[09-03-27-AT11]

EXERCISE 5.1

A *Find exact real number values without using a calculator.*

1. $\sin^{-1} 0$ **2.** $\cos^{-1} 0$

3. $\arccos(\sqrt{3}/2)$ **4.** $\arcsin(\sqrt{3}/2)$

5. $\tan^{-1} 1$ **6.** $\arctan \sqrt{3}$

7. $\cos^{-1} \frac{1}{2}$ **8.** $\sin^{-1}(\sqrt{2}/2)$

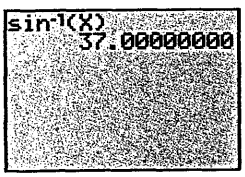
In Problems 9–14, evaluate to four significant digits using a calculator.

9. $\cos^{-1} 0.4038$ **10.** $\sin^{-1} 0.9103$

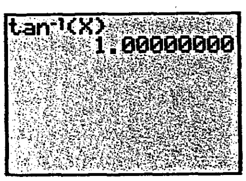
11. $\tan^{-1} 43.09$ **12.** $\arctan 103.7$

13. $\arcsin 1.131$ **14.** $\arccos 3.051$

15. Explain how to find the value of x that produces the result shown in the graphing utility window below, and find it. The utility is in degree mode. Give the answer to six decimal places.



16. Explain how to find the value of x that produces the result shown in the graphing utility window below, and find it. The utility is in radian mode. Give the answer to six decimal places.



B *Find exact real number values without using a calculator.*

17. $\arccos(-\frac{1}{2})$ **18.** $\arcsin(-\sqrt{2}/2)$

19. $\tan^{-1}(-1)$ **20.** $\arctan(-\sqrt{3})$

21. $\sin^{-1}(-\sqrt{3}/2)$ **22.** $\cos^{-1}(-1)$

23. $\cos^{-1}(-\sqrt{3}/2)$ **24.** $\sin^{-1}(-1)$

25. $\sin[\sin^{-1}(-0.6)]$ **26.** $\tan(\tan^{-1} 25)$

27. $\cos[\sin^{-1}(-\sqrt{2}/2)]$ **28.** $\sec[\sin^{-1}(-\sqrt{3}/2)]$

Evaluate to four significant digits using a calculator.

29. $\tan^{-1}(-4.038)$ **30.** $\arctan(-10.04)$

31. $\sec[\sin^{-1}(-0.0399)]$ **32.** $\cot[\cos^{-1}(-0.7003)]$

33. $\sqrt{2} + \tan^{-1} \sqrt[3]{5}$ **34.** $\sqrt{5} + \cos^{-1}(1 - \sqrt{2})$

Graph Problems 35 and 36 with the aid of a calculator. Plot points using x values −1.0, −0.8, −0.6, −0.4, −0.2, 0.0, 0.2, 0.4, 0.6, 0.8, and 1.0; then join the points with a smooth curve.

35. $y = \sin^{-1} x$ **36.** $y = \cos^{-1} x$

Find the exact degree measure of θ without a calculator.

37. $\theta = \arccos(-1/2)$ **38.** $\theta = \arcsin(-\sqrt{2}/2)$

39. $\theta = \tan^{-1}(-1)$ **40.** $\theta = \arctan(-\sqrt{3})$

41. $\theta = \sin^{-1}(-\sqrt{3}/2)$ **42.** $\theta = \cos^{-1}(-1)$

In Problems 43–48, find the degree measure of θ to two decimal places using a calculator.

43. $\theta = \tan^{-1} 3.0413$ **44.** $\theta = \cos^{-1} 0.7149$

45. $\theta = \arcsin(-0.8107)$ **46.** $\theta = \arccos(-0.7728)$

47. $\theta = \arctan(-17.305)$ **48.** $\theta = \tan^{-1}(-0.3031)$

49. Evaluate $\cos^{-1}[\cos(-0.3)]$ with a calculator set in radian mode. Explain why this does or does not illustrate a cosine–inverse cosine identity.

