EXERCISES 2.2

1–34 ■ Differentiate each function.

1. $f(x) = x^2 - 10x + 100$

2. $g(x) = x^{100} + 50x + 1$

3. $V(r) = \frac{4}{3}\pi r^3$

4. $s(t) = t^8 + 6t^7 - 18t^2 + 2t$

5. $F(x) = (16x)^3$

6. $G(y) = (y^2 + 1)(2y - 7)$

7. $Y(t) = 6t^{-9}$

8. $R(x) = \dfrac{\sqrt{10}}{x^7}$

9. $g(x) = x^2 + \dfrac{1}{x^2}$

10. $f(t) = \sqrt{t} - \dfrac{1}{\sqrt{t}}$

11. $h(x) = \dfrac{x + 2}{x - 1}$

12. $f(u) = \dfrac{1 - u^2}{1 + u^2}$

13. $G(s) = (s^2 + s + 1)(s^2 + 2)$

14. $H(t) = \sqrt[3]{t}\,(t + 2)$

15. $y = \dfrac{x^2 + 4x + 3}{\sqrt{x}}$

16. $y = \dfrac{\sqrt{x} - 1}{\sqrt{x} + 1}$

17. $y = \sqrt{5x}$

18. $y = x^{4/3} - x^{2/3}$

19. $y = \dfrac{1}{x^4 + x^2 + 1}$

20. $y = x^2 + x + x^{-1} + x^{-2}$

21. $y = ax^2 + bx + c$

22. $y = A + \dfrac{B}{x} + \dfrac{C}{x^2}$

23. $y = \dfrac{3t - 7}{t^2 + 5t - 4}$

24. $y = \dfrac{4t + 5}{2 - 3t}$

25. $y = x + \sqrt[5]{x^2}$

26. $y = x^4 - \sqrt[4]{x}$

27. $u = x^{\sqrt{2}}$

28. $u = \sqrt[3]{t^2} + 2\sqrt{t^3}$

29. $v = x\sqrt{x} + \dfrac{1}{x^2\sqrt{x}}$

30. $v = \dfrac{6}{\sqrt[3]{t^5}}$

31. $f(x) = \dfrac{x}{x + \dfrac{c}{x}}$

32. $f(x) = \dfrac{ax + b}{cx + d}$

33. $f(x) = \dfrac{x^5}{x^3 - 2}$

34. $s = \sqrt{t}\,(t^3 - \sqrt{t} + 1)$

35. The general polynomial of degree n has the form

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

where $a_n \neq 0$. Find the derivative of P.

36–39 ■ Find the equation of the tangent line to the given curve at the specified point.

36. $y = \dfrac{x}{x - 3}$, $(6, 2)$

37. $y = x + \dfrac{4}{x}$, $(2, 4)$

38. $y = x^{5/2}$, $(4, 32)$

39. $y = x + \sqrt{x}$, $(1, 2)$

40. (a) The curve $y = x/(1 + x^2)$ is called a **serpentine**. Find an equation of the tangent line to this curve at the point $(3, 0.3)$.

(b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.

41. (a) The curve $y = 1/(1 + x^2)$ is called a **witch of Agnesi**. Find an equation of the tangent line to this curve at the point $(-1, \frac{1}{2})$.

(b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.

42. (a) If $f(x) = x/(x^2 - 1)$, find $f'(x)$.

(b) Check to see that your answer to part (a) is reasonable by comparing the graphs of f and f'.

43. (a) If $f(x) = 3x^{15} - 5x^3 + 3$, find $f'(x)$.

(b) Check to see that your answer to part (a) is reasonable by comparing the graphs of f and f'.

44. Find the equations of the tangent lines to the curve $y = (x - 1)/(x + 1)$ that are parallel to the line $x - 2y = 1$.

