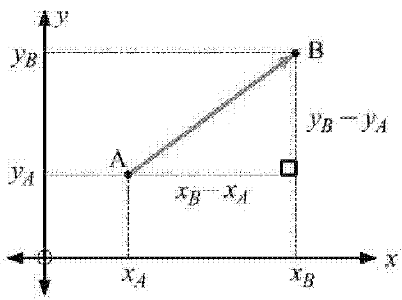
G VECTORS IN COORDINATE GEOMETRY

VECTORS BETWEEN TWO POINTS



In 2-D: consider points $A(x_A, y_A)$ and $B(x_B, y_B)$

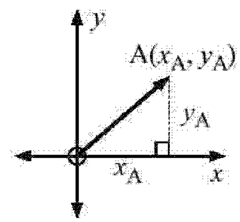
In going from A to B,

$x_B - x_A$ is the x-**step**, and

$y_B - y_A$ is the y-**step**.

Consequently

$$\overrightarrow{AB} = \begin{bmatrix} x_B - x_A \\ y_B - y_A \end{bmatrix}.$$



Notice that if O is (0, 0) and A is (x_A, y_A) then $\overrightarrow{OA}$ is $\begin{bmatrix} x_A \\ y_A \end{bmatrix}$.

In 3-D: if the points are $A(x_A, y_A, z_A)$ and $B(x_B, y_B, z_B)$

$$\overrightarrow{AB} = \begin{bmatrix} x_B - x_A \\ y_B - y_A \\ z_B - z_A \end{bmatrix} \qquad \overrightarrow{OA} = \begin{bmatrix} x_A \\ y_A \\ z_A \end{bmatrix} \qquad \text{and note} \qquad \begin{aligned} \overrightarrow{AB} &= \overrightarrow{OB} - \overrightarrow{OA} \\ &= \mathbf{b} - \mathbf{a} \end{aligned}$$

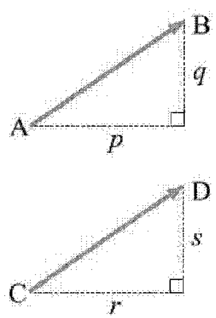
where **a**, **b** are the position vectors of A and B respectively.

DISTANCE BETWEEN TWO POINTS

The distance between two points A and B is the length of vector $\overrightarrow{AB}$ (or $\overrightarrow{BA}$), given by $|\overrightarrow{AB}|$.

Hence, the distance between points A and B is the length of vector $\overrightarrow{AB}$, given by $|\overrightarrow{AB}|$.

VECTOR EQUALITY



Two vectors are **equal** if they have the same length and direction.

Consequently, in 2-D their x-steps are equal i.e., $p = r$

and their y-steps are equal i.e., $q = s$

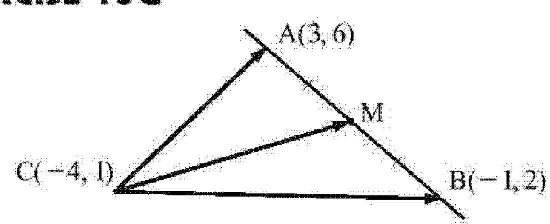
i.e., $\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} r \\ s \end{bmatrix} \Leftrightarrow p = r$ and $q = s$.

(where $\Leftrightarrow$ reads "if and only if")

In 3-D, $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \Leftrightarrow a = p,\ b = q$ and $c = r$.

EXERCISE 15G

1



Find:

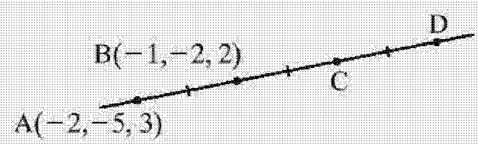
a the coordinates of M

b vectors $\overrightarrow{CA}$, $\overrightarrow{CM}$ and $\overrightarrow{CB}$.

c Verify that $\overrightarrow{CM} = \frac{1}{2}\overrightarrow{CA} + \frac{1}{2}\overrightarrow{CB}$.

Example 26

Find the coordinates of C and D in:



$\overrightarrow{AB} = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$

$$\begin{aligned} \overrightarrow{OC} &= \overrightarrow{OA} + \overrightarrow{AC} \\ &= \overrightarrow{OA} + 2\overrightarrow{AB} \\ &= \begin{bmatrix} -2 \\ -5 \\ 3 \end{bmatrix} + \begin{bmatrix} 2 \\ 6 \\ -2 \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \end{aligned}$$

C is (0, 1, 1)

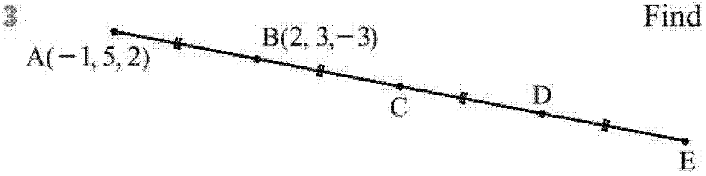
$$\begin{aligned} \overrightarrow{OD} &= \overrightarrow{OA} + \overrightarrow{AD} \\ &= \overrightarrow{OA} + 3\overrightarrow{AB} \\ &= \begin{bmatrix} -2 \\ -5 \\ 3 \end{bmatrix} + \begin{bmatrix} 3 \\ 9 \\ -3 \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix} \end{aligned}$$

∴ D is (1, 4, 0)

2 Find B if C is the centre of a circle with diameter AB:

- **a** A is (3, −2) and C(1, 4)
- **b** A is (0, 5) and C(−1, −2)
- **c** A is (−1, −4) and C(3, 0)

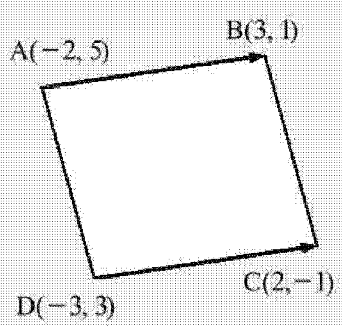
3



Find the coordinates of C, D and E.

