[5.4] Exercise

Simplify. Assume that all variables represent positive real numbers. See Examples 1 and 2.

1. $\sqrt{36} + \sqrt{100}$
2. $\sqrt{25} + \sqrt{81}$
3. $4\sqrt{12} - 7\sqrt{27}$
4. $3\sqrt{32} - 2\sqrt{8}$
5. $6\sqrt{18} + \sqrt{32} - 2\sqrt{50}$
6. $5\sqrt{8} - 3\sqrt{72} + 3\sqrt{50}$
7. $2\sqrt{63} - 2\sqrt{28} + 3\sqrt{7}$
8. $6\sqrt{27} - 2\sqrt{48} + \sqrt{75}$
9. $2\sqrt{5} - 3\sqrt{20} - 4\sqrt{45}$
10. $5\sqrt{54} + 2\sqrt{24} - 2\sqrt{96}$
11. $2\sqrt{40} + 6\sqrt{90} - 3\sqrt{160}$
12. $5\sqrt{28} - 3\sqrt{63} + 2\sqrt{112}$
13. $3\sqrt{2x} - \sqrt{8x} - \sqrt{72x}$
14. $4\sqrt{18k} - \sqrt{72k} + 4\sqrt{50k}$
15. $9\sqrt{3r} - 2\sqrt{12r} + 5\sqrt{27r}$
16. $-\sqrt{20z} + 2\sqrt{125z} - 3\sqrt{45z}$
17. $7q\sqrt{10} - 2q\sqrt{40} + 8q\sqrt{90}$
18. $3a\sqrt{7} + 2a\sqrt{28} - 5a\sqrt{63}$
19. $3\sqrt{72m^2} + 2\sqrt{32m^2} - 3\sqrt{18m^2}$
20. $9\sqrt{27p^2} - 4\sqrt{108p^2} - 2\sqrt{48p^2}$
21. $\sqrt[3]{54} - 2\sqrt[3]{16}$
22. $5\sqrt[3]{81} - 4\sqrt[3]{24}$
23. $2\sqrt[3]{27x} + 2\sqrt[3]{8x}$
24. $6\sqrt[3]{128m} - 3\sqrt[3]{16m}$
25. $\sqrt[3]{x^2y} - \sqrt[3]{8x^2y}$
26. $3\sqrt[3]{x^2y^2} - 2\sqrt[3]{64x^2y^2}$
27. $3x\sqrt[3]{xy^2} - 2\sqrt[3]{8x^4y^2}$
28. $6q^2\sqrt[3]{5q} - 2q\sqrt[3]{40q^4}$
29. $5\sqrt[4]{32} + 3\sqrt[4]{162}$
30. $2\sqrt[4]{512} - 4\sqrt[4]{32}$
31. $2\sqrt[4]{32a^3} + 5\sqrt[4]{2a^3}$
32. $-\sqrt[4]{16r} + 5\sqrt[4]{r}$
33. $3\sqrt[4]{x^5y} - 2x\sqrt[4]{xy}$
34. $2\sqrt[4]{m^9p^6} - 3m^2p\sqrt[4]{mp^2}$
35. $\dfrac{3}{\sqrt{2}} - \dfrac{\sqrt{2}}{2}$
36. $\dfrac{5}{\sqrt{6}} - \dfrac{\sqrt{6}}{2}$