45. At what point on the curve $y = x\sqrt{x}$ is the tangent line parallel to the line $3x - y + 6 = 0$?

46. For what values of x does the graph of $f(x) = 2x^3 - 3x^2 - 6x + 87$ have a horizontal tangent?

47. Find the points on the curve $y = x^3 - x^2 - x + 1$ where the tangent is horizontal.

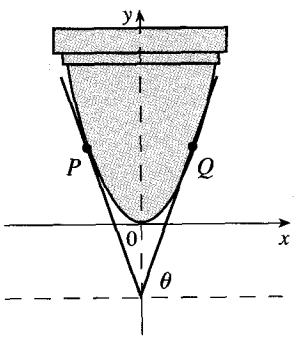
48. Draw a diagram to show that there are two tangent lines to the parabola $y = x^2$ that pass through the point $(0, -4)$. Find the coordinates of the points where these tangent lines intersect the parabola.

49. How many tangent lines to the curve $y = x/(x + 1)$ pass through the point $(1, 2)$? At which points do these tangent lines touch the curve?

50. Find the equations of both lines through the point $(2, -3)$ that are tangent to the parabola $y = x^2 + x$.

51. Show that the curve $y = 6x^3 + 5x - 3$ has no tangent line with slope 4.

52. A manufacturer of cartridges for stereo systems has designed a stylus with parabolic cross-section as shown in the figure. The equation of the parabola is $y = 16x^2$, where



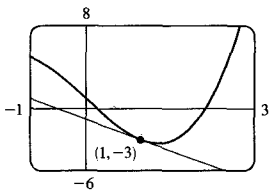
CHAPTER 2

Exercises 2.1 ■ page 121

1. 7, $7x - y - 12 = 0$

3. (a) -2, $2x + y + 1 = 0$

(b)



5. -2 m/s

7. $1 - 4a$

9. $-1/(2a - 1)^2$

11. $1/(3 - a)^{3/2}$

13. $f(x) = \sqrt{x}$, $a = 1$

15. $f(x) = x^9$, $a = 1$

17. $f(x) = \sin x$, $a = \pi/2$ **19.** $f'(x) = 5$, $\mathbb{R}$, $\mathbb{R}$

21. $f'(x) = 3x^2 - 2x + 2$, $\mathbb{R}$, $\mathbb{R}$

23. $g'(x) = 1/\sqrt{1 + 2x}$, $[-\frac{1}{2}, \infty)$, $(-\frac{1}{2}, \infty)$

25. $G'(x) = -10/(2 + x)^2$, $\{x \mid x \neq -2\}$, $\{x \mid x \neq -2\}$

27. $f'(x) = 4x^3$, $\mathbb{R}$, $\mathbb{R}$ **29.** 1, $2x$, $3x^2$, nx^{n-1}, $5x^4$

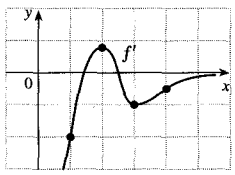
31. (a) $f'(x) = 1 + 2/x^2$

33. (a) -2

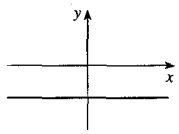
(b) 0.8

(c) -1

(d) -0.5



35.



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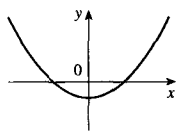

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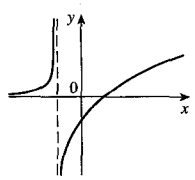
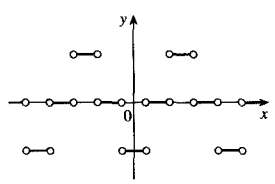
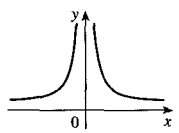


41.



43.



45. 3.296

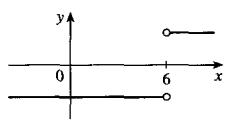
47. -5, 4, 8, 9, 5, -0.5, -8

49. (a) $1/(3a^{2/3})$

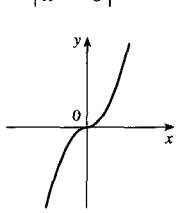
51. -1, 11 (vertical tangents), 4 (discontinuity), 8 (corner)

53. $f'(x) = \begin{cases} -1 & \text{if } x < 6 \\ 1 & \text{if } x > 6 \end{cases}$

or $f'(x) = \dfrac{x - 6}{|x - 6|}$



55. (a)