Example 27

Use vectors to show that ABCD is a parallelogram where A is (−2, 5), B(3, 1), C(2, −1) and D is (−3, 3).



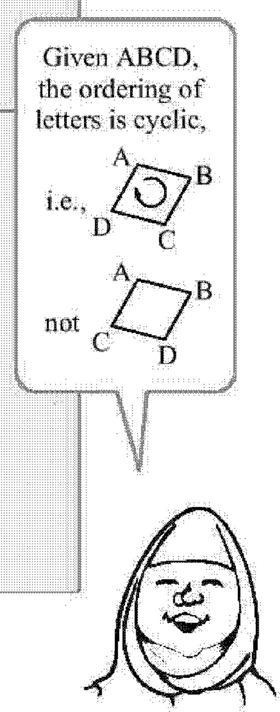
$\overrightarrow{AB} = \begin{bmatrix} 3-(-2) \\ 1-5 \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \end{bmatrix}$

$\overrightarrow{DC} = \begin{bmatrix} 2-(-3) \\ -1-3 \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \end{bmatrix}$

i.e., $\overrightarrow{AB} = \overrightarrow{DC}$

∴ side AB is parallel to side DC and is equal in length (magnitude) to side DC.

Hence ABCD is a parallelogram.

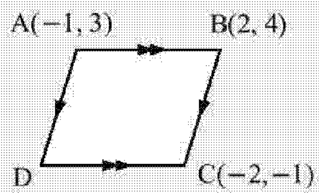


4 Use vectors to find whether or not ABCD is a parallelogram:

- **a** A(3, −1), B(4, 2), C(−1, 4) and D(−2, 1)
- **b** A(5, 0, 3), B(−1, 2, 4), C(4, −3, 6) and D(10, −5, 5)
- **c** A(2, −3, 2), B(1, 4, −1), C(−2, 6, −2) and D(−1, −1, 2).

Example 28

Use vector methods to find the remaining vertex of:



If D is (a, b) then

$$\overrightarrow{CD} = \begin{bmatrix} a-(-2) \\ b-(-1) \end{bmatrix} = \begin{bmatrix} a+2 \\ b+1 \end{bmatrix}$$

But $\overrightarrow{CD} = \overrightarrow{BA}$

$$\therefore \begin{bmatrix} a+2 \\ b+1 \end{bmatrix} = \begin{bmatrix} -1-2 \\ 3-4 \end{bmatrix}$$

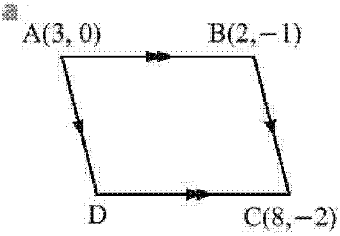
$\therefore \quad a+2=-3 \quad \text{and} \quad b+1=-1$

$\therefore \quad a=-5 \quad \text{and} \quad b=-2$

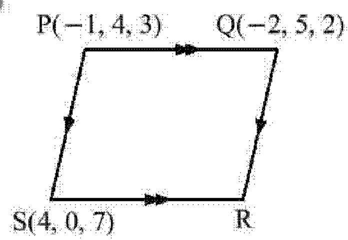
So, D is $(-5, -2)$.

5 Use vector methods to find the remaining vertex of:

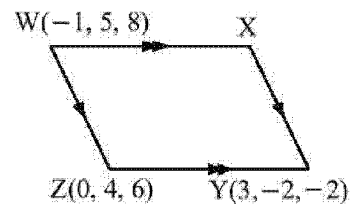
a



b



c

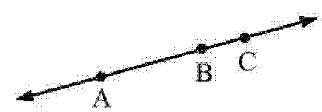


6 Find scalars r and s such that:

a $r\begin{bmatrix} 1 \\ -1 \end{bmatrix} + s\begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} -8 \\ -27 \end{bmatrix}$ **b** $r\begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} + s\begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix} = \begin{bmatrix} 7 \\ -19 \\ 2 \end{bmatrix}$

Three or more points are said to be **collinear** if they lie on the same straight line.

Notice that, A, B and C are collinear if $\overrightarrow{AB} = k\overrightarrow{BC}$ for some scalar k.



Example 29

Prove that A$(-1, 2, 3)$, B$(4, 0, -1)$ and C$(14, -4, -9)$ are collinear and hence find the ratio in which B divides CA.

$$\overrightarrow{AB} = \begin{bmatrix} 5 \\ -2 \\ -4 \end{bmatrix} \qquad \overrightarrow{BC} = \begin{bmatrix} 10 \\ -4 \\ -8 \end{bmatrix} = 2\begin{bmatrix} 5 \\ -2 \\ -4 \end{bmatrix} \qquad \therefore \ \overrightarrow{BC} = 2\overrightarrow{AB}$$

$\therefore$ BC is parallel to AB and since B is common to both, A, B and C are collinear.