37. $\dfrac{2\sqrt{5}}{3} + \dfrac{1}{\sqrt{5}}$
38. $\dfrac{4\sqrt{3}}{3} + \dfrac{2}{\sqrt{3}}$
39. $\sqrt{7} - \dfrac{1}{\sqrt{7}}$
40. $\sqrt{2} - \dfrac{1}{\sqrt{2}}$
41. $\dfrac{\sqrt{32}}{3} + \dfrac{2\sqrt{2}}{3} - \dfrac{1}{\sqrt{8}}$
42. $\dfrac{\sqrt{27}}{2} - \dfrac{3\sqrt{3}}{2} + \dfrac{2}{\sqrt{27}}$
43. $\sqrt{\dfrac{3}{5}} - \dfrac{2\sqrt{15}}{3} + \dfrac{1}{\sqrt{15}}$
44. $\sqrt{\dfrac{5}{8}} - 2\sqrt{18} + \dfrac{3}{\sqrt{18}}$
45. $3\sqrt{x} + \dfrac{2}{\sqrt{x}} + \sqrt{\dfrac{1}{x}}$
46. $\dfrac{5}{\sqrt{p}} - 2\sqrt{\dfrac{4}{p}} + \sqrt{p}$
47. $\dfrac{4}{\sqrt{y^3}} - 3\sqrt{y} + \sqrt{\dfrac{16}{y}}$
48. $\dfrac{9}{\sqrt{z^3}} + 2\sqrt{z} - \sqrt{\dfrac{1}{z}}$
49. $2\sqrt[3]{3} - \dfrac{5}{\sqrt[3]{9}}$
50. $-\sqrt[3]{5} + \dfrac{1}{\sqrt[3]{25}}$
51. $3\sqrt[3]{\dfrac{m^5}{9}} - 2m\sqrt[3]{\dfrac{m^2}{72}}$
52. $2a\sqrt[3]{\dfrac{a}{5}} - 6\sqrt[3]{\dfrac{a^4}{40}}$

5.5 Exercises

Multiply, then simplify the products. Assume that all variables represent positive real numbers. See Example 1.

1. $3(5 - \sqrt{6})$
2. $2(\sqrt{3} - \sqrt{7})$
3. $\sqrt{6}(3 + \sqrt{2})$
4. $\sqrt{2}(\sqrt{32} - \sqrt{9})$
5. $5(\sqrt{72} - \sqrt{8})$
6. $\sqrt{3}(\sqrt{12} + 2)$
7. $\sqrt{5}(\sqrt{15} - \sqrt{5})$
8. $\sqrt{2}(\sqrt{18} + \sqrt{3})$

9. $(\sqrt{7} + 3)(\sqrt{7} - 3)$
10. $(\sqrt{3} - 5)(\sqrt{3} + 5)$
11. $(\sqrt{2} - \sqrt{3})(\sqrt{2} + \sqrt{3})$
12. $(\sqrt{7} + \sqrt{3})(\sqrt{7} - \sqrt{3})$
13. $(\sqrt{8} - \sqrt{2})(\sqrt{8} + \sqrt{2})$
14. $(\sqrt{20} - \sqrt{5})(\sqrt{20} + \sqrt{5})$
15. $(\sqrt{2} + 1)(\sqrt{3} - 1)$
16. $(\sqrt{3} + 3)(\sqrt{5} - 2)$
17. $(\sqrt{11} - \sqrt{7})(\sqrt{2} + \sqrt{5})$
18. $(\sqrt{6} + \sqrt{2})(\sqrt{3} + \sqrt{2})$
19. $(\sqrt{7} + \sqrt{5})(\sqrt{6} - \sqrt{2})$
20. $(\sqrt{3} - \sqrt{2})(\sqrt{5} - \sqrt{6})$
21. $(2\sqrt{3} + \sqrt{5})(3\sqrt{3} - 2\sqrt{5})$
22. $(\sqrt{7} - \sqrt{11})(2\sqrt{7} + 3\sqrt{11})$
23. $(\sqrt{5} + 2)^2$
24. $(\sqrt{11} - 1)^2$
25. $(\sqrt{21} - \sqrt{5})^2$
26. $(\sqrt{6} - \sqrt{2})^2$
27. $(\sqrt{8} + \sqrt{2})^2$
28. $(\sqrt{27} - \sqrt{3})^2$
29. $(2 + \sqrt[3]{6})(2 - \sqrt[3]{6})$
30. $(\sqrt[3]{3} + 6)(\sqrt[3]{3} - 6)$
31. $(2 + \sqrt[3]{2})(4 - 2\sqrt[3]{2} + \sqrt[3]{4})$
32. $(\sqrt[3]{3} - 1)(\sqrt[3]{9} + \sqrt[3]{3} + 1)$
33. $(3\sqrt{x} - \sqrt{5})(2\sqrt{x} + 1)$
34. $(4\sqrt{p} + \sqrt{7})(\sqrt{p} - 9)$
35. $(3\sqrt{r} - \sqrt{s})(3\sqrt{r} + \sqrt{s})$
36. $(\sqrt{k} + 4\sqrt{m})(\sqrt{k} - 4\sqrt{m})$
37. $(5\sqrt{z} + 1)^2$
38. $(6\sqrt{a} - 5)^2$
39. $(\sqrt[3]{2y} - 5)(4\sqrt[3]{2y} + 1)$
40. $(\sqrt[3]{9z} - 2)(5\sqrt[3]{9z} + 7)$

