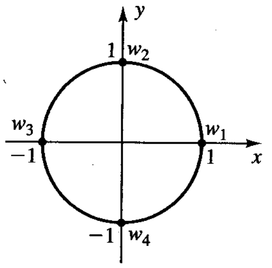
15. $2e^{(30°)i}, 2e^{(150°)i}, 2e^{(270°)i}$

17. $2^{1/10}e^{(27°)i}, 2^{1/10}e^{(99°)i}, 2^{1/10}e^{(171°)i}, 2^{1/10}e^{(243°)i}, 2^{1/10}e^{(315°)i}$

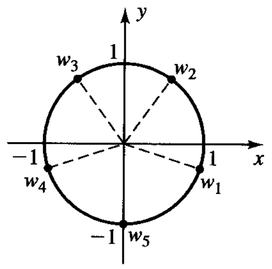
19. $w_1 = 2e^{(60°)i}, w_2 = 2e^{(180°)i}, w_3 = 2e^{(300°)i}$



21. $w_1 = 1e^{(0°)i}, w_2 = 1e^{(90°)i}, w_3 = 1e^{(180°)i}, w_4 = 1e^{(270°)i}$

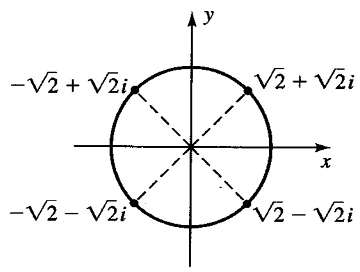


23. $w_1 = 1e^{(-18°)i}, w_2 = 1e^{(54°)i}, w_3 = 1e^{(126°)i}, w_4 = 1e^{(198°)i}$, $w_5 = 1e^{(270°)i}$



25. (A) $(-\sqrt{2} + \sqrt{2}i)^4 + 16 = -16 + 16 = 0$; three

(B) The four roots are equally spaced around the circle. Since there are 4 roots, the angle between successive roots on the circle is $360°/4 = 90°$.



(C) $(\sqrt{2} + \sqrt{2}i)^4 + 16 = -16 + 16 = 0$,
$(-\sqrt{2} - \sqrt{2}i)^4 + 16 = -16 + 16 = 0$,
$(\sqrt{2} - \sqrt{2}i)^4 + 16 = -16 + 16 = 0$

27. $x_1 = 3e^{(60°)i} = \dfrac{3}{2} + \dfrac{3\sqrt{3}}{2}i, x_2 = 3e^{(180°)i} = -3$,

$x_3 = 3e^{(300°)i} = \dfrac{3}{2} - \dfrac{3\sqrt{3}}{2}i$

29. $x_1 = 4e^{(0°)i} = 4, x_2 = 4e^{(120°)i} = -2 + 2\sqrt{3}i$,
$x_3 = 4e^{(240°)i} = -2 - 2\sqrt{3}i$

33. $1, 0.309 + 0.951i, -0.809 + 0.588i, -0.809 - 0.588i$,
$0.309 - 0.951i$

35. $0.855 + 1.481i, -1.710, 0.855 - 1.481i$