To find the ratio in which B divides CA, we find

$$\overrightarrow{CB} : \overrightarrow{BA} = -2\begin{bmatrix} 5 \\ -2 \\ -4 \end{bmatrix} : -\begin{bmatrix} 5 \\ -2 \\ -4 \end{bmatrix} = 2 : 1$$

$\therefore$ B divides CA internally in the ratio 2 : 1.

7 **a** Prove that A$(-2, 1, 4)$, B$(4, 3, 0)$ and C$(19, 8, -10)$ are collinear and hence find the ratio in which A divides CB.

b Prove that P$(2, 1, 1)$, Q$(5, -5, -2)$ and R$(-1, 7, 4)$ are collinear and hence find the ratio in which Q divides PR.

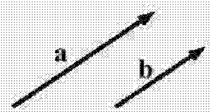
8 **a** A(2, −3, 4), B(11, −9, 7) and C(−13, a, b) are collinear. Find a and b.

b K(1, −1, 0), L(4, −3, 7) and M(a, 2, b) are collinear. Find a and b.

H PARALLELISM

PARALLELISM

If two non-zero vectors are **parallel**, then one is a scalar multiple of the other and vice versa.



Note:
- If **a** is parallel to **b**, then there exists a scalar, k say, such that $\mathbf{a} = k\mathbf{b}$.
- If $\mathbf{a} = k\mathbf{b}$ for some scalar k, then
  - ▸ **a** is parallel to **b**, and
  - ▸ $|\mathbf{a}| = |k|\,|\mathbf{b}|$.

Notice that $\mathbf{a} = \begin{bmatrix} 2 \\ 6 \\ -4 \end{bmatrix}$ is parallel to $\mathbf{b} = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$ and $\mathbf{c} = \begin{bmatrix} 4 \\ 12 \\ -8 \end{bmatrix}$ as $\mathbf{a} = 2\mathbf{b}$ and $\mathbf{a} = \frac{1}{2}\mathbf{c}$.

Also $\mathbf{a} = \begin{bmatrix} 2 \\ 6 \\ -4 \end{bmatrix}$ is parallel to $\mathbf{d} = \begin{bmatrix} -3 \\ -9 \\ 6 \end{bmatrix}$ as $\mathbf{a} = -\frac{2}{3}\mathbf{d}$.

Example 30

Find r and s given that $\mathbf{a} = \begin{bmatrix} 2 \\ -1 \\ r \end{bmatrix}$ is parallel to $\mathbf{b} = \begin{bmatrix} s \\ 2 \\ -3 \end{bmatrix}$.

Since **a** and **b** are parallel, then $\mathbf{a} = k\mathbf{b}$ for some scalar k

$$\therefore \begin{bmatrix} 2 \\ -1 \\ r \end{bmatrix} = k\begin{bmatrix} s \\ 2 \\ -3 \end{bmatrix}$$

$$\therefore \quad 2 = ks, \quad -1 = 2k \quad \text{and} \quad r = -3k$$

Consequently, $k = -\frac{1}{2}$ and $\therefore$ $2 = -\frac{1}{2}s$ and $r = -3\left(\frac{-1}{2}\right)$

$$\therefore \quad r = \tfrac{3}{2} \text{ and } s = -4$$

EXERCISE 15H

1 $\mathbf{a} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -6 \\ r \\ s \end{bmatrix}$ are parallel. Find r and s.

2 Find scalars a and b, given that $\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} a \\ 2 \\ b \end{bmatrix}$ are parallel.

3 **a** Find a vector of length 1 unit which is parallel to $\mathbf{a} = \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}$.
(**Hint:** Let the vector be $k\mathbf{a}$.)

b Find a vector of length 2 units which is parallel to $\mathbf{b} = \begin{bmatrix} -2 \\ -1 \\ 2 \end{bmatrix}$.

4 What can be deduced from the following?

a $\overrightarrow{AB} = 3\overrightarrow{CD}$ b $\overrightarrow{RS} = -\frac{1}{2}\overrightarrow{KL}$ c $\overrightarrow{AB} = 2\overrightarrow{BC}$ d $\overrightarrow{BC} = \frac{1}{3}\overrightarrow{AC}$

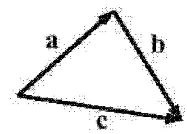
5 The position vectors of P, Q, R and S from O are $\begin{bmatrix}3\\2\\-1\end{bmatrix}$, $\begin{bmatrix}1\\4\\-3\end{bmatrix}$, $\begin{bmatrix}2\\-1\\2\end{bmatrix}$ and $\begin{bmatrix}-1\\-2\\3\end{bmatrix}$ respectively.

a Deduce that PR and QS are parallel.

b What is the relationship between the lengths of PR and QS?

TRIANGLE INEQUALITY

In any triangle, the sum of any two sides must always be greater than the third side. This is based on the well known result "the shortest distance between two points is a straight line".



6 Prove that $|\mathbf{a}+\mathbf{b}| \leqslant |\mathbf{a}| + |\mathbf{b}|$ using a geometrical argument.

[**Hint:** Consider a **a** is not parallel to **b** and use the triangle inequality
b **a** and **b** parallel c any other cases.]

