Concepts Review

1. The natural domain of the sine function is ____________; its range is ____________.

2. The period of the cosine function is ____________; the period of the sine function is ____________; the period of the tangent function is ____________.

3. Since $\sin(-x) = -\sin x$, the sine function is ____________, and since $\cos(-x) = \cos x$, the cosine function is ____________.

4. If $(-4, 3)$ lies on the terminal side of an angle θ whose vertex is at the origin and initial side is along the positive x-axis, then $\cos\theta =$ ____________.

Problem Set 2.3

1. Convert the following degree measures to radians (leave π in your answer).

(a) $30°$ (b) $45°$ (c) $-60°$
(d) $240°$ (e) $-370°$ (f) $10°$
(g) $22\frac{1}{2}°$ (h) $600°$ (i) $-120°$

2. Convert the following radian measures to degrees.

(a) $\frac{7}{6}\pi$ (b) $\frac{3}{4}\pi$ (c) $-\frac{1}{3}\pi$
(d) $\frac{4}{3}\pi$ (e) $-\frac{35}{18}\pi$ (f) $\frac{3}{18}\pi$
(g) $\frac{9}{8}\pi$ (h) $\frac{10}{3}\pi$ (i) $-\frac{4}{3}\pi$

[C] 3. Convert the following degree measures to radians ($1° = \pi/180 = 1.7453 \times 10^{-2}$ radians).

(a) $33.3°$ (b) $46°$ (c) $-66.6°$
(d) $240.11°$ (e) $-369°$ (f) $11°$
(g) $22.5°$ (h) $359°$ (i) $-121.35°$

[C] 4. Convert the following radian measures to degrees (1 radian $= 180/\pi = 57.296$ degrees).

(a) 3.141 (b) 6.28 (c) 5.00
(d) 0.001 (e) -0.1 (f) 36.0
(g) -2.00 (h) 1.234 (i) -10.0

[C] 5. Calculate (be sure that your calculator is in radian or degree mode as needed).

(a) $\dfrac{56.4\tan 34.2°}{\sin 34.1°}$ (b) $\dfrac{5.34\tan 21.3°}{\sin 3.1° + \cot 23.5°}$
(c) $\tan(0.452)$ (d) $\sin(-0.361)$
(e) $\cos(-0.361)$ (f) $\tan(-0.361)$

[C] 6. Calculate.

(a) $\dfrac{234.1\sin(1.56)}{\cos(0.34)}$ (b) $\sin^2(2.51) + \sqrt{\cos(0.51)}$

[C] 7. Calculate.

(a) $\dfrac{56.3\tan 34.2°}{\sin 56.1°}$ (b) $\left(\dfrac{\sin 35°}{\sin 26° + \cos 26°}\right)^3$

8. Verify the values of $\sin t$ and $\cos t$ in Figure 6.

9. Evaluate without using a calculator.

(a) $\tan\left(\frac{\pi}{6}\right)$ (b) $\sec(\pi)$ (c) $\sec\left(\frac{3\pi}{4}\right)$
(d) $\csc\left(\frac{\pi}{2}\right)$ (e) $\cot\left(\frac{\pi}{4}\right)$ (f) $\tan\left(-\frac{\pi}{4}\right)$

10. Evaluate without using a calculator.

(a) $\tan\left(\frac{\pi}{3}\right)$ (b) $\sec\left(\frac{\pi}{3}\right)$ (c) $\cot\left(\frac{\pi}{3}\right)$
(d) $\csc\left(\frac{\pi}{4}\right)$ (e) $\tan\left(-\frac{\pi}{6}\right)$ (f) $\cos\left(-\frac{\pi}{3}\right)$

11. Verify that the following are identities (see Example 2).

(a) $(1 + \sin z)(1 - \sin z) = \dfrac{1}{\sec^2 z}$
(b) $(\sec t - 1)(\sec t + 1) = \tan^2 t$
(c) $\sec t - \sin t \tan t = \cos t$
(d) $\dfrac{\sec^2 t - 1}{\sec^2 t} = \sin^2 t$