(b) All x

(c) $f'(x) = 2|x|$

57. (a) -5, 5 **59.** Does not exist **63.** 63°

Exercises 2.2 ■ page 132

1. $f'(x) = 2x - 10$ **3.** $V'(r) = 4\pi r^2$ **5.** $F'(x) = 12288x^2$

7. $Y'(t) = -54t^{-10}$ **9.** $g'(x) = 2x - (2/x^3)$

11. $h'(x) = -3/(x - 1)^2$

13. $G'(s) = (2s + 1)(s^2 + 2) + (s^2 + s + 1)(2s)$
$[= 4s^3 + 3s^2 + 6s + 2]$

15. $y' = \frac{3}{2}\sqrt{x} + (2/\sqrt{x}) - 3/(2x\sqrt{x})$ **17.** $y' = \sqrt{5}/(2\sqrt{x})$

19. $y' = -(4x^3 + 2x)/(x^4 + x^2 + 1)^2$ **21.** $y' = 2ax + b$

23. $y' = (-3t^2 + 14t + 23)/(t^2 + 5t - 4)^2$

25. $y' = 1 + 2/(5\sqrt[5]{x^3})$ **27.** $u' = \sqrt{2}\, x^{\sqrt{2}-1}$

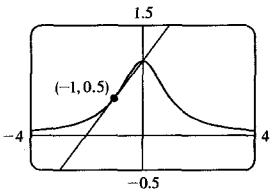
29. $v' = \frac{3}{2}\sqrt{x} - 5/(2x^3\sqrt{x})$ **31.** $f'(x) = 2cx/(x^2 + c)^2$

33. $f'(x) = 2x^4(x^3 - 5)/(x^3 - 2)^2$

35. $P'(x) = na_n x^{n-1} + (n - 1)a_{n-1}x^{n-2} + \cdots + 2a_2x + a_1$

37. $y = 4$ **39.** $3x - 2y + 1 = 0$

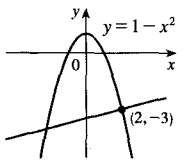
41. (a) $x - 2y + 2 = 0$ (b)

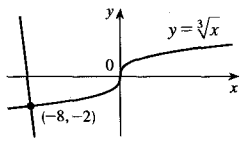


43. (a) $45x^{14} - 15x^2$ **45.** (4, 8)

47. (1, 0), $(-\frac{1}{3}, \frac{32}{27})$ **49.** 2, $(-2 \pm \sqrt{3}, (1 \mp \sqrt{3})/2)$

53. $x - 4y - 14 = 0$ **55.** $12x + y + 98 = 0$



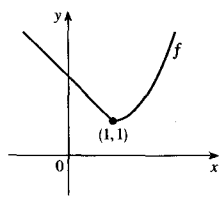


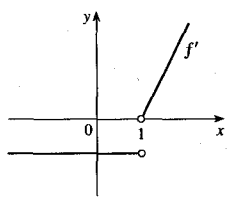
57. $(-\frac{1}{4}, \frac{1}{256})$ **59.** (a) -16 (b) $-\frac{20}{9}$ (c) 20

61. (a) 0 (b) $-\frac{2}{3}$

65. $y' = (x^4 + x + 1)(2x - 3)/(2\sqrt{x})$
$+ \sqrt{x}\,[(4x^3 + 1)(2x - 3) + 2(x^4 + x + 1)]$

67. No

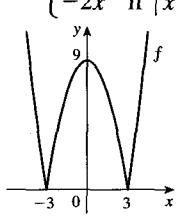


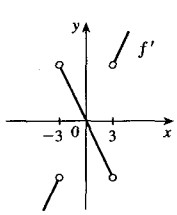


69. (a) Not differentiable at 3 or -3

$f'(x) = \begin{cases} 2x & \text{if } |x| > 3 \\ -2x & \text{if } |x| < 3 \end{cases}$

(b)





71. $a = -\frac{1}{2}$, $b = 2$ **75.** 1000