Section 1.1 Exercises

[08-10-28-ELA-11]

In Exercises 1–6, determine whether the given equation is linear in the variables x and y.

1. $2x - 3y = 4$
2. $3x - 4xy = 0$
3. $\frac{3}{y} + \frac{2}{x} - 1 = 0$
4. $x^2 + y^2 = 4$
5. $2 \sin x - y = 14$
6. $(\sin 2)x - y = 14$

In Exercises 7–10, find a parametric representation of the solution set of the given linear equation.

7. $2x - 4y = 0$
8. $3x - \frac{1}{2}y = 9$
9. $x + y + z = 1$
10. $13x_1 - 26x_2 + 39x_3 = 13$

In Exercises 11–16, use back-substitution to solve the given system.

11. $\begin{aligned} x_1 - x_2 &= 2 \\ x_2 &= 3 \end{aligned}$
12. $\begin{aligned} 2x_1 - 4x_2 &= 6 \\ 3x_2 &= 9 \end{aligned}$
13. $\begin{aligned} -x + y - z &= 0 \\ 2y + z &= 3 \\ \tfrac{1}{2}z &= 0 \end{aligned}$
14. $\begin{aligned} x - y &= 4 \\ 2y + z &= 6 \\ 3z &= 6 \end{aligned}$
15. $\begin{aligned} 5x_1 + 2x_2 + x_3 &= 0 \\ 2x_1 + x_2 &= 0 \end{aligned}$
16. $\begin{aligned} x_1 + x_2 + x_3 &= 0 \\ x_2 &= 0 \end{aligned}$

In Exercises 17–20, graph each system of equations as a pair of lines in the xy-plane. Solve each system and interpret your answer.

17. $\begin{aligned} 2x + y &= 4 \\ x - y &= 2 \end{aligned}$
18. $\begin{aligned} x + 3y &= 2 \\ -x + 2y &= 3 \end{aligned}$
19. $\begin{aligned} x - y &= 1 \\ -2x + 2y &= 5 \end{aligned}$
20. $\begin{aligned} \tfrac{1}{2}x - \tfrac{1}{3}y &= 1 \\ -2x + \tfrac{4}{3}y &= -4 \end{aligned}$

In Exercises 21–26, complete the following for each system of equations.

(a) Use a graphing utility to graph the equations in the system.
(b) Use the graphs to determine whether the system is consistent or inconsistent.
(c) If the system is consistent, approximate the solution.
(d) Solve the system algebraically.
(e) Compare the solution in part (d) with the approximation in part (c). What can you conclude?

21. $\begin{aligned} -3x - y &= 3 \\ 6x + 2y &= 1 \end{aligned}$
22. $\begin{aligned} 4x - 5y &= 3 \\ -8x + 10y &= 14 \end{aligned}$
23. $\begin{aligned} 2x - 8y &= 3 \\ \tfrac{1}{2}x + y &= 0 \end{aligned}$
24. $\begin{aligned} 9x - 4y &= 5 \\ \tfrac{1}{2}x + \tfrac{1}{3}y &= 0 \end{aligned}$
25. $\begin{aligned} 4x - 8y &= 9 \\ 0.8x - 1.6y &= 1.8 \end{aligned}$
26. $\begin{aligned} -5.3x + 2.1y &= 1.25 \\ 15.9x - 6.3y &= -3.75 \end{aligned}$

In Exercises 27–46, solve the given system of linear equations.

27. $\begin{aligned} x_1 - x_2 &= 0 \\ 3x_1 - 2x_2 &= -1 \end{aligned}$
28. $\begin{aligned} 3x + 2y &= 2 \\ 6x + 4y &= 14 \end{aligned}$
29. $\begin{aligned} 2u + v &= 120 \\ u + 2v &= 120 \end{aligned}$
30. $\begin{aligned} x_1 - 2x_2 &= 0 \\ 6x_1 + 2x_2 &= 0 \end{aligned}$
31. $\begin{aligned} 9x - 3y &= -1 \\ \tfrac{1}{5}x + \tfrac{2}{5}y &= -\tfrac{1}{3} \end{aligned}$
32. $\begin{aligned} \tfrac{2}{3}x_1 + \tfrac{1}{6}x_2 &= 0 \\ 4x_1 + x_2 &= 0 \end{aligned}$
33. $\begin{aligned} \frac{x-1}{2} + \frac{y+2}{3} &= 4 \\ x - 2y &= 5 \end{aligned}$
34. $\begin{aligned} \frac{x_1+3}{4} + \frac{x_2-1}{3} &= 1 \\ 2x_1 - x_2 &= 12 \end{aligned}$

35. $$\begin{aligned} 0.02x_1 - 0.05x_2 &= -0.19 \\ 0.03x_1 + 0.04x_2 &= 0.52 \end{aligned}$$

36. $$\begin{aligned} 0.05x_1 - 0.03x_2 &= 0.21 \\ 0.07x_1 + 0.02x_2 &= 0.17 \end{aligned}$$

