Exercises 2.6

Use the identities developed in this section to evaluate the following expressions.

1. $\sin\frac{5\pi}{12}\sin\frac{\pi}{12}$
2. $\sin\frac{\pi}{4}\cos\frac{\pi}{12}$
3. $\cos\frac{\pi}{8}\cos\frac{3\pi}{8}$
4. $\cos\frac{7\pi}{12}\cos\frac{\pi}{12}$
5. $\cos\frac{11\pi}{8}\sin\frac{3\pi}{8}$
6. $\sin\frac{\pi}{24}\sin\frac{5\pi}{24}$

Verify that each of the following equations is an identity.

7. $\cos x + \cos y \equiv 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right).$
8. $\cos x - \cos y \equiv -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right).$
9. $\sin x + \sin y \equiv 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right).$
10. $\sin x - \sin y \equiv 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right).$
11. $\sin 3x\sin 7x \equiv \frac{1}{2}[\cos 4x - \cos 10x].$
12. $2\cos 2\alpha\sin 4\alpha \equiv \sin 6\alpha + \sin 2\alpha.$
13. $\cos 4x\cos 2x \equiv \frac{1}{2}[\cos 6x + \cos 2x].$
14. $\sin 5x\sin(-x) \equiv \frac{1}{2}[\cos 6x - \cos 4x].$
15. $\sin 3\phi\cos 7\phi \equiv \frac{1}{2}[\sin 10\phi - \sin 4\phi].$
16. $\cos 3x\cos(-3x) \equiv \frac{1}{2}[1 + \cos 6x].$
17. $\cos 2\theta\,[2\cos 4\theta - 1] \equiv \cos 6\theta.$
18. $\sin 2\beta\,[1 + 2\cos 4\beta] \equiv \sin 6\beta.$
19. $2\sin 2x\cos 2x \equiv 4\cos x(\sin x - 2\sin^3 x).$
20. $\sin\frac{\phi}{4}\cos\frac{\phi}{4} \equiv \frac{1}{2}\sin\frac{\phi}{2}.$
21. $\cos 3x\cos\left(\frac{\pi}{2} - 3x\right)\sec\left(6x - \frac{\pi}{2}\right) \equiv \frac{1}{2}.$
22. $2\cos 2\theta \equiv \frac{\sin 4\theta}{\sin 2\theta}.$
23. $\sin(u+h) - \sin u \equiv 2\cos\left(u + \frac{h}{2}\right)\sin\frac{h}{2}.$
24. $4\sin x\sin 2x\sin 3x \equiv \sin 4x + \sin 2x - \sin 6x.$
25. $\sin 5\phi\cos 5\phi\csc 10\phi \equiv \frac{1}{2}.$
26. $4\cos x\cos 2x\cos 3x \equiv 1 + \cos 2x + \cos 4x + \cos 6x.$
27. $2\cos 2\alpha\cos 2\alpha \equiv 2 + 8\cos^4\alpha - 8\cos^2\alpha.$
28. $\sin 5x\cos x \equiv \sin 3x\cos 3x + \sin 2x\cos 2x.$
29. $2\cos 6\theta\cos 2\theta \equiv (2\cos 4\theta - 1)(\cos 4\theta + 1).$

Verify the following identities.

1. $\sec^2 x\,(1 - \sin^2 x) \equiv 1.$
2. $\left(\sin \frac{\phi}{2} - \cos \frac{\phi}{2}\right)^2 \equiv 1 - \sin \phi.$
3. $\sin 5y \equiv \sin 7y \cos 2y - \cos 7y \sin 2y.$
4. $\cos 2\beta \equiv \cos^4 \beta - \sin^4 \beta.$
5. $\sin \alpha + \cos \alpha \cot \alpha \equiv \csc \alpha.$
6. $\sin 2z \equiv \frac{2 \tan z}{\sec^2 z}.$
7. $\frac{1 - \cos 2\theta}{\sin 2\theta} \equiv \tan \theta.$
8. $\sec^2 \frac{y}{2} \equiv 2 \csc y \tan \frac{y}{2}.$
9. $\tan \beta\,(\sin \beta + \cot \beta \cos \beta) \equiv \sec \beta.$
10. $\sin 3\phi \sin \phi \equiv \sin^2 2\phi - \sin^2 \phi.$
11. $\cos (x + \pi) - \cos (x - \pi) \equiv 0.$
12. $2 \csc 2\gamma \equiv \cot \gamma + \tan \gamma.$
13. $\tan^2 \phi + \cot^2 \phi \equiv \frac{1 - 2 \sin^2 \phi + 2 \sin^4 \phi}{\sin^2 \phi - \sin^4 \phi}.$
14. $\csc^2 \frac{\beta}{2} \equiv \frac{2}{1 - \cos \beta}.$
15. $(\cos x \cos z)^2 - (\sin x \sin z)^2 \equiv \cos^2 z - \sin^2 x.$
16. $\cos^2 2\gamma - 2 \cos^2 \gamma \equiv -1.$
17. $\frac{\cos y + \sin^2 y \sec y}{\sec y} \equiv 1.$
18. $\tan \frac{\alpha}{2} \equiv \csc \alpha - \cot \alpha.$
19. $\cot (\theta + \phi) \equiv \frac{\cot \theta \cot \phi - 1}{\cot \theta + \cot \phi}.$
20. $\tan 3z \equiv \frac{3 \tan z - \tan^3 z}{1 - 3 \tan^2 z}.$
21. $(\cos \alpha + \sin \alpha)^4 \equiv 1 + 4 \cos^2 \alpha \sin^2 \alpha + 4 \cos \alpha \sin \alpha.$
22. $\cos 4\alpha \equiv 1 - 8 \sin^2 \alpha \cos^2 \alpha.$
23. $(\sin \alpha \cos \theta)^2 - (\cos \alpha \sin \theta)^2 \equiv \sin^2 \alpha - \sin^2 \theta.$
24. $\frac{1 + \sin 2x + \cos 2x}{1 - \cos 2x + \sin 2x} \equiv \cot x.$
25. $\frac{1 - \tan\frac{\alpha}{2}}{1 + \tan\frac{\alpha}{2}} \equiv \sec \alpha - \tan \alpha.$