Section 1.3

1. Convert the following to logarithmic form.

(a) $2^4 = 16$ **(b)** $3^{-2} = \frac{1}{9}$

(c) $100 = 10^2$ **(d)** $a^3 = y$

(e) $2^x = p$ **(f)** $x^4 = 2 - k$

2. Convert the following to index form.

(a) $3 = \log_5 125$ **(b)** $-2 = \log_2\left(\frac{1}{4}\right)$

(c) $\log_4 64 = 3$ **(d)** $\log_x 3 = 4$

(e) $\log_3 y = n$ **(f)** $p + 1 = \log_2 (4y)$

3. Check whether the logarithm $\log_x (5 - 2x)$ is defined when:

(a) $x = 2$ **(b)** $x = 0.5$ **(c)** $x = 3$

(d) $x = 2.5$ **(e)** $x = 1$ **(f)** $x = \sqrt{2}$

4. Solve the following equations.

(a) $\log_2 x = 3$ **(b)** $\log_x 9 = 2$

(c) $x = \log_4 8$ **(d)** $\log_3 (x - 2) = 1$

(e) $\log_2 (2x + 1) = -3$ **(f)** $\log_9 \sqrt{27} = x$

(g) $\log_x (6x - 8) = 2$ **(h)** $\log_x 8 = \frac{3}{2}$

5. Evaluate.

(a) $\log_4 4 - 3 \log_2 2$ **(b)** $\log_2 1 + 2 \log_5 5$

(c) $(3 - \log_3 3)^3$ **(d)** $\left(\frac{3 \log_x x + 2}{4 - 2 \log_5 1}\right)^2$

(e) $\log_2 (6 - 5 \log_7 7)$

6. Given that $\log_4 x = 2$ and $\log_2 y = 3$, evaluate $\frac{x}{y}$.

7. Given that $\log_3 x = a$ and $\log_{\sqrt{3}} y = b$, express xy^2 as a power of 3.

8. Solve the following simultaneous equations.

(a) $\log_x 16 = 4$ and $\log_2 y = x$ **(b)** $\log_y x = 2$ and $xy = 8$

***9.** Given that $\log_4 y = a$ and $\log_8 (2y) = b$, show that $2a = 3b - 1$.

10. If $\log_2 (\log_3 x) = \log_5 5$, find x.

From New Additional Mathematics, Ho Soo Thong & Khoe Nyak Hi

NOTE: This book uses the notation lg to mean log base 10

Section 1.4

1. Evaluate the following logarithms.

(a) $\log_2 4$ **(b)** $\lg\left(\frac{1}{10}\right)$ **(c)** $\log_2 8$

(d) $\log_3 27$ **(e)** $\log_2\left(\frac{1}{4}\right)$ **(f)** $\log_9 \sqrt{3}$

2. Evaluate the following expressions.

(a) $\log_8 2 + \log_8 4$ **(b)** $\log_3 36 - \log_3 12$

(c) $\log_2 60 - \log_2 15$ **(d)** $\log_3 4 + \log_3 2 - \log_3 72$

(e) $\log_6 54 - 2\log_6 3$ **(f)** $\log_5 4 + 2\log_5 3 - 2\log_5 6$

3. Simplify and express each of the following as a single logarithm.

(a) $\log_a 8 - 2\log_a 4$ **(b)** $2\log_x 5 - 3\log_x 2 + \log_x 4$

(c) $\lg\left(\frac{8}{75}\right) - 2\lg\left(\frac{3}{5}\right) + 4\lg\left(\frac{3}{2}\right)$