50. Evaluate $\sin^{-1}[\sin(-2)]$ with a calculator set in radian mode. Explain why this does or does not illustrate a sine–inverse sine identity.

51. The identity $\sin(\sin^{-1} x) = x$ is valid for $-1 \le x \le 1$.
(A) Graph $y = \sin(\sin^{-1} x)$ for $-1 \le x \le 1$.
(B) What happens if you graph $y = \sin(\sin^{-1} x)$ over a wider interval, say $-2 \le x \le 2$? Explain.

52. The identity $\cos(\cos^{-1} x) = x$ is valid for $-1 \le x \le 1$.
(A) Graph $y = \cos(\cos^{-1} x)$ for $-1 \le x \le 1$.
(B) What happens if you graph $y = \cos(\cos^{-1} x)$ over a wider interval, say $-2 \le x \le 2$? Explain.

C *In Problems 53–56, find exact real number values without using a calculator.*

53. $\sin[\arccos \frac{1}{2} + \arcsin(-1)]$

54. $\cos[\cos^{-1}(-\sqrt{3}/2) - \sin^{-1}(-\frac{1}{2})]$

55. $\sin[2\sin^{-1}(-\frac{4}{5})]$

56. $\cos\left(\dfrac{\cos^{-1}\frac{1}{3}}{2}\right)$

In Problems 57–60, write each as an algebraic expression in x free of trigonometric or inverse trigonometric functions.

57. $\sin(\cos^{-1} x), \quad -1 \le x \le 1$

58. $\cos(\sin^{-1} x), \quad -1 \le x \le 1$

59. $\tan(\arcsin x), \quad -1 \le x \le 1$

60. $\cos(\arctan x)$

Verify each identity in Problems 61 and 62.

61. $\tan^{-1}(-x) = -\tan^{-1} x$

62. $\sin^{-1}(-x) = -\sin^{-1} x$

63. Let $f(x) = \cos^{-1}(2x - 3)$.
(A) Explain how you would find the domain of f and find it.
(B) Graph f over the interval $0 \le x \le 3$ and explain the result.

64. Let $g(x) = \sin^{-1}\left(\dfrac{x+1}{2}\right)$.
(A) Explain how you would find the domain of g and find it.
(B) Graph g over the interval $-4 \le x \le 2$ and explain the result.

65. Let $h(x) = 3 + 5\sin(x - 1), \; -\pi/2 \le x \le 1 + \pi/2$.
(A) Find $h^{-1}(x)$.
(B) Explain how x must be restricted in $h^{-1}(x)$.

66. Let $f(x) = 4 + 2\cos(x - 3), \; -\pi/2 \le x \le 3 + \pi/2$.
(A) Find $f^{-1}(x)$.
(B) Explain how x must be restricted in $f^{-1}(x)$.

67. The identity $\sin^{-1}(\sin x) = x$ is valid for $-\pi/2 \le x \le \pi/2$.
(A) Graph $y = \sin^{-1}(\sin x)$ for $-\pi/2 \le x \le \pi/2$.
(B) What happens if you graph $y = \sin^{-1}(\sin x)$ over a larger interval, say $-2\pi \le x \le 2\pi$? Explain.

68. The identity $\cos^{-1}(\cos x) = x$ is valid for $0 \le x \le \pi$.
(A) Graph $y = \cos^{-1}(\cos x)$ for $0 \le x \le \pi$.
(B) What happens if you graph $y = \cos^{-1}(\cos x)$ over a larger interval, say $-2\pi \le x \le 2\pi$? Explain.