I UNIT VECTORS

A **unit vector** is any vector which has a length of one unit.

For example,

- $\begin{bmatrix}1\\0\\0\end{bmatrix}$ is a unit vector as its length is $\sqrt{1^2+0^2+0^2} = 1$

- $\begin{bmatrix}\frac{1}{\sqrt{2}}\\0\\-\frac{1}{\sqrt{2}}\end{bmatrix}$ is a unit vector as its length is $\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + 0^2 + \left(-\frac{1}{\sqrt{2}}\right)^2} = 1$

- $\mathbf{i} = \begin{bmatrix}1\\0\\0\end{bmatrix}$, $\mathbf{j} = \begin{bmatrix}0\\1\\0\end{bmatrix}$ and $\mathbf{k} = \begin{bmatrix}0\\0\\1\end{bmatrix}$ are special unit vectors in the direction of the positive X, Y and Z-axes respectively.

Notice that $\mathbf{a} = \begin{bmatrix}a_1\\a_2\\a_3\end{bmatrix} \Leftrightarrow \mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$.

component form unit vector form

Thus, $\mathbf{a} = \begin{bmatrix}2\\3\\-5\end{bmatrix}$ can be written as $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$ and vice versa.

We call **i**, **j** and **k** the **base** vectors as any vector can be written as a linear combination of the vectors **i**, **j** and **k**.

EXERCISE 15I

1 Express the following vectors in component form and find their length:

a $\mathbf{i} - \mathbf{j} + \mathbf{k}$ b $3\mathbf{i} - \mathbf{j} + \mathbf{k}$ c $\mathbf{i} - 5\mathbf{k}$ d $\frac{1}{2}(\mathbf{j} + \mathbf{k})$

2 Find k for the unit vectors:

a $\begin{bmatrix} 0 \\ k \end{bmatrix}$ **b** $\begin{bmatrix} k \\ 0 \end{bmatrix}$ **c** $\begin{bmatrix} k \\ 1 \end{bmatrix}$ **d** $\begin{bmatrix} -\frac{1}{2} \\ k \\ \frac{1}{4} \end{bmatrix}$ **e** $\begin{bmatrix} k \\ \frac{2}{3} \\ -\frac{1}{3} \end{bmatrix}$

Example 31

Find the length of $2\mathbf{i} - 5\mathbf{j}$.

As $2\mathbf{i} - 5\mathbf{j} = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$, its length is

$\sqrt{2^2 + (-5)^2}$

$= \sqrt{29}$ units

3 Find the length of the vectors:

a $3\mathbf{i} + 4\mathbf{j}$ **b** $2\mathbf{i} - \mathbf{j} + \mathbf{k}$ **c** $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ **d** $-2.36\mathbf{i} + 5.65\mathbf{j}$

4 Find the unit vector in the direction of: **a** $\mathbf{i} + 2\mathbf{j}$ **b** $2\mathbf{i} - 3\mathbf{k}$ **c** $-2\mathbf{i} - 5\mathbf{j} - 2\mathbf{k}$

Example 32

Find a vector $\mathbf{b}$ of length 7 in the opposite direction to the vector $\mathbf{a} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$.

The unit vector in the direction of $\mathbf{a}$ is $\dfrac{1}{\sqrt{4+1+1}}\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{6}}\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$.

We now multiply this unit vector by -7. Thus $\mathbf{b} = -\frac{7}{\sqrt{6}}\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$.

5 Find a vector $\mathbf{b}$ if:

a it has the same direction as $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and has length 3 units

b it has opposite direction to $\begin{bmatrix} -1 \\ -4 \end{bmatrix}$ and has length 2 units

c it has the same direction as $\begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix}$ and has length 6 units

d it has opposite direction to $\begin{bmatrix} -1 \\ -2 \\ -2 \end{bmatrix}$ and has length 5 units

Note:

- vector $\mathbf{b}$ of length k, $k > 0$ in the same direction as $\mathbf{a}$ is $\mathbf{b} = \dfrac{k}{|\mathbf{a}|}\mathbf{a}$
- vector $\mathbf{b}$ of length k, $k > 0$ in the opposite direction to $\mathbf{a}$ is $\mathbf{b} = -\dfrac{k}{|\mathbf{a}|}\mathbf{a}$
- vector $\mathbf{b}$ of length k, $k > 0$ which is parallel to $\mathbf{a}$ is $\mathbf{b} = \pm\dfrac{k}{|\mathbf{a}|}\mathbf{a}$

EXERCISE 15C.3

1 **a** $\begin{bmatrix}-3\\-15\end{bmatrix}$ **b** $\begin{bmatrix}-1\\2\end{bmatrix}$ **c** $\begin{bmatrix}0\\14\end{bmatrix}$ **d** $\begin{bmatrix}5\\-3\end{bmatrix}$ **e** $\begin{bmatrix}\frac{5}{2}\\\frac{11}{2}\end{bmatrix}$ **f** $\begin{bmatrix}-7\\7\end{bmatrix}$

g $\begin{bmatrix}5\\11\end{bmatrix}$ **h** $\begin{bmatrix}3\\\frac{17}{3}\end{bmatrix}$ **2** **a** $\begin{bmatrix}8\\-1\end{bmatrix}$ **b** $\begin{bmatrix}8\\-1\end{bmatrix}$ **c** $\begin{bmatrix}8\\-1\end{bmatrix}$