(d) $2\lg(x+2) + \lg(x+1) - \lg(x^2+3x+2)$

4. Evaluate the following:

(a) $\log_a \sqrt{a}$ **(b)** $\log_a \frac{1}{a^3}$

(c) $\frac{\log_a 9}{2\log_a 27}$ **(d)** $\log_{\sqrt{a}} a^2$

5. Given that $\log_a 3 = 0.477$ and $\log_a 5 = 0.699$, evaluate the following:

(a) $\log_a 15$ **(b)** $\log_a 3\sqrt{5}$ **(c)** $\log_a 0.6$

(d) $\frac{\log_a 25}{\log_a 3a}$ **(e)** $\log_a(5a^2)$ **(f)** $\log_a\left(\frac{9}{5a}\right)$

6. Given that $\log_4 3 = a$ and $\log_4 5 = b$, express the following in terms of a and b.

(a) $\log_4 45$ **(b)** $\log_4 20$ **(c)** $\log_4 75$

(d) $\log_4(0.6)$ **(e)** $\log_4(0.75)$ **(f)** $\log_4(1.8)$

7. Given that $\lg x = p$ and $\lg y = q$, express the following in terms of p and q.

(a) $\lg(xy^2)$ **(b)** $\lg\left(\frac{10x}{y}\right)$ **(c)** $\lg\sqrt{10x^3y}$

(d) $\lg\left(\frac{100\sqrt{x}}{y^2}\right)$ **(e)** xy **(f)** $\lg(y^x)$

8. Find y in terms of x when

(a) $\lg y = 1 + 3\lg x$, **(b)** $\lg(y+1) = 2 - \frac{1}{2}\lg x$,

(c) $2\log_3 y - 4 = 3\log_3(x+2)$, **(d)** $3 + \log_2(x+y) = \log_2(x-2y)$.

9. If $\log_2(y+1) = 2\log_2 x + c$ and $y = 3$ when $x = 2$, find y in terms of x.

10. Express each of the following as a single logarithm.

(a) $2 + \log_3 5$ **(b)** $3 - 2\lg 5$ **(c)** $3\log_a 2 - 4 + \log_a a^3$

Section 1.5

For questions 1 to 10, solve for x.

1. $\log_2 (x - 1) = \log_2 (4x - 7)$

2. $\log_3 (x + 2) + \log_3 (x - 2) = \log_3 (2x - 1)$

3. $\lg 18 + \lg\left(\frac{1}{3}x\right) - \lg (x + 1) = 0$

4. $\log_3 x + \log_3 (x + 2) = 1$

5. $2 \lg 5 - \lg (x + 2) = 1 - \lg (2x - 1)$

6. $\log_2 (x - 2) + \log_2 (8 - x) - \log_2 (x - 5) = 3$

7. $\log_2 (x - 1)^2 = 2 + \log_2 (x + 2)$

8. $\log_3 (x + 2) + \log_3 (10 - x) - 3 = 0$

9. $3 \log_x 2 + \log_x 18 = 2$

10. $\log_p 2 + \log_p (x - 1) = 0$

11. Evaluate.

(a) $\log_3 5 \,.\, \log_5 27$ **(b)** $\dfrac{\log_5 4 \,.\, \log_2 10}{\log_{25}\sqrt{10}}$

12. If a, b and c are positive numbers other than 1, show that

$$\log_b a \,.\, \log_c b \,.\, \log_a c = 1.$$

13. By using the substitution $y = \log_3 x$ or otherwise, solve the equation

(a) $\log_3 x + 2 = 3 \log_x 3$, **(b)** $\log_3 x^3 = (\log_3 x)^3$.

14. Solve the following equations.

(a) $\log_3 x = 9 \log_x 3$ **(b)** $4 \log_4 x - 9 \log_x 4 = 0$

(c) $\log_2 x = \log_4 (x + 6)$ **(d)** $\log_5 (5 - 4x) = \log_{\sqrt{5}} (2 - x)$

***15.** If $2 \log_a x = 1 + \log_a (7x - 10a)$, find x in terms of a.

***16.** Find x for which $27 \times 3^{\lg x} = 9^{1 + \lg (x - 20)}$.

NOTE: The notation lg means log base

Section 1.6

In this exercise, give all answers correct to 3 significant figures where necessary.

