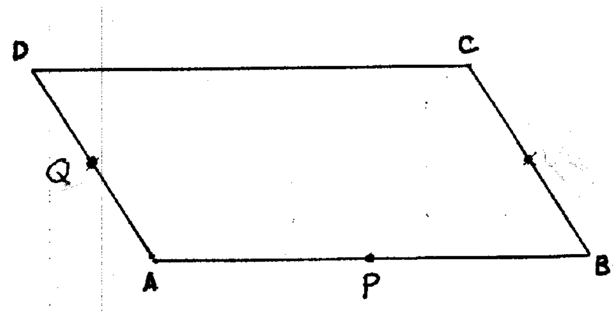
[08-12-04-T11]
Review

[1] ABCD is a parallelogram. P and Q are the midpoints of sides AB, AD respectively.
Show that $\overrightarrow{\mathbf{PD}} + \overrightarrow{\mathbf{QB}} = \frac{1}{2}\overrightarrow{\mathbf{AC}}$



[2] If $\vec{p} = 3\vec{e}_1 - 5\vec{e}_2$ and $\vec{q} = -\vec{e}_1 - \vec{e}_2$ find $|\vec{p} - 2\vec{q}|$.

[3] Let $\vec{u} = \langle 5, -4 \rangle$, $\vec{v} = \langle -10, 4 \rangle$. Write $\vec{a} = \langle -7, 2 \rangle$ as a linear combination of vectors $\vec{u}$ and $\vec{v}$. That is, find scalars m and n such that $\vec{a} = m\vec{u} + n\vec{v}$

[4] Find the cosine of the angle formed by the vectors $\langle 3, 4 \rangle$ and $\langle 8, 15 \rangle$. (Answer with a fraction that is fully reduced).

[5] Prove that for two vectors $\vec{a}$ and $\vec{b}$ not equal to zero, if $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then $\vec{a}$ is perpendicular to $\vec{b}$.

[6] For points $A(-7, 2)$ and $B(3, -5)$, write the position vector of $\overrightarrow{\mathrm{AB}}$ in component form using the basic vectors $\vec{e}_1$ and $\vec{e}_2$ of the x-axis and the y-axis.

[7] Relative to an origin O, the position vectors of points $A(\vec{a})$, $B(\vec{b})$, and $C(\vec{c})$ are $2\vec{p} - 2\vec{q}$, $3\vec{p} + m\vec{q}$, and $(2 + m)\vec{p} + 6\vec{q}$, where $\vec{a}$ and $\vec{b}$ are non-parallel vectors. Given that A, B, and C are collinear, find the possible values of m.

[8] The equation of line ℓ_1 is $\vec{c} = \langle 1, 1 \rangle + t\langle 1, 2 \rangle$ and the equation of line ℓ_2 is $\vec{d} = \langle 1, 5 \rangle + s\langle 3, -4 \rangle$. Find the coordinates of the point at which ℓ_1 intersects ℓ_2.

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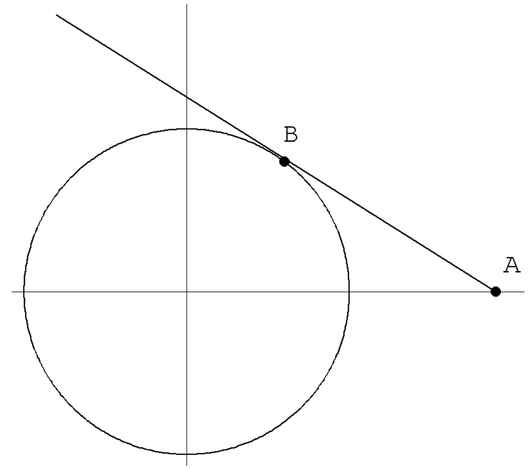
[9] Let a circle have its center at the origin of a coordinate system. Point $B(3, 4)$ is on the circle. Can a line be drawn from $A(10, 0)$ tangent to the circle at point B? Give a proof for your answer.



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Answers

■ Answers

[1] Proof

[2] $\sqrt{34}$

[3] $m = \frac{2}{5}$, $n = \frac{9}{10}$

[4] $\text{Cos}\,\theta = \frac{84}{85}$

[5] Proof

[6] $\overrightarrow{AB} = 10\,\vec{e}_1 - 7\,\vec{e}_2$

[7] $m = -4 \vee m = 2$

[8] The point $(\frac{11}{5}, \frac{17}{5})$

[9] Such a line does not exist.