Section 1.3 Exercises

Polynomial Curve Fitting

In Exercises 1–6, (a) determine the polynomial whose graph passes through the given points, and (b) sketch the graph of the polynomial showing the given points.

1. $(2, 5), (3, 2), (4, 5)$
2. $(2, 4), (3, 4), (4, 4)$
3. $(2, 4), (3, 6), (5, 10)$
4. $(-1, 3), (0, 0), (1, 1), (4, 58)$
5. $(1997, 5), (1998, 7), (1999, 12)$ (Let $z = x - 1998$.)
6. $(1997, 150), (1998, 180), (1999, 240), (2000, 360)$ (Let $z = x - 1998$.)
7. Try to fit the graph of a polynomial function to the given values. What happens, and why?

x	1	2	3	3	4
y	1	1	2	3	4

8. The graph of a function f passes through the points $(0, 1)$, $\left(2, \frac{1}{3}\right)$, and $\left(4, \frac{1}{5}\right)$. Find a quadratic function that passes through these points.
9. Find a polynomial function p of degree 2 or less that passes through the points $(0, 1)$, $(2, 3)$, and $(4, 5)$. Then sketch the graph of $y = 1/p(x)$ and compare this graph with the graph of the polynomial found in Exercise 8.
10. ***Calculus*** The graph of a parabola passes through the points $(0, 1)$ and $\left(\frac{1}{2}, \frac{1}{2}\right)$ and has a horizontal tangent at $\left(\frac{1}{2}, \frac{1}{2}\right)$. Find an equation for the parabola and sketch its graph.
11. ***Calculus*** A cubic polynomial has horizontal tangents at $(1, -2)$ and $(-1, 2)$. Find an equation for the cubic and sketch its graph.
12. Find an equation of the circle passing through the points $(1, 3)$, $(-2, 6)$, and $(4, 2)$.
13. The U.S. census lists the population of the United States as 203.3 million in 1970, 226.5 million in 1980, and 248.7 million in 1990. Fit a second-degree polynomial to these three points and use your result to predict the population in 2000 and 2005.
14. The U.S. population for the years 1920, 1930, 1940, and 1950 is given in the following table.

Year	1920	1930	1940	1950
Population (in millions)	106	123	132	151

(a) Find a cubic polynomial that fits these data and use your result to estimate the population in 1960.
(b) How does your estimate compare with the actual 1960 population of 179 million?

15. Use $\sin 0 = 0$, $\sin(\pi/2) = 1$, and $\sin \pi = 0$ to estimate $\sin(\pi/3)$.
16. Use $\log_2 1 = 0, \log_2 2 = 1$, and $\log_2 4 = 2$ to estimate $\log_2 3$.
17. ***Guided Proof*** Prove that if a polynomial function $p(x) = a_0 + a_1x + a_2x^2$ is zero for $x = -1$, $x = 0$, and $x = 1$, then $a_0 = a_1 = a_2 = 0$.
    Getting Started: Write a system of linear equations and solve the system for a_0, a_1, and a_2.
    (i) Substitute $x = -1, 0$, and 1 into $p(x)$.
    (ii) Set the result equal to 0.
    (iii) Solve the resulting system of linear equations in variables a_0, a_1, and a_2.
18. The statement in Exercise 17 can be generalized: If a polynomial function $p(x) = a_0 + a_1x + \cdots + a_{n-1}x^{n-1}$ is zero for more than $n - 1$ x-values, then $a_0 = a_1 = \cdots = a_{n-1} = 0$. Use this result to prove that there is at most one polynomial function of degree $n - 1$ (or less) whose graph passes through n points in the plane with distinct x-coordinates.

Network Analysis

19. Water is flowing through a network of pipes (in thousands of cubic meters per hour), as shown in Figure 1.15.
    (a) Solve this system for the water flow represented by x_i, $i = 1, 2, \ldots, 7$.
    (b) Find the network flow pattern when $x_6 = x_7 = 0$.
    (c) Find the network flow pattern when $x_5 = 1000$ and $x_6 = 0$.

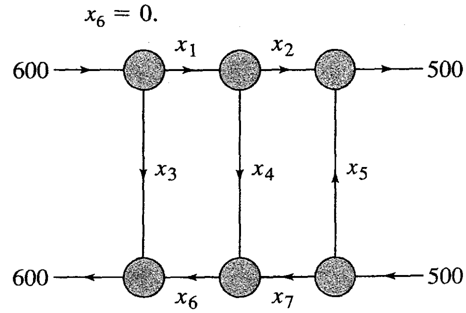


Figure 1.15

20. The flow of traffic (in vehicles per hour) through a network of streets is shown in Figure 1.16.

(a) Solve this system for x_i, $i = 1, 2, \ldots, 5$.
(b) Find the traffic flow when $x_2 = 200$ and $x_3 = 50$.
(c) Find the traffic flow when $x_2 = 150$ and $x_3 = 0$.