Applications

69. Sonic Boom An aircraft flying faster than the speed of sound produces sound waves that pile up behind the aircraft in the form of a cone. The cone of a level flying aircraft intersects the ground in the form of a hyperbola, and along this curve we experience a *sonic boom* (see the figure). In Section 2.4 it was shown that

$$\sin\frac{\theta}{2} = \frac{\text{Speed of sound}}{\text{Speed of aircraft}} \qquad (1)$$

where θ is the cone angle. The ratio

$$M = \frac{\text{Speed of aircraft}}{\text{Speed of sound}} \qquad (2)$$

is the Mach number. A Mach number of 2.3 would indicate an aircraft moving at 2.3 times the speed of sound. From equations (1) and (2) we obtain

$$\sin\frac{\theta}{2} = \frac{1}{M}$$

(A) Write θ in terms of M.
(B) Find θ to the nearest degree for $M = 1.7$ and for $M = 2.3$.

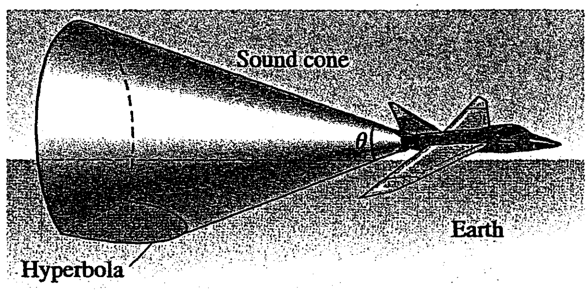


Figure for 69

70. Space Science A spacecraft traveling in a circular orbit h miles above the earth observes horizons in each direction (see the figure at the top of the next page), where r is the radius of the earth (3,959 mi).
(A) Express θ in terms of h and r.
(B) Find θ, in degrees (to one decimal place), for $h = 425.4$ mi.
(C) Find the length (to the nearest mile) of the arc subtended by angle θ found in part (B). What percentage (to one decimal place) of the great circle containing the arc does the arc represent? (A great circle is any circle on the surface of the earth having the center of the earth as its center.)

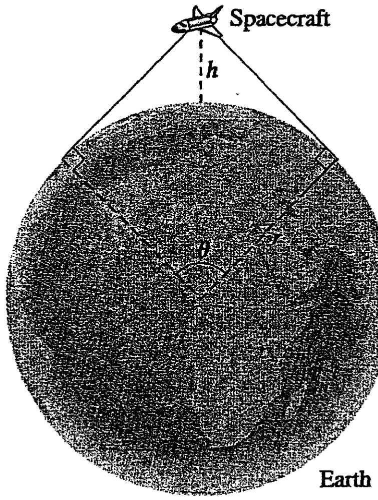


Figure for 70

*71. Precalculus: Sports A particular soccer field is 110 yd by 60 yd, and the goal is 8 yd wide at the end of the field (see the figure). A player is dribbling the ball along a line parallel to and 5 yd inside the side line. Assuming the player has a clear shot all along this line, is there an optimal distance x from the end of the field where the shot should be taken? That is, is there a distance x for which θ is maximum? Parts (A)–(D) will attempt to answer this question.

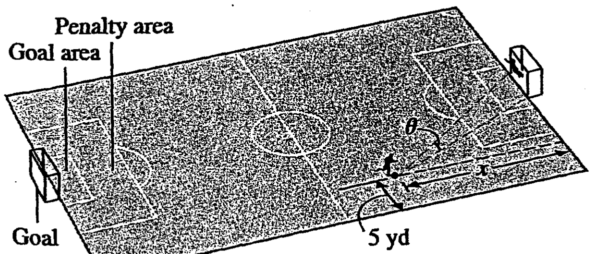


Figure for 71

(A) Discuss what you think happens to θ as x varies from 0 yd to 55 yd.

(B) Show that

$$\theta = \tan^{-1}\left(\frac{8x}{x^2 + 609}\right)$$

$$\left[\textit{Hint:}\ \ \tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} \text{ is useful.}\right]$$

(C) Complete Table 1 (to two decimal places) and select the value of x that gives the maximum value of θ in the table. (If your calculator has a table-generating feature, use it.)