1. Evaluate.

(a) $\lg 9$ **(b)** $\lg 24.3$ **(c)** $2 \ln 30$

(d) $\dfrac{4}{\lg 6}$ **(e)** $3 \ln 4 - \ln 6$ **(f)** $\ln 5 - \lg 3$

(g) $\lg 5 + 2 \lg 3$ **(h)** $\lg (\lg 90)$ **(i)** $(\ln 8)^2$

(j) $\ln (\lg 7)$ **(k)** $\dfrac{\lg 5}{\lg 2} + 1$ **(l)** $\dfrac{\ln 5}{\ln 2} + 1$

2. Evaluate the following by first converting them to common or natural logarith

(a) $\log_5 7$ **(b)** $\log_3 11$ **(c)** $\log_4 (5.3)$

(d) $\log_{\frac{1}{2}} 9$ **(e)** $\log_6 \pi$ **(f)** $\log_{2.5} (6.7)$

3. For each of the following, find y in terms of x.

(a) $10^y = x + 1$ **(b)** $e^{1-y} = 3x$ **(c)** $\ln (y + 1) = x$

(d) $2 \lg y = x - 2$ **(e)** $e^{2y} = x - 4$ **(f)** $\ln (x + y) - 4x = ($

4. Solve for x.

(a) $\lg x = 0.61$ **(b)** $(\ln x)^2 = 3$ **(c)** $\ln x = \lg 2$

(d) $\lg 3x = 9$ **(e)** $\ln 2 \cdot \ln 4x = 3$ **(f)** $\lg (x - 2) = (\lg 3)^2$

(g) $\ln 4x = \lg 3 \cdot \lg 5$ **(h)** $\lg (2x + 1) = \log_2 3$ **(i)** $\lg (x - 1) = \ln (e^2 - 1)$

5. Evaluate $10^{\log_{10} 5}$, $e^{\log_e 4}$ and $3^{\log_3 7}$. Deduce the value of $4^{\log_4 3}$ and verify your answer using a calculator.

6. Solve the equations, giving answers correct to 4 significant figures.

(a) $\ln (\lg 3x) = 0.2$ **(b)** $\ln (4 - x) = \dfrac{1}{\lg 2} + \dfrac{2}{\lg 3}$ **(c)** $2 \log_3 x = \log_4 x + 1$

Section 1.7

Solve the equations in questions 1 to 15.

1. $5^x = 9$ **2.** $(1.6)^x = 21$ **3.** $2(3^x) = 5$

4. $4 - 7^{2x} = 1$ **5.** $e^x = 7$ **6.** $e^{3x} = 14$

7. $4e^{2x} = 21$ **8.** $e^{4x} - 125 = 0$ **9.** $3^{x+1} = 12$

10. $4^{2x-3} = 20$ **11.** $e^{1+x} = 19$ **12.** $e^{\frac{1}{2}x} = 0.7$

13. $(4.1)^x = \pi$ **14.** $6^{\frac{2}{x}} = 4$ **15.** $e^{x^2} = 312$

16. Given $3^{x+1} \cdot 2^{x-2} = 21$, show that $6^x = 28$. Hence or otherwise, find x.

17. Find x, correct to 3 significant figures, in each of the following:
(a) $5^{x-1} \cdot 3^{x+2} = 10$ **(b)** $2^{2x} \cdot 5^{x+1} = 7$
(c) $4(3^{2x}) = e^x$ **(d)** $3^x \cdot 10^{2x} = 4 \cdot 20^{x-2}$

18. Using the substitution $y = e^x$, solve the following equations.
(a) $2e^{2x} - 3e^x = 2$ **(b)** $e^x = 7 - 12e^{-x}$ **(c)** $e^{3x} + 2e^x = 3e^{2}$

