

EXERCISES 2.5

1-4 ■ Find dy/dx and $dy/dx|_{x=1}$ in two ways: (a) using the Chain Rule and (b) without using the Chain Rule, as in Example 2.

1. $y = u^2, \quad u = x^2 + 2x + 3$

2. $y = u^2 - 2u + 3, \quad u = 5 - 6x$

3. $y = u^3, \quad u = x + (1/x)$

4. $y = u - u^2, \quad u = \sqrt{x} + \sqrt[3]{x}$

5-48 ■ Find the derivative of the function.

5. $F(x) = (x^2 + 4x + 6)^5$

6. $F(x) = (x^3 - 5x)^4$

7. $G(x) = (3x - 2)^{10}(5x^2 - x + 1)^{12}$

8. $g(t) = (6t^2 + 5)^3(t^3 - 7)^4$

9. $f(t) = (2t^2 - 6t + 1)^{-8}$

10. $f(t) = \frac{1}{(t^2 - 2t - 5)^4}$

11. $g(x) = \sqrt{x^2 - 7x}$

12. $k(x) = \sqrt[3]{1 + \sqrt{x}}$

37. $y = \frac{1 + \sin 2x}{1 - \sin 2x}$

38. $y = x \sin \frac{1}{x}$

39. $y = \tan^2(x^3)$

40. $y = (\sin \sqrt{x^2 + 1})^{\sqrt{2}}$

41. $y = \cos^2(\cos x) + \sin^2(\cos x)$

42. $y = \sin(\sin(\sin x))$

43. $y = \sqrt{x + \sqrt{x}}$

44. $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$

45. $f(x) = [x^3 + (2x - 1)^3]^3$

46. $g(t) = \sqrt[4]{(1 - 3t)^4 + t^4}$

47. $y = \sin(\tan \sqrt{\sin x})$

48. $y = \sqrt{\cos(\sin^2 x)}$

49-52 ■ Find the equation of the tangent line to the curve at the given point.

49. $y = (x^3 - x^2 + x - 1)^{10}, \quad (1, 0)$

50. $y = \sqrt{x + (1/x)}, \quad (1, \sqrt{2})$

51. $y = \frac{8}{\sqrt{4 + 3x}}, \quad (4, 2)$

52. $y = \sin x + \cos 2x, \quad (\pi/6, 1)$

13. $h(t) = \left(t - \frac{1}{t}\right)^{3/2}$

15. $F(y) = \left(\frac{y-6}{y+7}\right)^3$

17. $f(z) = \frac{1}{\sqrt[3]{2z-1}}$

19. $y = (2x - 5)^4(8x^2 - 5)^{-3}$

21. $y = \tan 3x$

23. $y = \cos(x^3)$

25. $y = (1 + \cos^2 x)^6$

27. $y = \cos(\tan x)$

29. $y = \sec^2 2x - \tan^2 2x$

31. $y = \csc(x/3)$

33. $y = \sin^3 x + \cos^3 x$

35. $y = \sin \frac{1}{x}$

14. $F(s) = \sqrt{s^3 + 1}(s^2 + 1)^4$

16. $s(t) = \sqrt[4]{\frac{t^3 + 1}{t^3 - 1}}$

18. $f(x) = \frac{x}{\sqrt{7 - 3x}}$

20. $y = (x^2 + 1)\sqrt[3]{x^2 + 2}$

22. $y = 4 \sec 5x$

24. $y = \cos^3 x$

26. $y = \tan(x^2) + \tan^2 x$

28. $y = \sin(\sin x)$

30. $y = \sqrt{1 + 2 \tan x}$

32. $y = \cot \sqrt[3]{1 + x^2}$

34. $y = \sin^2(\cos 4x)$

36. $y = \frac{\sin^2 x}{\cos x}$

53. (a) Find an equation of the tangent line to the curve $y = \tan(\pi x^2/4)$ at the point $(1, 1)$.



(b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.

54. (a) The curve $y = |x|/\sqrt{2 - x^2}$ is called a **bullet-nose curve**. Find an equation of the tangent line to this curve at the point $(1, 1)$.