TABLE 1

x (yd)	10	15	20	25	30	35
θ (deg)	6.44					

(D) Graph the equation in part (B) in a graphing utility for $0 \le x \le 55$. Describe what the graph shows. Use a built-in routine to find the maximum θ and the distance x that produces it.

*72. Precalculus: Related Rates The figure represents a circular courtyard surrounded by a high stone wall. A flood light, located at E, shines into the courtyard. A person walks from the center C along CD to D, at the rate of 6 ft/sec.

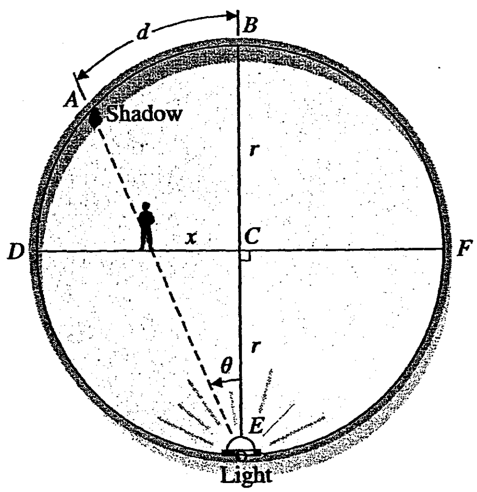


Figure for 72

(A) Do you think that the shadow moves along the circular wall at a constant rate, or does it speed up or slow down as the person walks from C to D?

(B) Show that if the person walks x feet from C along CD, then the shadow will move a distance d given by

$$d = 2r\theta = 2r\tan^{-1}\frac{x}{r} \qquad (1)$$

Matched Problem 3 Use a calculator to evaluate the following as real numbers to three decimal places.

(A) $\cot^{-1} 2.314$ (B) $\sec^{-1}(-1.549)$

Answers to Matched Problems

1. (A) $5\pi/6$ (B) $-\pi/6$ 2. $-\frac{4}{3}$
3. (A) 0.408 (B) 2.273

EXERCISE 5.2

A *Find the exact real number value of each without using a calculator.*

1. $\cot^{-1}\sqrt{3}$
2. $\cot^{-1} 0$
3. $\operatorname{arccsc} 1$
4. $\operatorname{arcsec} 2$
5. $\sec^{-1}\sqrt{2}$
6. $\csc^{-1} 2$
7. $\sin(\cot^{-1} 0)$
8. $\cos(\cot^{-1} 1)$
9. $\tan(\csc^{-1}\frac{5}{4})$
10. $\cot(\sec^{-1}\frac{5}{3})$

B

11. $\cot^{-1}(-1)$
12. $\sec^{-1}(-1)$
13. $\operatorname{arsec}(-2)$
14. $\operatorname{arccsc}(-\sqrt{2})$
15. $\operatorname{arccsc}(-2)$
16. $\operatorname{arccot}(-\sqrt{3})$
17. $\csc^{-1}\frac{1}{2}$
18. $\sec^{-1}(-\frac{1}{2})$
19. $\cos[\csc^{-1}(-\frac{5}{3})]$
20. $\tan[\cot^{-1}(-1/\sqrt{3})]$
21. $\cot[\sec^{-1}(-\frac{5}{4})]$
22. $\sin[\cot^{-1}(-\frac{3}{4})]$
23. $\cos[\sec^{-1}(-2)]$
24. $\sin[\csc^{-1}(-2)]$
25. $\cot(\cot^{-1} 33.4)$
26. $\sec[\sec^{-1}(-44)]$
27. $\csc[\csc^{-1}(-4)]$
28. $\cot[\cot^{-1}(-7.3)]$

Use a calculator to evaluate the following as real numbers to three decimal places.

29. $\cot^{-1} 3.065$
30. $\cot^{-1} 7.306$
31. $\sec^{-1}(-1.963)$
32. $\sec^{-1} 2.041$
33. $\csc^{-1} 1.172$
34. $\csc^{-1}(-1.938)$
35. $\cot^{-1}(-5.104)$
36. $\cot^{-1}(-12.236)$

Find the exact degree measure of θ without using a calculator.