19. Express the following in the form $\ln x = ax + b$ and find a and b.
(a) $x^3 = e^{6x-1}$ **(b)** $xe^{-x} = 2.46$ **(c)** $(xe^x)^2 = 30e^{-x}$

20. Given $y = 5e^{0.2x}$, find **(a)** y when $x = 3$, **(b)** x when $y = 12$.

21. Solve the simultaneous equations $\ln(3x - y) = 2\ln 6 - \ln 9$ and $\frac{(e^x)^2}{e^y} = e$.

22. Using suitable substitutions, solve for x.
(a) $9^x - 4 = 3^{x+1}$ **(b)** $2e^x = 7\sqrt{e^x} - 3$

CHAPTER 1

Exercise 1.1 *(p. 2)*

1. **(a)** 1 **(b)** 4 **(c)** 144 **(d)** $\frac{1}{2}$ **(e)** 5 **(f)** $\frac{1}{3}$

2. **(a)** 54 **(b)** 4

3. **(a)** $8y$ **(b)** $\frac{1}{2}y^2$ **(c)** $\frac{12}{y}$ **(d)** $\frac{32}{y^3}$ **(e)** $\frac{1}{2}y^2$ **(f)** $y^3 - \frac{1}{y^2}$

4. **(a)** $18y^2$ **(b)** $\frac{1}{6}yz$ **(c)** $\frac{z^2}{y}$

Exercise 1.2 *(p. 5)*

1. **(a)** $\frac{3}{2}$ **(b)** $\frac{5}{2}$ **(c)** -2 **(d)** $-\frac{5}{2}$ **(e)** $\frac{1}{2}$ **(f)** 2
(g) 2, -2 **(h)** 2 **(i)** $\frac{2}{3}$ **(j)** $\frac{5}{4}$ **(k)** 2 **(l)** -6, 2
(m) -2, 4 **(n)** -2, 1

2. $a = 3$, $n = 2$

3. **(a)** $x = -\frac{4}{9}$, $y = \frac{1}{9}$ **(b)** $x = 2$, $y = 1$ **(c)** $x = 1$, $y = -1$

4. $m = 4$, $n = 3$ **5.** 3

6. **(a)** 0, 1 **(b)** 0, 2

7. **(a)** 0, 1 **(b)** 1, 3 **(c)** $-\frac{1}{2}$, $\frac{1}{2}$ **(d)** 0 **(e)** -2, 0 **(f)** 3

9. **(a)** 2 **(b)** ± 2 **(c)** 6 **(d)** 3 **(e)** 2 **(f)** 2

10. $r = 3$, $k = \frac{2}{3}$ **11.** $x = 1$, $y = -1$ or $x = 2$, $y = 1$ **12.** 4

Exercise 1.3 *(p. 9)*

1. **(a)** $4 = \log_2 16$ **(b)** $-2 = \log_3\left(\frac{1}{9}\right)$ **(c)** $2 = \log_{10} 100$
(d) $3 = \log_a y$ **(e)** $x = \log_2 p$ **(f)** $4 = \log_x(2 - k)$

2. **(a)** $5^3 = 125$ **(b)** $2^{-2} = \frac{1}{4}$ **(c)** $4^3 = 64$
(d) $x^4 = 3$ **(e)** $3^n = y$ **(f)** $2^{p+1} = 4y$

3. **(c)**, **(d)** and **(e)** are not defined

4. **(a)** 8 **(b)** 3 **(c)** $\frac{3}{2}$ **(d)** 5
(e) $-\frac{7}{16}$ **(f)** $\frac{3}{4}$ **(g)** 2, 4 **(h)** 4