12. Verify that the following are identities (see Example 2).

(a) $\sin^2 v + \dfrac{1}{\sec^2 v} = 1$
(b) $\cos 3t = 4\cos^3 t - 3\cos t$ *Hint*: Use a double-angle identity.
(c) $\sin 4x = 8\sin x\cos^3 x - 4\sin x\cos x$ *Hint*: Use a double-angle identity twice.
(d) $(1 + \cos\theta)(1 - \cos\theta) = \sin^2\theta$

13. Verify the following are identities.

(a) $\dfrac{\sin u}{\csc u} + \dfrac{\cos u}{\sec u} = 1$
(b) $(1 - \cos^2 x)(1 + \cot^2 x) = 1$
(c) $\sin t(\csc t - \sin t) = \cos^2 t$
(d) $\dfrac{1 - \csc^2 t}{\csc^2 t} = \dfrac{-1}{\sec^2 t}$

14. Sketch the graphs of the following on $[-\pi, 2\pi]$.

(a) $y = \sin 2x$ (b) $y = 2\sin t$
(c) $y = \cos\left(x - \frac{\pi}{4}\right)$ (d) $y = \sec t$

15. Sketch the graphs of the following on $[-\pi, 2\pi]$.

(a) $y = \csc t$ (b) $y = 2\cos t$
(c) $y = \cos 3t$ (d) $y = \cos\left(t + \frac{\pi}{3}\right)$

Determine the period, amplitude, and shifts (both horizontal and vertical) and draw a graph over the interval $-5 \le x \le 5$ for the functions listed in Problems 16–23.

16. $y = 3\cos\dfrac{x}{2}$ 17. $y = 2\sin 2x$

18. $y = \tan x$ 19. $y = 2 + \dfrac{1}{6}\cot(2x)$

20. $y = 3 + \sec(x - \pi)$ 21. $y = 21 + 7\sin(2x + 3)$

22. $y = 3\cos\left(x - \dfrac{\pi}{2}\right) - 1$ 23. $y = \tan\left(2x - \dfrac{\pi}{3}\right)$

24. Which of the following represent the same graph? Check your result analytically using trigonometric identities.

(a) $y = \sin\left(x + \frac{\pi}{2}\right)$ (b) $y = \cos\left(x + \frac{\pi}{2}\right)$

(c) $y = -\sin(x + \pi)$ (d) $y = \cos(x - \pi)$

(e) $y = -\sin(\pi - x)$ (f) $y = \cos\left(x - \frac{\pi}{2}\right)$

(g) $y = -\cos(\pi - x)$ (h) $y = \sin\left(x - \frac{\pi}{2}\right)$

25. Which of the following are odd functions? Even functions? Neither?

(a) $t \sin t$ (b) $\sin^2 t$ (c) $\csc t$

(d) $|\sin t|$ (e) $\sin(\cos t)$ (f) $x + \sin x$

26. Which of the following are odd functions? Even functions? Neither?

(a) $\cot t + \sin t$ (b) $\sin^3 t$ (c) $\sec t$

(d) $\sqrt{\sin^4 t}$ (e) $\cos(\sin t)$ (f) $x^2 + \sin x$

Use the half-angle identities to find the exact values in Problems 27–31.

27. $\cos^2 \frac{\pi}{3} =$ **28.** $\sin^2 \frac{\pi}{6} =$

29. $\sin^3 \frac{\pi}{6} =$ **30.** $\cos^2 \frac{\pi}{12} =$

31. $\sin^2 \frac{\pi}{8} =$

32. Find identities analogous to the addition identities for each expression.

(a) $\sin(x - y)$ (b) $\cos(x - y)$ (c) $\tan(x - y)$

33. Use the addition identity for the tangent to show that $\tan(t + \pi) = \tan t$ for all t in the domain of $\tan t$.

34. Show that $\cos(x - \pi) = -\cos x$ for all x.

[≈] [C] **35.** Suppose that a tire on a truck has an outer radius of 2.5 feet. How many revolutions per minute does the tire make when the truck is traveling 60 miles per hour?

[≈] **36.** How far does a wheel of radius 2 feet roll along level ground in making 150 revolutions? (See Example 3.)

[≈] [C] **37.** A belt passes around two wheels, as shown in Figure 15. How many revolutions per second does the small wheel make when the large wheel makes 21 revolutions per second?

