Exercises 6.2

Convert the following complex numbers from rectangular form to polar form.

1. $-1 + i$
2. $1 - \sqrt{3}\,i$
3. $-4i$
4. $3 + \sqrt{3}\,i$
5. $6 - 2\sqrt{3}\,i$
6. $4 - 4i$
7. $-1 + \sqrt{3}\,i$
8. -9

Graph each of the following complex numbers. Then convert the complex number from polar form to rectangular form.

9. $5\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$
10. $3\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$
11. $4\left[\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right]$
12. $10\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$
13. $8\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$
14. $6\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$
15. $-6\left[\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right]$
16. $4[\cos(-\pi) + i\sin(-\pi)]$

Exercises 6.2

1. $\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$
3. $4\left[\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right]$
5. $4\sqrt{3}\left[\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right]$
7. $2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$
9. $\frac{5}{\sqrt{2}} + \frac{5}{\sqrt{2}}i$
11. $2 - 2\sqrt{3}\,i$
13. $-4\sqrt{2} - 4\sqrt{2}\,i$
15. $-3\sqrt{3} + 3i$

Exercises 6.3

Find the indicated products, powers, and quotients.

1. $\left[5\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)\right]\cdot\left[2\left(\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}\right)\right]$
2. $\left[\sqrt{2}\left(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right)\right]\cdot\left[13\left(\cos(-\pi)+i\sin(-\pi)\right)\right]$
3. $\left[\sqrt{6}\left(\cos\frac{\pi}{9}+i\sin\frac{\pi}{9}\right)\right]\cdot\left[\sqrt{18}\left(\cos\frac{\pi}{5}+i\sin\frac{\pi}{5}\right)\right]$
4. $\left[3\left(\cos\left(\frac{-\pi}{2}\right)+i\sin\left(\frac{-\pi}{2}\right)\right)\right]\cdot\left[7\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)\right]$
5. $\left[3\left(\cos\frac{2\pi}{5}+i\sin\frac{2\pi}{5}\right)\right]^3$
6. $\left[\sqrt{5}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)\right]^6$
7. $\left[\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right]^8$
8. $\left[2\left(\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}\right)\right]^{10}$
9. $\left[8\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)\right]\div\left[4\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)\right]$
10. $\left[35\left(\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}\right)\right]\div\left[7\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)\right]$
11. $\left[4\left(\cos\frac{\pi}{5}+i\sin\frac{\pi}{5}\right)\right]\div\left[16\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\right]$
12. $\left[3\left(\cos\frac{\pi}{7}+i\sin\frac{\pi}{7}\right)\right]\div\left[\sqrt{3}\left(\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}\right)\right]$
13. $(1-i)^8$
14. $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^6$
15. $(3-\sqrt{3i})^4$
16. $(-2-2i)^8$
17. $(1-i)^8\div(3-\sqrt{3}i)^4$
18. $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^6\div(-2-2i)^8$

Exercises 6.3

1. $10\left(\cos\frac{13\pi}{6}+i\sin\frac{13\pi}{6}\right)$
3. $6\sqrt{3}\left(\cos\frac{14\pi}{45}+i\sin\frac{14\pi}{45}\right)$
5. $27\left(\cos\frac{6\pi}{5}+i\sin\frac{6\pi}{5}\right)$
7. 1
9. $2\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$
11. $\frac{1}{4}\left[\cos\left(-\frac{2\pi}{15}\right)+i\sin\left(-\frac{2\pi}{15}\right)\right]$
13. 16
15. $144\left[\cos\left(-\frac{2\pi}{3}\right)+i\sin\left(-\frac{2\pi}{3}\right)\right]$
17. $\frac{1}{9}\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$

Exercises 6.4

1. Find the 3 third roots of unity. Express them in rectangular form.
2. Find the 8 eighth roots of unity. Express them in rectangular form.
3. Find the 7 seventh roots of unity.
4. Find the 10 tenth roots of unity.
5. Find the 3 cube roots of $2 - 2i$.
6. Find the 5 fifth roots of $-4 - 4i$.
7. Find the 7 seventh roots of $5\left(\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}\right)$.
8. Find the 6 sixth roots of $13\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$.
9. Find the 6 sixth roots of $-64i$.
10. Find the 3 cube roots of $27i$. Express them in rectangular form.
11. Find the 3 cube roots of $-i$. Express them in rectangular form.
12. Find the 2 square roots of $-25i$. Express them in rectangular form.

Exercises 6.4

1. $1, -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} - \frac{\sqrt{3}}{2}i$
3. $1, \cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7}, \cos\frac{4\pi}{7} + i\sin\frac{4\pi}{7}, \cos\frac{6\pi}{7} + i\sin\frac{6\pi}{7},$
   $\cos\frac{8\pi}{7} + i\sin\frac{8\pi}{7}, \cos\frac{10\pi}{7} + i\sin\frac{10\pi}{7}, \cos\frac{12\pi}{7} + i\sin\frac{12\pi}{7}$
5. $\sqrt{2}\left[\cos\left(-\frac{\pi}{12}\right) + i\sin\left(-\frac{\pi}{12}\right)\right], \sqrt{2}\left(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right),$
   $\sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$.
7. $\sqrt[7]{5}\left(\cos\frac{\pi}{35} + i\sin\frac{\pi}{35}\right), \sqrt[7]{5}\left(\cos\frac{11\pi}{35} + i\sin\frac{11\pi}{35}\right),$
   $\sqrt[7]{5}\left(\cos\frac{21\pi}{35} + i\sin\frac{21\pi}{35}\right), \sqrt[7]{5}\left(\cos\frac{31\pi}{35} + i\sin\frac{31\pi}{35}\right),$
   $\sqrt[7]{5}\left(\cos\frac{41\pi}{35} + i\sin\frac{41\pi}{35}\right), \sqrt[7]{5}\left(\cos\frac{51\pi}{35} + i\sin\frac{51\pi}{35}\right),$
   $\sqrt[7]{5}\left(\cos\frac{61\pi}{35} + i\sin\frac{61\pi}{35}\right)$
9. $\sqrt{2}\left[\cos\left(-\frac{\pi}{12}\right) + i\sin\left(-\frac{\pi}{12}\right)\right], \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right),$
   $\sqrt{2}\left(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right), \sqrt{2}\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right),$
   $\sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right), \sqrt{2}\left(\cos\frac{19\pi}{12} + i\sin\frac{19\pi}{12}\right)$
11. $\frac{\sqrt{3}}{2} - \frac{i}{2}, i, -\frac{\sqrt{3}}{2} - \frac{i}{2}$