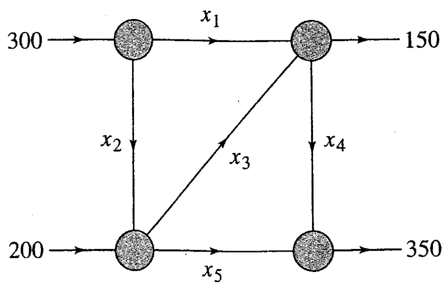


Figure 1.16

21. The flow of traffic (in vehicles per hour) through a network of streets is shown in Figure 1.17.

(a) Solve this system for x_i, $i = 1, 2, 3, 4$.
(b) Find the traffic flow when $x_4 = 0$.
(c) Find the traffic flow when $x_4 = 100$.

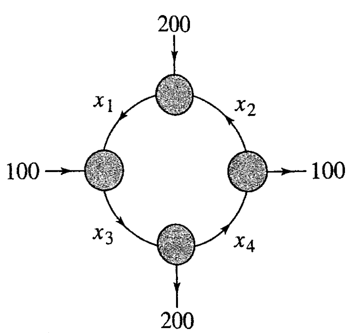


Figure 1.17

22. The flow of traffic (in vehicles per hour) through a network of streets is shown in Figure 1.18. (See top of next column.)

(a) Solve this system for x_i, $i = 1, 2, \ldots, 5$.
(b) Find the traffic flow when $x_3 = 0$ and $x_5 = 100$.
(c) Find the traffic flow when $x_3 = x_5 = 100$.

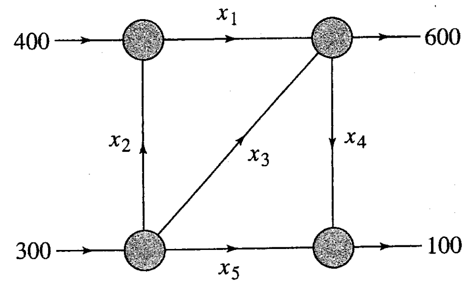


Figure 1.18

23. Determine the currents I_1, I_2, and I_3 for the electrical network shown in Figure 1.19.

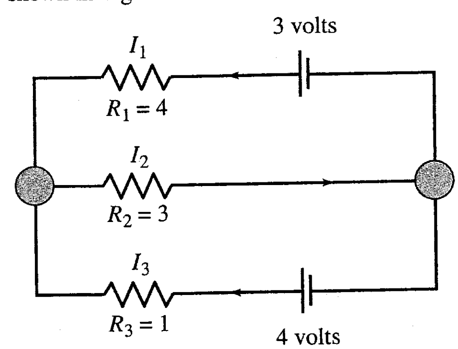


Figure 1.19

24. Determine the currents I_1, I_2, and I_3 for the electrical network shown in Figure 1.20.

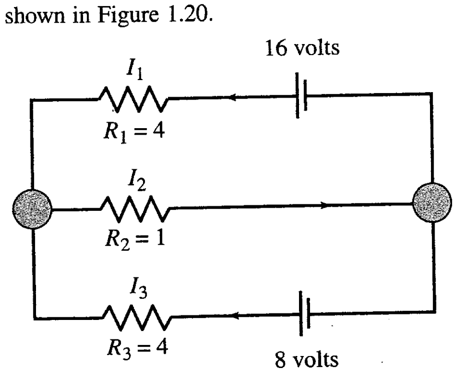


Figure 1.20

25. (a) Determine the currents I_1, I_2, and I_3 for the electrical network shown in Figure 1.21.
(b) How is the result affected when A is changed to 2 volts and B is changed to 6 volts?