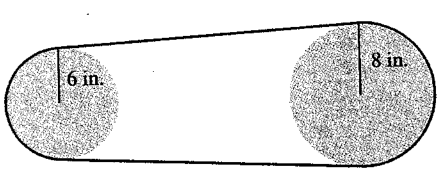


Figure 15

38. The **angle of inclination** α of a line is the smallest positive angle from the positive x-axis to the line ($\alpha = 0$ for a horizontal line). Show that the slope m of the line is equal to $\tan \alpha$.

39. Find the angle of inclination of the following lines (see Problem 38).

(a) $y = \sqrt{3}\,x - 7$ (b) $\sqrt{3}\,x + 3y = 6$

40. Let ℓ_1 and ℓ_2 be two nonvertical lines with slopes m_1 and m_2, respectively. If θ, the angle from ℓ_1 to ℓ_2, is not a right angle, then

$$\tan \theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

Show this using the fact that $\theta = \theta_2 - \theta_1$ in Figure 16.

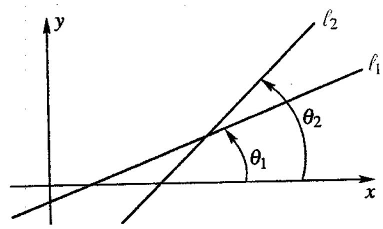


Figure 16

[C] **41.** Find the angle (in radians) from the first line to the second (see Problem 40).

(a) $y = 2x, y = 3x$ (b) $y = \frac{x}{2}, y = -x$

(c) $2x - 6y = 12, 2x + y = 0$

42. Derive the formula $A = \frac{1}{2}r^2 t$ for the area of a sector of a circle. Here r is the radius and t is the radian measure of the central angle (see Figure 17).

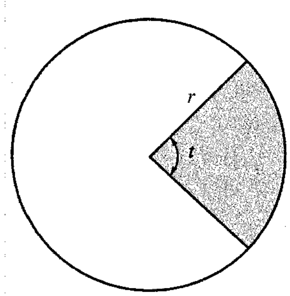


Figure 17

43. Find the area of the sector of a circle of radius 5 centimeters and central angle 2 radians (see Problem 42).

44. A regular polygon of n sides is inscribed in a circle of radius r. Find formulas for the perimeter, P, and area, A, of the polygon in terms of n and r.

45. An isosceles triangle is topped by a semicircle, as shown in Figure 18. Find a formula for the area A of the whole figure in terms of the side length r and angle t (radians). (We say that A is a function of the two independent variables r and t.)

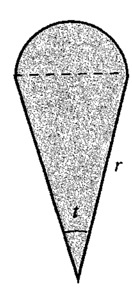
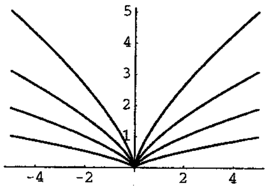


Figure 18

41. (a)



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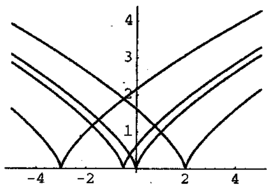

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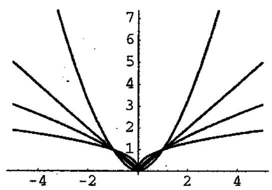


Problem Set 2.3

1. **(a)** $\frac{\pi}{6}$; **(b)** $\frac{\pi}{4}$; **(c)** $-\frac{\pi}{3}$; **(d)** $\frac{4\pi}{3}$; **(e)** $-\frac{37\pi}{18}$; **(f)** $\frac{\pi}{18}$; **(g)** $\frac{\pi}{8}$; **(h)** $\frac{10\pi}{3}$; **(i)** $-\frac{2\pi}{3}$

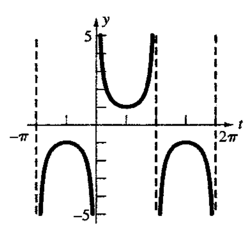
3. **(a)** 0.5812; **(b)** 0.8029; **(c)** −1.1624; **(d)** 4.1907; **(e)** −6.4403; **(f)** 0.1920; **(g)** 0.3927; **(h)** 6.2657; **(i)** −2.1180

5. **(a)** 68.37; **(b)** 0.8845; **(c)** 0.4855; **(d)** −0.3532; **(e)** 0.9355; **(f)** −0.3775