37. $\theta = \operatorname{arcsec}(-2)$
38. $\theta = \operatorname{arccsc}(-\sqrt{2})$
39. $\theta = \cot^{-1}(-1)$
40. $\theta = \operatorname{arccot}(-1/\sqrt{3})$
41. $\theta = \csc^{-1}(-2/\sqrt{3})$
42. $\theta = \sec^{-1}(-1)$

Find the degree measure to two decimal places using a calculator.

43. $\theta = \cot^{-1} 0.3288$
44. $\theta = \sec^{-1} 1.3989$
45. $\theta = \operatorname{arccsc}(-1.2336)$
46. $\theta = \operatorname{arcsec}(-1.2939)$
47. $\theta = \operatorname{arccot}(-0.0578)$
48. $\theta = \cot^{-1}(-3.2994)$

C *Find exact values for each problem without using a calculator.*

49. $\tan[\csc^{-1}(-\frac{5}{3}) + \tan^{-1}\frac{1}{4}]$
50. $\tan[\tan^{-1} 4 - \sec^{-1}(-\sqrt{5})]$
51. $\tan[2\cot^{-1}(-\frac{3}{4})]$
52. $\tan[2\sec^{-1}(-\sqrt{5})]$

In Problems 53–58, write as an algebraic expression in x free of trigonometric or inverse trigonometric functions.

53. $\sin(\cot^{-1} x)$
54. $\cos(\cot^{-1} x)$
55. $\csc(\sec^{-1} x)$
56. $\tan(\csc^{-1} x)$
57. $\sin(2\cot^{-1} x)$
58. $\sin(2\sec^{-1} x)$
59. Show that $\sec^{-1} x = \cos^{-1}(1/x)$ for $x \ge 1$ and $x \le -1$.
60. Show that $\csc^{-1} x = \sin^{-1}(1/x)$ for $x \ge 1$ and $x \le -1$.

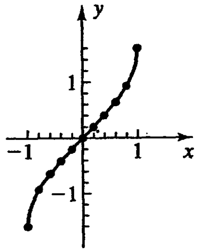
Problems 61–64 require the use of a graphing utility. Use an appropriate inverse trigonometric identity to graph each function in the viewing window $-5 \le x \le 5$, $-\pi \le y \le \pi$.

61. $y = \sec^{-1} x$
62. $y = \csc^{-1} x$
63. $y = \cot^{-1} x$ [Use two viewing windows, one for $-5 \le x \le 0$, and the other for $0 \le x \le 5$.]
64. $y = \cot^{-1} x$ [Using one viewing window, $-5 \le x \le 5$, graph $y_1 = \pi(x < 0) + \tan^{-1}(1/x)$, where $<$ is selected from the TEST menu. The expression $(x < 0)$ assumes a value of 1 for $x < 0$ and 0 for $x \ge 0$.]

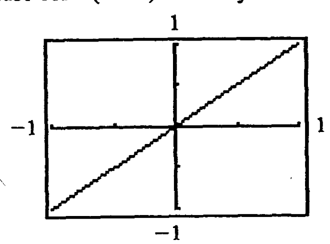
CHAPTER 5

Exercise 5.1

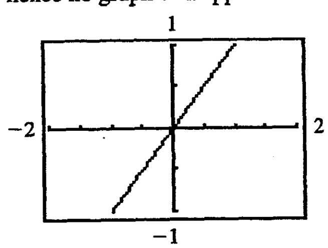
1. 0 **3.** $\pi/6$ **5.** $\pi/4$ **7.** $\pi/3$ **9.** 1.155
11. 1.548 **13.** Not defined
15. $x = \sin 37 = 0.601815$ **17.** $2\pi/3$ **19.** $-\pi/4$
21. $-\pi/3$ **23.** $5\pi/6$ **25.** -0.6 **27.** $\sqrt{2}/2$
29. -1.328 **31.** 1.001 **33.** 2.456
35.