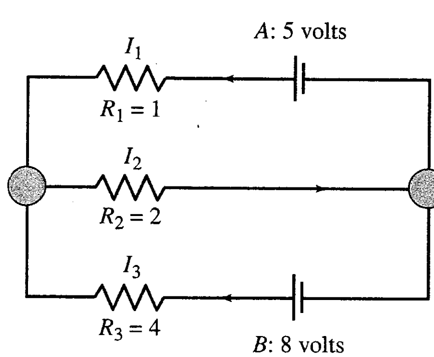
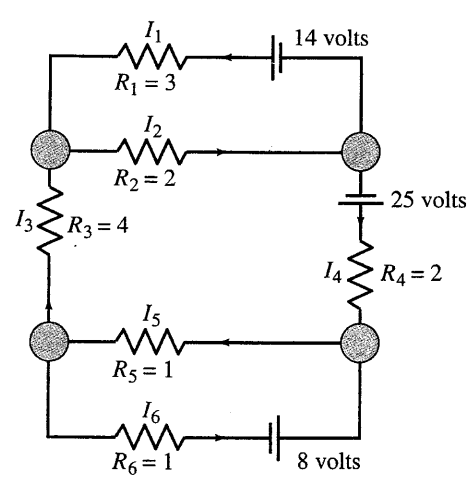
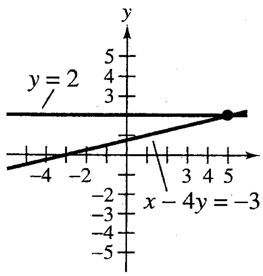


Figure 1.21

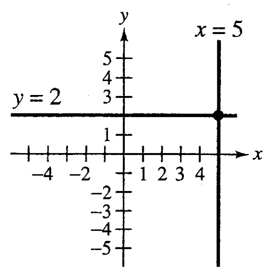
26. Determine the currents I_1, I_2, I_3, I_4, I_5, and I_6 for the electrical network shown in Figure 1.22.



$$x - 4y = -3$$
$$y = 2$$



$$x = 5$$
$$y = 2$$



SECTION 1.2 (*page 24*)

1. 3×3 **3.** 1×5 **5.** 4×5

7. Reduced row-echelon form

9. Not in row-echelon form

11. Not in row-echelon form

13. $x_1 = 0$, $x_2 = 2$

15. $x_1 = 2$, $x_2 = -1$, $x_3 = -1$

17. $x_1 = -26$, $x_2 = 13$, $x_3 = -7$, $x_4 = 4$

19. $x = 3$, $y = 2$

21. No solution

23. $x = 4$, $y = -2$

25. $x_1 = 4$, $x_2 = -3$, $x_3 = 2$

27. $x_1 = 1 + 2t$, $x_2 = 2 + 3t$, $x_3 = t$

29. $x = 100 + 96t - 3s$, $y = s$, $z = 54 + 52t$, $w = t$

31. $x = 0$, $y = 2 - 4t$, $z = t$

33. $x_1 = 23.5361 + 0.5278t$, $x_2 = 18.5444 + 4.1111t$, $x_3 = 7.4306 + 2.1389t$, $x_4 = t$

35. $x_1 = 2$, $x_2 = -2$, $x_3 = 3$, $x_4 = -5$, $x_5 = 1$

37. $x_1 = 0$, $x_2 = -t$, $x_3 = t$

39. $x_1 = -t$, $x_2 = s$, $x_3 = 0$, $x_4 = t$

41. (a) Two equations in two unknowns
(b) All real $k \neq -\frac{4}{3}$
(c) Two equations in three unknowns
(d) All real k

43. (a) $a + b + c = 0$
(b) $a + b + c \neq 0$
(c) Not possible

45. (a) $x = \frac{8}{3} - \frac{5}{6}t$, $y = -\frac{8}{3} + \frac{5}{6}t$, $z = t$
(b) $x = \frac{18}{7} - \frac{11}{14}t$, $y = -\frac{20}{7} + \frac{13}{14}t$, $z = t$
(c) $x = 3 - t$, $y = -3 + t$, $z = t$
(d) Each system has an infinite number of solutions.

47. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

49. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & k \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

51. (a) True (b) True
(c) False (d) True

53. $ad - bc \neq 0$

55. $\lambda = 1, 3$

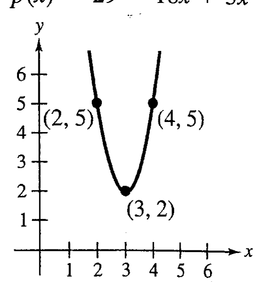
57. Yes, it is possible:
$x_1 + x_2 + x_3 = 0$
$x_1 + x_2 + x_3 = 1$

59. $\alpha = \pi/3, 5\pi/3, \beta = \pi/2$ (*Hint:* Let $u = \cos \alpha$ and $v = \sin \beta$.)

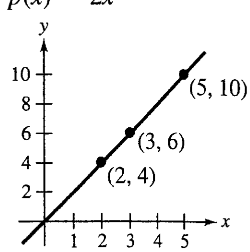
61. The rows have been interchanged.