7. **(a)** 46.097; **(b)** 0.0789

9. **(a)** $\frac{\sqrt{3}}{3}$; **(b)** −1; **(c)** $-\sqrt{2}$; **(d)** 1; **(e)** 1; **(f)** −1

15. **(a)**



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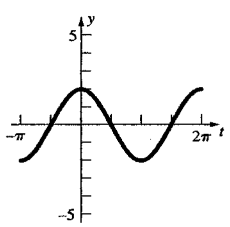

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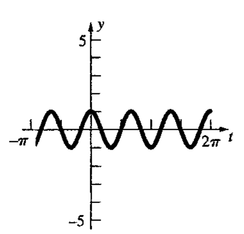
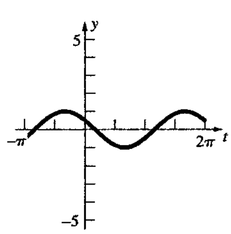
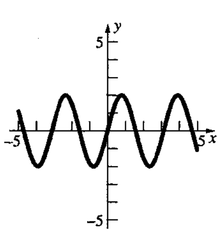


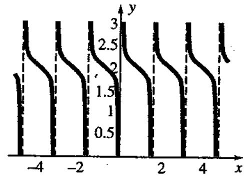
(d)



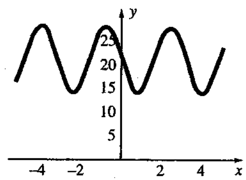
17. Period $= \pi$; Amplitude $= 2$



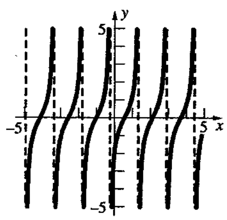
19. Period $= \frac{\pi}{2}$; shift: 2 units up



21. Period $= \pi$; amplitude $= 7$; shift: 21 units up, $\frac{3}{2}$ units left



23. Period $= \frac{\pi}{2}$; Shift $\frac{\pi}{6}$ units right



25. **(a)** Even; **(b)** Even; **(c)** Odd; **(d)** Even; **(e)** Even; **(f)** Odd

27. $\frac{1}{4}$ **29.** $\frac{1}{8}$ **31.** $\frac{2-\sqrt{2}}{4}$

35. 336 rev/min

37. 28 rev/sec

39. **(a)** $\frac{\pi}{3}$; **(b)** $\frac{5\pi}{6}$

41. **(a)** 0.1419; **(b)** 1.8925; **(c)** 1.7127

43. 25 cm^2 **45.** $r^2 \sin\frac{t}{2}\cos\frac{t}{2} + \frac{\pi r^2}{2}\sin^2\frac{t}{2}$

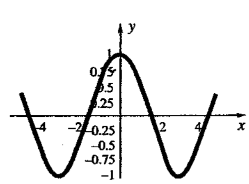
47. $S_1(n) = \frac{n(n+1)}{2}$ **49.** $S_3(n) = \frac{1}{4}n^2(n+1)^2$

51. 67.5°F

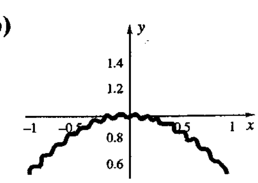
53. As t increases, the point on the rim of the wheel will move around the circle of radius 2.
(a) $x(2) \approx 1.902$; $y(2) \approx 0.618$; $x(6) \approx -1.176$; $y(6) \approx -1.618$; $x(10) = 0$; $y(10) = 2$; $x(0) = 0$; $y(0) = 2$
(b) $x(t) = -2\sin(\frac{\pi}{5}t)$, $y(t) = 2\cos(\frac{\pi}{5}t)$
(c) The point is at (2, 0) when $\frac{\pi}{5}t = \frac{\pi}{2}$; that is, when $t = \frac{5}{2}$

55. **(c)**
$$A_1\sin(\omega t + \phi_1) + A_2\sin(\omega t + \phi_2) + A_3\sin(\omega t + \phi_3) = (A_1\cos\phi_1 + A_2\cos\phi_2 + A_3\cos\phi_3)\sin\omega t + (A_1\sin\phi_1 + A_2\sin\phi_2 + A_3\sin\phi_3)\cos\omega t$$

57. **(a)**



(b)



(c)