37. 120° **39.** $-45°$ **41.** $-60°$ **43.** 71.80°
45. $-54.16°$ **47.** $-86.69°$
49. 0.3; does not illustrate a cosine–inverse cosine identity, because $\cos^{-1}(\cos x) = x$ only if $0 \le x \le \pi$.
51. (A)



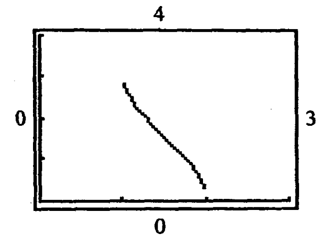
(B) The domain of $\sin^{-1}$ is restricted to $-1 \le x \le 1$; hence no graph will appear for other values of x.



53. $-\frac{1}{2}$ **55.** $-\frac{24}{25}$ **57.** $\sqrt{1 - x^2}$ **59.** $\dfrac{x}{\sqrt{1 - x^2}}$

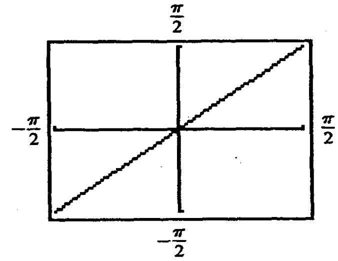
63. (A) $\cos^{-1} x$ has domain $-1 \le x \le 1$; therefore, $\cos^{-1}(2x - 3)$ has domain $-1 \le 2x - 3 \le 1$, or $1 \le x \le 2$.

(B) The graph appears only for the domain values $1 \le x \le 2$.

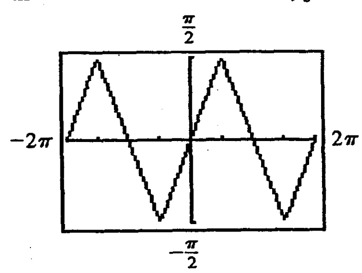


65. (A) $h^{-1}(x) = 1 + \sin^{-1}[(x - 3)/5]$
(B) $\sin^{-1} x$ has domain $-1 \le x \le 1$; therefore, $1 + \sin^{-1}[(x - 3)/5]$ has domain $-1 \le (x - 3)/5 \le 1$, or $-2 \le x \le 8$.

67. (A)



(B) The domain for $\sin x$ is $(-\infty, \infty)$ and the range is $[-1, 1]$, which is the domain for $\sin^{-1} x$. Thus, $y = \sin^{-1}(\sin x)$ has a graph over the interval $(-\infty, \infty)$, but $\sin^{-1}(\sin x) = x$ has a graph only on the restricted domain of $\sin x$, $[-\pi/2, \pi/2]$.



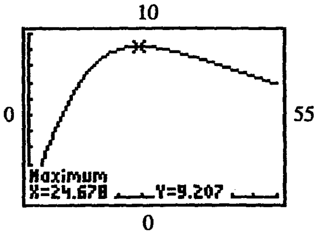
69. (A) $\theta = 2\sin^{-1}(1/M), M > 1$ (B) 72°; 52°
71. (A) It appears that θ increases, and then decreases.

71. (C) From the table, Max $\theta = 9.21°$ when $x = 25$ yd.

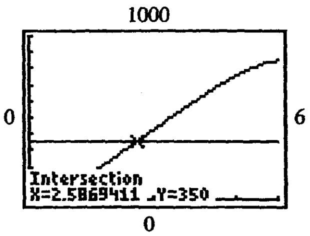
TABLE 1

x (yd)	10	15	20	25
θ (deg)	6.44	8.19	9.01	9.21
x (yd)	30	35		
θ (deg)	9.04	8.68		

(D) The angle θ increases rapidly until a maximum is reached, and then declines more slowly.
Max $\theta = 9.21°$ when $x = 24.68$ yd.