(b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.

55. (a) If $f(x) = \sqrt{1 - x^2}/x$, find $f'(x)$.



(b) Check to see that your answer to part (a) is reasonable by comparing the graphs of f and f' .

56. (a) If $f(x) = 1/(\cos^2 \pi x + 9 \sin^2 \pi x)$, find $f'(x)$.



(b) Check to see that your answer to part (a) is reasonable by comparing the graphs of f and f' .

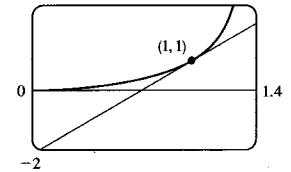
57. Find all points on the graph of the function $f(x) = 2 \sin x + \sin^2 x$ at which the tangent line is horizontal.

58. Find the x -coordinates of all points on the curve $y = \sin 2x - 2 \sin x$ at which the tangent line is horizontal.

59. Suppose that $F(x) = f(g(x))$ and $g(3) = 6$, $g'(3) = 4$, $f'(3) = 2$, and $f'(6) = 7$. Find $F'(3)$.

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1. $4u(x+1)$, 48 3. $3u^2(1 - 1/x^2)$, 0
 5. $F'(x) = 10(x^2 + 4x + 6)^4(x+2)$
 7. $G'(x) = 6(3x-2)^9(5x^2-x+1)^{11}(85x^2-51x+9)$
 9. $f'(t) = -16(2t^2-6t+1)^{-9}(2t-3)$
 11. $g'(x) = (2x-7)/(2\sqrt{x^2-7x})$
 13. $h'(t) = \frac{3}{2}(t-1/t)^{1/2}(1+1/t^2)$
 15. $F'(y) = 39(y-6)^2/(y+7)^4$
 17. $f'(z) = -\frac{2}{5}(2z-1)^{-6/5}$
 19. $y' = 8(2x-5)^3(8x^2-5)^{-4}(-4x^2+30x-5)$
 21. $y' = 3\sec^2 3x$ 23. $y' = -3x^2 \sin(x^3)$
 25. $y' = -12 \cos x \sin x (1 + \cos^2 x)^5$
 27. $y' = -\sin(\tan x) \sec^2 x$ 29. $y' = 0$
 31. $y' = -(1/3) \csc(x/3) \cot(x/3)$
 33. $y' = 3 \sin x \cos x (\sin x - \cos x)$
 35. $y' = -\cos(1/x)/x^2$ 37. $y' = 4(\cos 2x)/(1 - \sin 2x)^2$
 39. $y' = 6x^2 \tan(x^3) \sec^2(x^3)$ 41. $y' = 0$
 43. $y' = [1 + 1/(2\sqrt{x})]/(2\sqrt{x + \sqrt{x}})$
 45. $f'(x) = 9[x^3 + (2x-1)^3]^2(9x^2 - 8x + 2)$
 47. $y' = \cos(\tan \sqrt{\sin x}) (\sec^2 \sqrt{\sin x}) [1/(2\sqrt{\sin x})] (\cos x)$
 49. $y = 0$ 51. $3x + 16y = 44$
 53. (a) $y = \pi x - \pi + 1$ (b) $\frac{3}{3}$



55. (a) $-1/(x^2\sqrt{1-x^2})$
 57. $((\pi/2) + 2n\pi, 3)$, $((3\pi/2) + 2n\pi, -1)$, n any integer
 59. 28 61. $v(t) = (5\pi/2) \cos(10\pi t)$ cm/s
 63. (a) $dB/dt = (7\pi/54) \cos(2\pi t/5.4)$ (b) 0.16
 65. (a) On $(0, \infty)$ (b) $G'(x) = h'(\sqrt{x})/(2\sqrt{x})$
 67. (a) $F'(x) = -\sin x f'(\cos x)$ (b) $G'(x) = -\sin(f(x))f'(x)$
 69. 0 75. $f'(x) = x/|x|$
 77. $h'(x) = |2x-1| + 2x(2x-1)/|2x-1|$