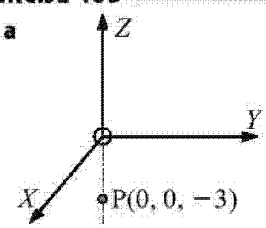
EXERCISE 15C.4

1 **a** $\sqrt{13}$ units **b** $\sqrt{17}$ units **c** $5\sqrt{2}$ units **d** $\sqrt{10}$ units **e** $\sqrt{29}$ units

2 **a** $\sqrt{10}$ units **b** $2\sqrt{10}$ units **c** $2\sqrt{10}$ units **d** $3\sqrt{10}$ units **e** $3\sqrt{10}$ units **f** $2\sqrt{5}$ units **g** $8\sqrt{5}$ units **h** $8\sqrt{5}$ units **i** $\sqrt{5}$ units **j** $\sqrt{5}$ units

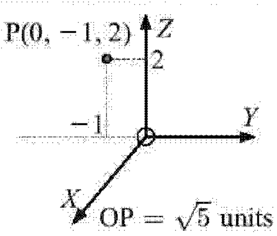
EXERCISE 15D

1 **a**



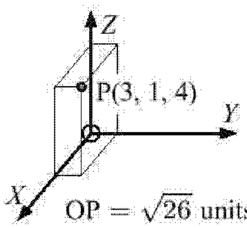
OP = 3 units

b



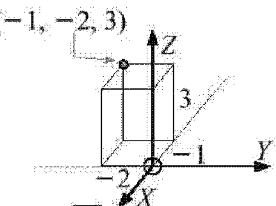
OP = $\sqrt{5}$ units

c



OP = $\sqrt{26}$ units

d



OP = $\sqrt{14}$ units

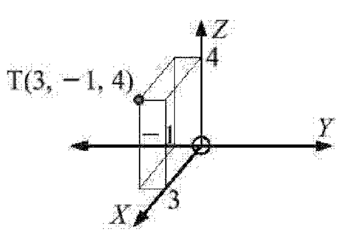
2 **a** **i** $\sqrt{14}$ units **ii** $(-\frac{1}{2}, \frac{1}{2}, 2)$ **b** **i** $\sqrt{14}$ units **ii** $(1, -\frac{1}{2}, \frac{3}{2})$
c **i** $\sqrt{21}$ units **ii** $(1, -\frac{1}{2}, 0)$ **d** **i** $\sqrt{14}$ units **ii** $(1, \frac{1}{2}, -\frac{3}{2})$

4 **a** isosceles **b** right angled **c** right angled **d** straight line **5** (0, 3, 5), $r = \sqrt{3}$ units

6 **a** $(0, y, 0)$ **b** (0, 2, 0) and (0, −4, 0)

EXERCISE 15E.1

1 **a**



b $\overrightarrow{OT} = \begin{bmatrix}3\\-1\\4\end{bmatrix}$ **c** OT = $\sqrt{26}$ units

2 **a** $\overrightarrow{AB} = \begin{bmatrix}4\\-1\\-3\end{bmatrix}$, $\overrightarrow{BA} = \begin{bmatrix}-4\\1\\3\end{bmatrix}$ **b** AB = $\sqrt{26}$ units, BA = $\sqrt{26}$ units

3 $\overrightarrow{OA} = \begin{bmatrix}3\\1\\0\end{bmatrix}$, $\overrightarrow{OB} = \begin{bmatrix}-1\\1\\2\end{bmatrix}$, $\overrightarrow{AB} = \begin{bmatrix}-4\\0\\2\end{bmatrix}$

4 **a** $\overrightarrow{NM} = \begin{bmatrix}5\\-4\\-1\end{bmatrix}$ **b** $\overrightarrow{MN} = \begin{bmatrix}-5\\4\\1\end{bmatrix}$ **c** MN = $\sqrt{42}$ units

5 **a** $\overrightarrow{OA} = \begin{bmatrix}-1\\2\\5\end{bmatrix}$, OA = $\sqrt{30}$ units

b $\overrightarrow{AC} = \begin{bmatrix}-2\\-1\\-5\end{bmatrix}$, AC = $\sqrt{30}$ units

c $\overrightarrow{CB} = \begin{bmatrix}5\\-1\\3\end{bmatrix}$, CB = $\sqrt{35}$ units

6 **a** $\sqrt{13}$ units **b** $\sqrt{14}$ units **c** 3 units

EXERCISE 15E.2

1 **a** $a = 5$, $b = 6$, $c = -6$ **b** $a = 4$, $b = 2$, $c = 1$

2 **a** $a = \frac{1}{3}$, $b = 2$, $c = 1$ **b** $a = 1$, $b = 2$ **c** $a = 1$, $b = -1$, $c = 2$

3 **a** $r = 2$, $s = 4$, $t = -7$ **b** $r = -4$, $s = 0$, $t = 3$

4 **a** $\overrightarrow{AB} = \begin{bmatrix}2\\-5\\-1\end{bmatrix}$, $\overrightarrow{DC} = \begin{bmatrix}2\\-5\\-1\end{bmatrix}$

b ABCD is a parallelogram **5** **a** S = (−2, 8, −3)