Complete each factorization. Assume that all variables represent positive real numbers.

41. $2\sqrt{3} + 2 = 2(\quad)$
42. $3 - 3\sqrt{2} = 3(\quad)$
43. $3\sqrt{2} + 3\sqrt{5} = 3(\quad)$
44. $5\sqrt{3} + 5\sqrt{7} = 5(\quad)$
45. $2\sqrt{20} - 4\sqrt{7} = 4(\quad)$
46. $6\sqrt{5} - \sqrt{8} = 2(\quad)$
47. $5x + \sqrt{50x^3} = 5x(\quad)$
48. $12p^3 - \sqrt{72p^5} = 6p^2(\quad)$

Write each expression in lowest terms. Assume that all variables represent positive real numbers. See Example 2.

49. $\dfrac{30 - 20\sqrt{6}}{10}$
50. $\dfrac{15 - 6\sqrt{5}}{12}$
51. $\dfrac{3 - 3\sqrt{5}}{3}$
52. $\dfrac{-5 + 5\sqrt{2}}{5}$
53. $\dfrac{16 - 4\sqrt{8}}{12}$
54. $\dfrac{12 - 9\sqrt{72}}{18}$
55. $\dfrac{6p - \sqrt{24p^3}}{3p}$
56. $\dfrac{11y - \sqrt{242y^5}}{22y}$

Rationalize the denominators in each of the following. Assume that all variables represent positive real numbers. See Example 3.

57. $\dfrac{3}{4 + \sqrt{5}}$
58. $\dfrac{4}{3 - \sqrt{7}}$
59. $\dfrac{6}{\sqrt{5} - 1}$
60. $\dfrac{8}{\sqrt{5} + 3}$
61. $\dfrac{2}{\sqrt{2} + \sqrt{5}}$
62. $\dfrac{-5}{\sqrt{3} - \sqrt{7}}$
63. $\dfrac{\sqrt{8}}{3 - \sqrt{2}}$
64. $\dfrac{\sqrt{27}}{2 + \sqrt{3}}$
65. $\dfrac{1}{2 - 3\sqrt{2}}$
66. $\dfrac{5}{2\sqrt{3} + 1}$
67. $\dfrac{2}{3\sqrt{5} + 2\sqrt{3}}$
68. $\dfrac{-1}{3\sqrt{2} - 2\sqrt{7}}$