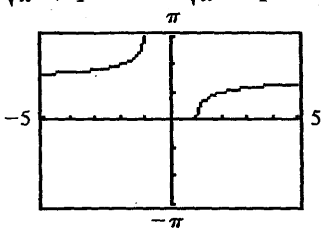


73. (B) 248 ft^3
(C) 2.6 ft

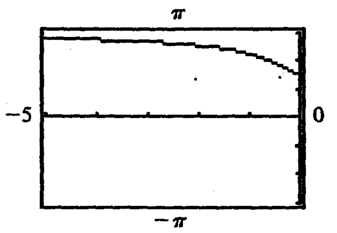


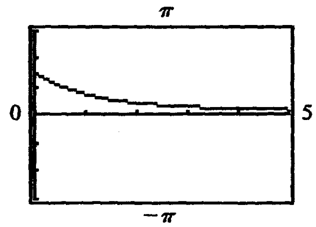
Exercise 5.2

1. $\pi/6$ **3.** $\pi/2$ **5.** $\pi/4$ **7.** 1 **9.** $\frac{4}{3}$
11. $3\pi/4$ **13.** $2\pi/3$ **15.** $-\pi/6$ **17.** Not defined
19. $\frac{4}{5}$ **21.** $-\frac{4}{3}$ **23.** $-\frac{1}{2}$ **25.** 33.4 **27.** -4
29. 0.315 **31.** 2.105 **33.** 1.022 **35.** 2.948
37. 120° **39.** 135° **41.** $-60°$ **43.** 71.80°
45. $-54.16°$ **47.** 93.31° **49.** $-\frac{8}{19}$ **51.** $\frac{24}{7}$
53. $\frac{1}{\sqrt{x^2+1}}$ **55.** $\frac{|x|}{\sqrt{x^2-1}}$ **57.** $\frac{2x}{x^2+1}$
61.



63.





Exercise 5.3

1. $2\pi/3, 4\pi/3$
3. $2\pi/3 + 2k\pi, 4\pi/3 + 2k\pi$, k any integer
5. 45°, 135°
7. $45° + k(360°), 135° + k(360°)$, k any integer
9. 104.9314° **11.** 1.1593, 5.1239
13. $0.6696 + 2k\pi, 2.4720 + 2k\pi$, k any integer
15. $\pi/2, 3\pi/2$
17. $90° + k(180°), 45° + k(180°)$, k any integer
19. $\pi/2$ **21.** 180° **23.** 90°, 270°
25. $\pi/6, 5\pi/6, 7\pi/6, 11\pi/6$ **27.** 104.5°
29. 0.9987, 5.284
31. $0.9987 + 2k\pi, -0.9987 + 2k\pi$, k any integer
33. $\cos^{-1}(-0.7334)$ has exactly one value, 2.3941; the equation $\cos x = -0.7334$ has infinitely many solutions, which are found by adding $2\pi k$, k any integer, to each solution in one period of $\cos x$.
35. 0, $\pi/2$ **37.** 0 **39.** 0.002613 sec **41.** 33.21°
43. 64.1° **45.** $(r, \theta) = (1, 30°), (1, 150°)$
47. $\theta = 45°$

Exercise 5.4

1. 0.4502 **3.** 0.6167 **5.** 0.9987, 5.2845
7. $0.9987 + 2k\pi, 5.2845 + 2k\pi$, k any integer
9. $(-1.5099, 1.8281)$
11. $[0.4204, 1.2346], [2.9752, \infty)$
13. Because $\sin^2 x - 2\sin x + 1 = (\sin x - 1)^2$, and the latter is greater than or equal to 0 for all real x.
15. 0.006104, 0.006137 **17.** $[-2\pi, 2\pi]$
19. $[0.2974, 1.6073] \cup [2.7097, 3.1416]$
21. 1.78 rad **23.** 35.64 ft^2
25. (A) $L = 12.4575$ mm (B) $y = 2.6495$ mm