EXERCISE 15F

1 **a** $\mathbf{x} = \frac{1}{2}\mathbf{q}$ **b** $\mathbf{x} = 2\mathbf{n}$ **c** $\mathbf{x} = -\frac{1}{3}\mathbf{p}$ **d** $\mathbf{x} = \frac{1}{2}(\mathbf{r} - \mathbf{q})$ **e** $\mathbf{x} = \frac{1}{5}(4\mathbf{s} - \mathbf{t})$ **f** $\mathbf{x} = 3(4\mathbf{m} - \mathbf{n})$

2 **a** $\mathbf{y} = \begin{bmatrix}-1\\\frac{3}{2}\end{bmatrix}$ **b** $\mathbf{y} = \begin{bmatrix}2\\4\end{bmatrix}$ **c** $\mathbf{y} = \begin{bmatrix}\frac{3}{2}\\-\frac{1}{2}\end{bmatrix}$ **d** $\mathbf{y} = \begin{bmatrix}\frac{5}{4}\\\frac{3}{4}\end{bmatrix}$

4 **a** $\mathbf{x} = \begin{bmatrix}4\\-6\\-5\end{bmatrix}$ **b** $\mathbf{x} = \begin{bmatrix}1\\-\frac{2}{3}\\\frac{5}{3}\end{bmatrix}$ **c** $\mathbf{x} = \begin{bmatrix}\frac{3}{2}\\-1\\\frac{5}{2}\end{bmatrix}$

5 $\overrightarrow{AB} = \begin{bmatrix}3\\4\\-2\end{bmatrix}$, AB = $\sqrt{29}$ units

7 **a** $\overrightarrow{BD} = \frac{1}{2}\mathbf{a}$ **b** $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ **c** $\overrightarrow{BA} = -\mathbf{b} + \mathbf{a}$ **d** $\overrightarrow{OD} = \mathbf{b} + \frac{1}{2}\mathbf{a}$ **e** $\overrightarrow{AD} = \mathbf{b} - \frac{1}{2}\mathbf{a}$ **f** $\overrightarrow{DA} = \frac{1}{2}\mathbf{a} - \mathbf{b}$

8 **a** $\begin{bmatrix}-1\\5\\-1\end{bmatrix}$ **b** $\begin{bmatrix}-3\\4\\-2\end{bmatrix}$ **c** $\begin{bmatrix}-3\\6\\-5\end{bmatrix}$

9 **a** $\begin{bmatrix}3\\1\\-2\end{bmatrix}$ **b** $\begin{bmatrix}1\\-3\\4\end{bmatrix}$ **c** $\begin{bmatrix}1\\4\\-9\end{bmatrix}$ **d** $\begin{bmatrix}2\\-4\\10\end{bmatrix}$ **e** $\begin{bmatrix}3\\2\\-5\end{bmatrix}$

f $\begin{bmatrix}-1\\\frac{3}{2}\\-\frac{7}{2}\end{bmatrix}$ **g** $\begin{bmatrix}1\\-4\\7\end{bmatrix}$ **h** $\begin{bmatrix}4\\2\\-2\end{bmatrix}$

10 **a** $\sqrt{11}$ units **b** $\sqrt{14}$ units **c** $\sqrt{38}$ units **d** $\sqrt{3}$ units **e** $\begin{bmatrix}\sqrt{11}\\-3\sqrt{11}\\2\sqrt{11}\end{bmatrix}$ **f** $\begin{bmatrix}-\frac{1}{\sqrt{11}}\\\frac{1}{\sqrt{11}}\\\frac{3}{\sqrt{11}}\end{bmatrix}$

EXERCISE 15G

1 **a** M(1, 4) **b** $\overrightarrow{CA} = \begin{bmatrix}7\\5\end{bmatrix}$, $\overrightarrow{CM} = \begin{bmatrix}5\\3\end{bmatrix}$, $\overrightarrow{CB} = \begin{bmatrix}3\\1\end{bmatrix}$

2 **a** B(−1, 10) **b** B(−2, −9) **c** B(7, 4)

3 C(5, 1, −8), D(8, −1, −13), E(11, −3, −18)

4 **a** parallelogram **b** parallelogram **c** not parallelogram

5 **a** D(9, −1) **b** R(3, 1, 6) **c** X(2, −1, 0)

6 **a** $r = 2$, $s = -5$ **b** $r = 4$, $s = -1$

7 **a** −7 : 2 **b** −1 : 2

8 **a** $a = 7$, $b = -1$ **b** $a = -\frac{7}{2}$, $b = -\frac{21}{2}$

EXERCISE 15H

1 $r = 3$, $s = -9$

2 $a = -6$, $b = -4$

3 **a** $\begin{bmatrix}\frac{2}{3}\\-\frac{1}{3}\\-\frac{2}{3}\end{bmatrix}$ **b** $\begin{bmatrix}-\frac{4}{3}\\-\frac{2}{3}\\\frac{4}{3}\end{bmatrix}$

4 **a** $\overrightarrow{AB} \parallel \overrightarrow{CD}$, AB = 3 CD **b** $\overrightarrow{RS} \parallel \overrightarrow{KL}$, RS = $\frac{1}{2}$KL opposite direction

c A, B and C are collinear and AB = 2 BC

d A, B and C are collinear and AC = 3 BC

5 **a** $\overrightarrow{PR} = \begin{bmatrix} -1 \\ -3 \\ 3 \end{bmatrix}$, $\overrightarrow{QS} = \begin{bmatrix} -2 \\ -6 \\ 6 \end{bmatrix}$ **b** $PR = \frac{1}{2} QS$