37. $$\begin{aligned} x + y + z &= 6 \\ 2x - y + z &= 3 \\ 3x - z &= 0 \end{aligned}$$

38. $$\begin{aligned} x + y + z &= 2 \\ -x + 3y + 2z &= 8 \\ 4x + y &= 4 \end{aligned}$$

39. $$\begin{aligned} 3x_1 - 2x_2 + 4x_3 &= 1 \\ x_1 + x_2 - 2x_3 &= 3 \\ 2x_1 - 3x_2 + 6x_3 &= 8 \end{aligned}$$

40. $$\begin{aligned} 5x_1 - 3x_2 + 2x_3 &= 3 \\ 2x_1 + 4x_2 - x_3 &= 7 \\ x_1 - 11x_2 + 4x_3 &= 3 \end{aligned}$$

41. $$\begin{aligned} 2x_1 + x_2 - 3x_3 &= 4 \\ 4x_1 + 2x_3 &= 10 \\ -2x_1 + 3x_2 - 13x_3 &= -8 \end{aligned}$$

42. $$\begin{aligned} x_1 + 4x_3 &= 13 \\ 4x_1 - 2x_2 + x_3 &= 7 \\ 2x_1 - 2x_2 - 7x_3 &= -19 \end{aligned}$$

43. $$\begin{aligned} x - 3y + 2z &= 18 \\ 5x - 15y + 10z &= 18 \end{aligned}$$

44. $$\begin{aligned} x_1 - 2x_2 + 5x_3 &= 2 \\ 3x_1 + 2x_2 - x_3 &= -2 \end{aligned}$$

45. $$\begin{aligned} x + y + z + w &= 6 \\ 2x + 3y - w &= 0 \\ -3x + 4y + z + 2w &= 4 \\ x + 2y - z + w &= 0 \end{aligned}$$

46. $$\begin{aligned} x_1 + 3x_4 &= 4 \\ 2x_2 - x_3 - x_4 &= 0 \\ 3x_2 - 2x_4 &= 1 \\ 2x_1 - x_2 + 4x_3 &= 5 \end{aligned}$$

C In Exercises 47–50, use a computer or graphing calculator to solve the given system of linear equations.

47. $$\begin{aligned} x_1 + 0.5x_2 + 0.33x_3 + 0.25x_4 &= 1.1 \\ 0.5x_1 + 0.33x_2 + 0.25x_3 + 0.2x_4 &= 1.2 \\ 0.33x_1 + 0.25x_2 + 0.2x_3 + 0.17x_4 &= 1.3 \\ 0.25x_1 + 0.2x_2 + 0.17x_3 + 0.14x_4 &= 1.4 \end{aligned}$$

48. $$\begin{aligned} 123.5x + 61.3y - 32.4z &= 34.3 \\ 54.7x - 45.6y + 98.2z &= 45.8 \\ 42.4x - 89.3y + 12.9z &= 35.8 \end{aligned}$$

49. $$\begin{aligned} \tfrac{1}{2}x_1 - \tfrac{3}{7}x_2 + \tfrac{2}{9}x_3 &= 4 \\ \tfrac{2}{3}x_1 + \tfrac{4}{9}x_2 - \tfrac{2}{5}x_3 &= 7 \\ \tfrac{4}{5}x_1 - \tfrac{1}{8}x_2 + \tfrac{4}{3}x_3 &= -5 \end{aligned}$$

50. $$\begin{aligned} \tfrac{1}{8}x - \tfrac{1}{7}y + \tfrac{1}{6}z - \tfrac{1}{5}w &= 1 \\ \tfrac{1}{7}x + \tfrac{1}{6}y - \tfrac{1}{5}z + \tfrac{1}{4}w &= 1 \\ \tfrac{1}{6}x - \tfrac{1}{5}y + \tfrac{1}{4}z - \tfrac{1}{3}w &= 1 \\ \tfrac{1}{5}x + \tfrac{1}{4}y - \tfrac{1}{3}z + \tfrac{1}{2}w &= 1 \end{aligned}$$

In Exercises 51–54, state why each system of equations must have at least one solution. Then solve the system and determine if it has exactly one solution or an infinite number of solutions.

51. $$\begin{aligned} 4x + 3y + 17z &= 0 \\ 5x + 4y + 22z &= 0 \\ 4x + 2y + 19z &= 0 \end{aligned}$$

52. $$\begin{aligned} 2x + 3y &= 0 \\ 4x + 3y - z &= 0 \\ 8x + 3y + 3z &= 0 \end{aligned}$$

53. $$\begin{aligned} 5x + 5y - z &= 0 \\ 10x + 5y + 2z &= 0 \\ 5x + 15y - 9z &= 0 \end{aligned}$$

54. $$\begin{aligned} 12x + 5y + z &= 0 \\ 12x + 4y - z &= 0 \end{aligned}$$

True or False? In Exercises 55 and 56, determine whether the statement is true or false. If it is true, give a reason or cite an appropriate statement in the text. If it is false, provide an example that shows that the statement is not true in all cases or cite an appropriate statement in the text.