5. **(a)** -2 **(b)** 2 **(c)** 8 **(d)** $\frac{25}{16}$ **(e)** 0

6. 2 **7.** 3^{a+b}

8. **(a)** $x = 2$, $y = 4$ **(b)** $x = 4$, $y = 2$ **10.** 9

Exercise 1.4 *(p. 14)*

1. (a) 2 (b) -1 (c) 3 (d) 3 (e) -2 (f) $\frac{1}{4}$

2. (a) 1 (b) 1 (c) 2 (d) -2 (e) 1 (f) 0

3. (a) $-\log_a 2$ (b) $\log_x\left(\frac{25}{2}\right)$ (c) $\lg\left(\frac{3}{2}\right)$ (d) $\lg(x+2)$

4. (a) $\frac{1}{2}$ (b) -3 (c) $\frac{1}{3}$ (d) 4

5. (a) 1.176 (b) 0.826 5 (c) -0.222 (d) 0.947 (e) 2.699 (f) -0.745

6. (a) $2a+b$ (b) $1+b$ (c) $a+2b$ (d) $a-b$ (e) $a-1$ (f) $2a-b$

7. (a) $p+2q$ (b) $1+p-q$ (c) $\frac{1}{2}(1+3p+q)$

(d) $2+\frac{1}{2}p-2q$ (e) 10^{p+q} (f) $q(10^p)$

8. (a) $y=10x^3$ (b) $y=\frac{100}{\sqrt{x}}-1$ (c) $y=9(x+2)^{\frac{3}{2}}$ (d) $y=-0.7x$

9. $y=x^2-1$

10. (a) $\log_3 45$ (b) $\lg 40$ (c) $\log_a\left(\frac{8}{a}\right)$

Exercise 1.5 *(p. 18)*

1. 2 **2.** 3 **3.** $\frac{1}{5}$ **4.** 1 **5.** $\frac{9}{8}$

6. 6 **7.** $-1, 7$ **8.** 1, 7 **9.** 12 **10.** $1\frac{1}{2}$.

11. (a) 3 (b) 8 **13.** (a) $3, \frac{1}{27}$ (b) $1, 3^{\sqrt{3}}, 3^{-\sqrt{3}}$

14. (a) $27, \frac{1}{27}$ (b) $8, \frac{1}{8}$ (c) 3 (d) $-1, 1$

15. $2a, 5a$ **16.** 40

Exercise 1.6 *(p. 20)*

1. (a) 0.954 (b) 1.39 (c) 6.80 (d) 5.14 (e) 2.37 (f) 1.13
(g) 1.65 (h) 0.291 (i) 4.32 (j) -0.168 (k) 3.32 (l) 3.32

2. (a) 1.21 (b) 2.18 (c) 1.20 (d) -3.17 (e) 0.639 (f) 2.08

3. (a) $y=\lg(x+1)$ (b) $y=1-\ln 3x$ (c) $y=e^x-1$

(d) $y=10^{\frac{1}{2}x-1}$ (e) $y=\frac{1}{2}\ln(x-4)$ (f) $y=e^{4x}-x$

4. (a) 4.07 (b) 5.65, 0.177 (c) 1.35 (d) 3.33×10^8 (e) 18.9
(f) 3.69 (g) 0.349 (h) 18.7 (i) 72.5

5. 5; 4; 7; 3 **6.** (a) 5.550 (b) $-1\,829$ (c) 2.484

Exercise 1.7 *(p. 22)*

1. 1.37 **2.** 6.48 **3.** 0.834 **4.** 0.282 **5.** 1.95 **6.** 0.880
7. 0.829 **8.** 1.21 **9.** 1.26 **10.** 2.58 **11.** 1.94 **12.** -0.713
13. 0.811 **14.** 2.58 **15.** ±2.40 **16.** 1.86
17. (a) 0.633 (b) 0.112 (c) -1.16 (d) -1.70
18. (a) 0.693 (b) 1.10, 1.39 (c) 0, 0.693

19. (a) $a=2, b=-\frac{1}{3}$ (b) $a=1, b=0.9$ (c) $a=-1.5, b=1.7$

20. (a) 9.11 (b) 4.38 **21.** $x=3, y=5$
22. (a) 1.26 (b) $-1.39, 2.20$