EXERCISE 15I

1 **a** $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$, $\sqrt{3}$ units **b** $\begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$, $\sqrt{11}$ units

c $\begin{bmatrix} 1 \\ 0 \\ -5 \end{bmatrix}$, $\sqrt{26}$ units **d** $\begin{bmatrix} 0 \\ \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$, $\frac{1}{\sqrt{2}}$ units

2 **a** $k = \pm 1$ **b** $k = \pm 1$ **c** $k = 0$ **d** $k = \pm\frac{\sqrt{11}}{4}$ **e** $k = \pm\frac{2}{3}$

3 **a** 5 units **b** $\sqrt{6}$ units **c** 3 units **d** $\doteqdot 6.12$ units

4 **a** $\frac{1}{\sqrt{5}}(\mathbf{i} + 2\mathbf{j})$ **b** $\frac{1}{\sqrt{13}}(2\mathbf{i} - 3\mathbf{k})$ **c** $\frac{1}{\sqrt{33}}(-2\mathbf{i} - 5\mathbf{j} - 2\mathbf{k})$

5 **a** $\frac{3}{\sqrt{5}}\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ **b** $-\frac{2}{\sqrt{17}}\begin{bmatrix} -1 \\ -4 \end{bmatrix}$ **c** $\frac{6}{\sqrt{18}}\begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix}$ **d** $-\frac{5}{3}\begin{bmatrix} -1 \\ -2 \\ -2 \end{bmatrix}$

EXERCISE 15J.1

1 **a** 7 **b** 22 **c** 29 **d** 66 **e** 52 **f** 3 **g** 5 **h** 1

2 **a** 2 **b** 2 **c** 14 **d** 14 **e** 4 **f** 4

3 **a** 1 **b** 1 **c** 0

5 **a** $t = 6$ **b** $t = -8$ **c** $t = 0$ or 2 **d** $t = -\frac{3}{2}$

6 **a** $t = -\frac{3}{2}$ **b** $t = -\frac{6}{7}$ **c** $t = \frac{-1 \pm \sqrt{5}}{2}$ **d** impossible

7 Show $\mathbf{a} \bullet \mathbf{b} = \mathbf{b} \bullet \mathbf{c} = \mathbf{a} \bullet \mathbf{c} = 0$ **8** **b** $t = -\frac{5}{6}$

9 $\overrightarrow{AB} \bullet \overrightarrow{AC} = 0$, $\therefore$ $\measuredangle$BAC is a right angle

10 **b** $AB = \sqrt{14}$ units, $BC = \sqrt{14}$ units, ABCD is a rhombus

c 0, the diagonals of a rhombus are perpendicular.

11 **a** 101.3° or 78.7° **b** 116.6° or 63.4°

c 63.4° or 116.6° **d** 71.6° or 108.4°

12 **a** 5 **b** -9

13 **a** $k\begin{bmatrix} -2 \\ 5 \end{bmatrix}$, $k \neq 0$ **b** $k\begin{bmatrix} -2 \\ 1 \end{bmatrix}$, $k \neq 0$ **c** $k\begin{bmatrix} 1 \\ 3 \end{bmatrix}$, $k \neq 0$

d $k\begin{bmatrix} 3 \\ 4 \end{bmatrix}$, $k \neq 0$ **e** $k\begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $k \neq 0$

EXERCISE 15J.2

1 **a** -1 **b** 109.5° (acute 70.5°) **c** $\begin{bmatrix} -\frac{1}{3} \\ -\frac{1}{3} \\ -\frac{1}{3} \end{bmatrix}$ **d** $\frac{1}{\sqrt{3}}$

2 $\measuredangle ABC \doteqdot 62.5°$, the exterior angle 117.5°

3 **a** 54.7° **b** 60° **c** 35.3°

4 **a** 30.3° **b** 54.2° **5** **a** $M(\frac{3}{2}, \frac{5}{2}, \frac{3}{2})$ **b** 51.5°

6 **a** $t = 0$ or -3 **b** $r = -2$, $s = 5$, $t = -4$

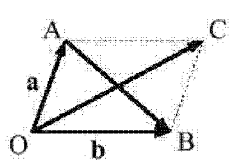
7 **a** 74.5° **b** 72.45°

8 $\mathbf{a} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{c} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ will do

$\mathbf{a} \bullet \mathbf{b} = \mathbf{a} \bullet \mathbf{c}$, but $\mathbf{b} \neq \mathbf{c}$

10 **a** **Hint:** Square both sides.

b Consider the parallelogram. Find $\overrightarrow{AB}$ and $\overrightarrow{OC}$, etc.



11 -7

12 $\mathbf{a} \bullet \mathbf{b}$ is a scalar and so $\mathbf{a} \bullet \mathbf{b} \bullet \mathbf{c}$ is a scalar 'dotted' with a vector which is meaningless.