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1. 16 **3.** $-13\sqrt{3}$ **5.** $12\sqrt{2}$ **7.** $5\sqrt{7}$ **9.** $-16\sqrt{5}$ **11.** $10\sqrt{10}$
13. $-5\sqrt{2x}$ **15.** $20\sqrt{3r}$ **17.** $27q\sqrt{10}$ **19.** $17m\sqrt{2}$ **21.** $-\sqrt[3]{2}$ **23.** $10\sqrt[3]{x}$
25. $-\sqrt[3]{x^2y}$ **27.** $-x\sqrt[3]{xy^2}$ **29.** $19\sqrt[4]{2}$ **31.** $9\sqrt[4]{2a^3}$ **33.** $x\sqrt[4]{xy}$ **35.** $\sqrt{2}$
37. $13\sqrt{5}/15$ **39.** $6\sqrt{7}/7$ **41.** $7\sqrt{2}/4$ **43.** $-2\sqrt{15}/5$ **45.** $(3x\sqrt{x} + 3\sqrt{x})/x$ or $3\sqrt{x}(x + 1)/x$ **47.** $(4\sqrt{y} - 3y^2\sqrt{y} + 4y\sqrt{y})/y^2$ or $\sqrt{y}(4 - 3y^2 + 4y)/y^2$ **49.** $\sqrt[3]{3}/3$
51. $2m\sqrt[3]{3m^2}/3$ **53.** $12\sqrt{5} + 5\sqrt{3}$ centimeters **55.** $37\sqrt{2} + 10\sqrt{3}$ yards **57.** $\sqrt{3} \approx 1.732$ and $2\sqrt{3} \approx 3.464$; also, $\sqrt{12} \approx 3.464$, suggesting (but not really proving) that $\sqrt{12} = 2\sqrt{3}$
59. $-12p^3 + 6p$ **61.** $15a^2 + 11ab - 14b^2$ **63.** $25x^2 - 10xy + y^2$ **65.** $49r^2 - 4s^2$
67. $4x - 5$

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1. $15 - 3\sqrt{6}$ **3.** $3\sqrt{6} + 2\sqrt{3}$ **5.** $20\sqrt{2}$ **7.** $5\sqrt{3} - 5$ **9.** -2 **11.** -1
13. 6 **15.** $\sqrt{6} - \sqrt{2} + \sqrt{3} - 1$ **17.** $\sqrt{22} + \sqrt{55} - \sqrt{14} - \sqrt{35}$ **19.** $\sqrt{42} - \sqrt{14} + \sqrt{30} - \sqrt{10}$ **21.** $8 - \sqrt{15}$ **23.** $9 + 4\sqrt{5}$ **25.** $26 - 2\sqrt{105}$ **27.** 18
29. $4 - \sqrt[3]{36}$ **31.** 10 **33.** $6x + 3\sqrt{x} - 2\sqrt{5x} - \sqrt{5}$ **35.** $9r - s$ **37.** $25z + 10\sqrt{z} + 1$ **39.** $4\sqrt[3]{4y^2} - 19\sqrt[3]{2y} - 5$ **41.** $2(\sqrt{3} + 1)$ **43.** $3(\sqrt{2} + \sqrt{5})$
45. $4(\sqrt{5} - \sqrt{7})$ **47.** $5x(1 + \sqrt{2x})$ **49.** $3 - 2\sqrt{6}$ **51.** $1 - \sqrt{5}$
53. $(4 - 2\sqrt{2})/3$ **55.** $(6 - 2\sqrt{6p})/3$ **57.** $3(4 - \sqrt{5})/11$ **59.** $3(\sqrt{5} + 1)/2$
61. $(-2\sqrt{2} + 2\sqrt{5})/3$ **63.** $(6\sqrt{2} + 4)/7$ **65.** $-(2 + 3\sqrt{2})/14$ **67.** $(6\sqrt{5} - 4\sqrt{3})/33$ or $2(3\sqrt{5} - 2\sqrt{3})/33$ **69.** $-2 + \sqrt{6} + \sqrt{2} - \sqrt{3}$ **71.** $2\sqrt{6} + 2\sqrt{5} - 3\sqrt{2} - \sqrt{15}$
73. $\sqrt{m} - 2$ **75.** $(3x + 6\sqrt{xy})/(x - 4y)$ **77.** $(m - 2\sqrt{3mr} + 3r)/(m - 3r)$
79. $3/(2\sqrt{3})$ **81.** $17/[2(6 + \sqrt{2})]$ **83.** $-2/[3(\sqrt{5} - \sqrt{7})]$ **85.** $(9a - b)/[b(3\sqrt{a} - \sqrt{b})]$
89. $\sqrt[3]{25} + \sqrt[3]{15} + \sqrt[3]{9}$ **91.** $\{2\}$ **93.** $\{1, 4\}$ **95.** $\{-1/3, 3/2\}$