SECTION 1.3 (*page 35*)

1. (a) $p(x) = 29 - 18x + 3x^2$
(b)



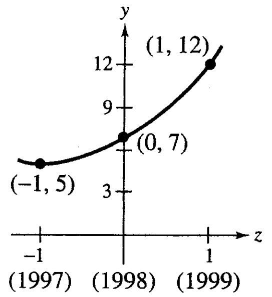
3. (a) $p(x) = 2x$
(b)



5. (a) $p(z) = 7 + \frac{7}{2}z + \frac{3}{2}z^2$

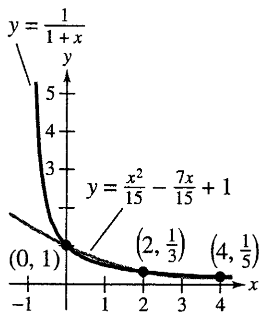
$p(x) = 7 + \frac{7}{2}(x - 1998) + \frac{3}{2}(x - 1998)^2$

(b)

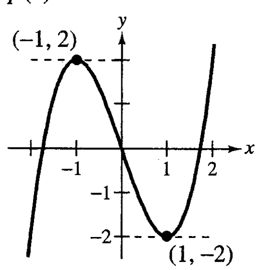


7. y is not a function of x because the x-value of 3 is repeated.

9. $p(x) = 1 + x$



11. $p(x) = -3x + x^3$



13. $p(x) = 203.3 + 2.37x - 0.005x^2$

2000: $x = 30$ ⟶ $p = 269.9$

2005: $x = 35$ ⟶ $p = 280.125$

15. $p(x) = -\dfrac{4}{\pi^2}x^2 + \dfrac{4}{\pi}x$

$\sin\dfrac{\pi}{3} \approx \dfrac{8}{9} \approx 0.889$

(Actual value is $\sqrt{3}/2 \approx 0.866$.)

17. Solve the system:

$$\begin{aligned} p(-1) &= a_0 - a_1 + a_2 = 0 \\ p(0) &= a_0 \qquad\qquad\;\; = 0 \\ p(1) &= a_0 + a_1 + a_2 = 0 \\ &\; a_0 = a_1 = a_2 = 0 \end{aligned}$$

19. (a) $x_1 = s$, $x_2 = t$, $x_3 = 600 - s$, $x_4 = s - t$, $x_5 = 500 - t$, $x_6 = s$, $x_7 = t$

(b) $x_1 = 0$, $x_2 = 0$, $x_3 = 600$, $x_4 = 0$, $x_5 = 500$, $x_6 = 0$, $x_7 = 0$

(c) $x_1 = 0$, $x_2 = -500$, $x_3 = 600$, $x_4 = 500$, $x_5 = 1000$, $x_6 = 0$, $x_7 = -500$

21. (a) $x_1 = 100 + t$, $x_2 = -100 + t$, $x_3 = 200 + t$, $x_4 = t$

(b) $x_1 = 100$, $x_2 = -100$, $x_3 = 200$, $x_4 = 0$

(c) $x_1 = 200$, $x_2 = 0$, $x_3 = 300$, $x_4 = 100$

23. $I_1 = 0$, $I_2 = 1$, $I_3 = 1$

25. (a) $I_1 = 1$, $I_2 = 2$, $I_3 = 1$ (b) $I_1 = 0$, $I_2 = 1$, $I_3 = 1$

Review Exercises – Chapter 1 (*page 38*)

1. $x = -\frac{1}{4} + \frac{1}{2}s - \frac{3}{2}t$, $y = s$, $z = t$

3. Row-echelon form (not reduced)

5. Not in row-echelon form

7. $x_1 = -2t$, $x_2 = t$, $x_3 = 0$

9. $x = \frac{1}{2}$, $y = \frac{3}{2}$

11. $x = -12$, $y = -8$

13. $x = 0$, $y = 0$

15. No solution

17. $x_1 = -\frac{1}{2}$, $x_2 = \frac{4}{5}$

19. $x = 0$, $y = 0$

21. $x = 2$, $y = -3$, $z = 3$

23. $x = \frac{1}{2}$, $y = -\frac{1}{3}$, $z = 1$

25. $x = 4 + 3t$, $y = 5 + 2t$, $z = t$

27. $x = \frac{3}{2} - 2t$, $y = 1 + 2t$, $z = t$

29. $x_1 = 1$, $x_2 = 4$, $x_3 = -3$, $x_4 = -2$