EXERCISE 15K.1

1 **a** [2, 5, 11] **b** [2, 4, 1] **c** $-\mathbf{i} - \mathbf{j} - \mathbf{k}$ **d** $\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}$

2 **a** $\mathbf{a} \times \mathbf{b} = [-11, -2, 5]$,

$\mathbf{a} \bullet (\mathbf{a} \times \mathbf{b}) = 0 = \mathbf{b} \bullet (\mathbf{a} \times \mathbf{b})$

$\mathbf{a} \times \mathbf{b}$ is a vector perpendicular to both **a** and **b**

3 **a** $\mathbf{i} \times \mathbf{i} = \mathbf{0}$ $\mathbf{j} \times \mathbf{j} = \mathbf{0}$ $\mathbf{k} \times \mathbf{k} = \mathbf{0}$

b $\mathbf{i} \times \mathbf{j} = \mathbf{k}$ $\mathbf{j} \times \mathbf{i} = -\mathbf{k}$ $\mathbf{j} \times \mathbf{k} = \mathbf{i}$ $\mathbf{k} \times \mathbf{j} = -\mathbf{i}$

$\mathbf{i} \times \mathbf{k} = -\mathbf{j}$ $\mathbf{k} \times \mathbf{i} = \mathbf{j}$

$\mathbf{a} \times \mathbf{a} = \mathbf{0}$ $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$

5 **a** $\begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$ **b** 17 **c** 17

7 **a** $\begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}$ **b** $\begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix}$ **c** $\begin{bmatrix} 2 \\ 4 \\ -1 \end{bmatrix}$ **d** $\begin{bmatrix} 2 \\ 4 \\ -1 \end{bmatrix}$

8 $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) + (\mathbf{a} \times \mathbf{c})$

11 **a** $\mathbf{a} \times \mathbf{b}$ **b** $\mathbf{0}$ **c** $\mathbf{0}$

12 **a** $k\begin{bmatrix} -4 \\ 1 \\ 3 \end{bmatrix}$ **b** $k\begin{bmatrix} 6 \\ 22 \\ -15 \end{bmatrix}$ **c** $(-\mathbf{i} + \mathbf{j} - 2\mathbf{k})n$

d $(5\mathbf{i} + \mathbf{j} + 4\mathbf{k})n$ $n, k \in \mathcal{R}$, $n, k \neq 0$

13 $k\begin{bmatrix} 4 \\ -5 \\ -7 \end{bmatrix}$, $k \neq 0$ $\frac{\sqrt{10}}{6}\begin{bmatrix} 4 \\ -5 \\ -7 \end{bmatrix}$ *or* $-\frac{\sqrt{10}}{6}\begin{bmatrix} 4 \\ -5 \\ -7 \end{bmatrix}$

14 **a** $\begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix}$ **b** $\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$

EXERCISE 15K.2

1 **a** $\mathbf{i} \times \mathbf{k} = -\mathbf{j}$, $\mathbf{k} \times \mathbf{i} = \mathbf{j}$

2 **a** $\mathbf{a} \bullet \mathbf{b} = -1$ $\mathbf{a} \times \mathbf{b} = [1, 5, 1]$

b $\cos\theta = -\frac{1}{\sqrt{28}}$ **c** $\sin\theta = \frac{\sqrt{27}}{\sqrt{28}}$ **d** $\sin\theta = \frac{\sqrt{27}}{\sqrt{28}}$

4 **a** $\overrightarrow{OA} = [2, 3, -1]$ $\overrightarrow{OB} = [-1, 1, 2]$

b $\overrightarrow{OA} \times \overrightarrow{OB} = [7, -3, 5]$ $|\overrightarrow{OA} \times \overrightarrow{OB}| = \sqrt{83}$

c Area ΔOAB $= \frac{1}{2}|\overrightarrow{OA}||\overrightarrow{OB}|\sin\theta$

$= \frac{1}{2}|\overrightarrow{OA} \times \overrightarrow{OB}| = \frac{\sqrt{83}}{2}$ units²

5 **a** $\overrightarrow{OC}$ is parallel to $\overrightarrow{AB}$ **b** $\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{c}$

EXERCISE 15K.3

1 **a** $\frac{\sqrt{101}}{2}$ units² **b** $\frac{\sqrt{133}}{2}$ units² **c** $\frac{\sqrt{69}}{2}$ units²

2 $8\sqrt{2}$ units² **3** **a** D(−4, 1, 3) **b** $\sqrt{307}$ units²

4 **a** 4 units³ **b** $(\sqrt{42} + 2\sqrt{3} + 3\sqrt{2} + 6)$ units²

5 **a** (3, 1, 0), (1, 3, 3), (4, 2, 3), (4, 3, 3) **b** $\doteqdot 79.01°$

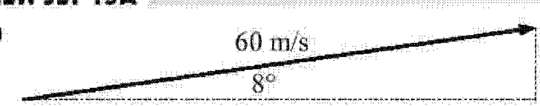
c 9 units³ **6** $k = 2 \pm 2\sqrt{33}$

7 $S = \frac{1}{2}\{|\mathbf{a} \times \mathbf{b}| + |\mathbf{a} \times \mathbf{c}| + |\mathbf{b} \times \mathbf{c}| + |(\mathbf{b} - \mathbf{a}) \times (\mathbf{c} - \mathbf{a})|\}$

9 **a** Yes **b** No **10** $k = \frac{23}{10}$

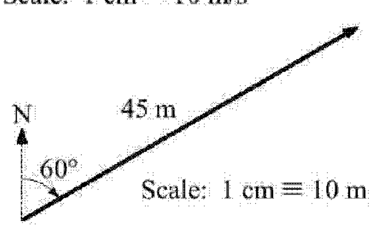
REVIEW SET 15A

1 **a**



Scale: 1 cm ≡ 10 m/s

b



Scale: 1 cm ≡ 10 m