55. (a) A system of one linear equation in two variables is always consistent.

(b) A system of two linear equations in three unknowns is always consistent.

(c) If a linear system is consistent, then it has an infinite number of solutions.

56. (a) A system of linear equations can have exactly two solutions.

(b) Two systems of linear equations are equivalent if they have the same solution set.

(c) A system of three linear equations in two unknowns is always inconsistent.

57. Find a system of two equations in two unknowns, x_1 and x_2, that has the solution set given by the parametric representation $x_1 = t$ and $x_2 = 3t - 4$, where t is any real number. Then show that the solutions to your system can also be written as $x_1 = \frac{4}{3} + \frac{t}{3}$ and $x_2 = t$.

58. Find a system of two equations in three unknowns, x_1, x_2, and x_3, that has the solution set given by the parametric representation $x_1 = t$, $x_2 = s$, and $x_3 = 3 + s - t$, where s and t are any real numbers. Then show that the solutions to your system can also be written as $x_1 = 3 + s - t$, $x_2 = s$, and $x_3 = t$.

In Exercises 59 and 60, solve the given system of equations by letting $X = 1/x$, $Y = 1/y$, and $Z = 1/z$.

59. $$\begin{aligned} \frac{12}{x} - \frac{12}{y} &= 7 \\ \frac{3}{x} + \frac{4}{y} &= 0 \end{aligned}$$

60. $$\begin{aligned} \frac{2}{x} + \frac{1}{y} - \frac{3}{z} &= 4 \\ \frac{4}{x} \phantom{+ \frac{1}{y}} + \frac{2}{z} &= 10 \\ -\frac{2}{x} + \frac{3}{y} - \frac{13}{z} &= -8 \end{aligned}$$

In Exercises 61 and 62, solve the given system of linear equations for x and y.

61. $$\begin{aligned} (\cos\theta)x + (\sin\theta)y &= 1 \\ (-\sin\theta)x + (\cos\theta)y &= 0 \end{aligned}$$

62. $$\begin{aligned} (\cos\theta)x + (\sin\theta)y &= 1 \\ (-\sin\theta)x + (\cos\theta)y &= 1 \end{aligned}$$

In Exercises 63–68, determine the value(s) of k such that the given system of linear equations has the indicated number of solutions.

63. An infinite number of solutions

$$\begin{aligned} 4x + ky &= 6 \\ kx + y &= -3 \end{aligned}$$

64. An infinite number of solutions

$$\begin{aligned} kx + y &= 4 \\ 2x - 3y &= -12 \end{aligned}$$

65. Exactly one solution

$$\begin{aligned} x + ky &= 0 \\ kx + y &= 0 \end{aligned}$$

66. No solution

$$\begin{aligned} x + ky &= 2 \\ kx + y &= 4 \end{aligned}$$

67. No solution

$$\begin{aligned} x + 2y + kz &= 6 \\ 3x + 6y + 8z &= 4 \end{aligned}$$

68. Exactly one solution

$$\begin{aligned} kx + 2ky + 3kz &= 4k \\ x + y + z &= 0 \\ 2x - y + z &= 1 \end{aligned}$$

69. Determine the values of k such that the following system of linear equations does not have a unique solution.

$$\begin{aligned} x + y + kz &= 3 \\ x + ky + z &= 2 \\ kx + y + z &= 1 \end{aligned}$$

70. Find values of a, b, and c such that the following system of linear equations has (a) exactly one solution, (b) an infinite number of solutions, and (c) no solution.

$$\begin{aligned} x + 5y + z &= 0 \\ x + 6y - z &= 0 \\ 2x + ay + bz &= c \end{aligned}$$

W **71.** Consider the following system of linear equations in x and y.

$$\begin{aligned} a_1x + b_1y &= c_1 \\ a_2x + b_2y &= c_2 \\ a_3x + b_3y &= c_3 \end{aligned}$$

Describe the graphs of these three equations in the xy-plane when the system has (a) exactly one solution, (b) an infinite number of solutions, and (c) no solution.

W **72.** Explain why the system of linear equations in Exercise 71 must be consistent if the constant terms c_1, c_2, and c_3 are all zero.

73. Show that if $ax^2 + bx + c = 0$ for all x, then $a = b = c = 0$.

74. Consider the following system of linear equations in x and y.

$$\begin{aligned} ax + by &= e \\ cx + dy &= f \end{aligned}$$

Under what conditions will the system have exactly one solution?

In Exercises 75 and 76, graph the lines determined by the given system of linear equations. Then use Gaussian elimination to solve the system. At each step of the elimination process, graph the corresponding lines. What do you observe about these lines?

75.
$$\begin{aligned} x - 4y &= -3 \\ 5x - 6y &= 13 \end{aligned}$$

76.
$$\begin{aligned} 2x - 3y &= 7 \\ -4x + 6y &= -14 \end{aligned}$$

SECTION 1.1 (*page 11*)

1. Linear **3.** Not linear **5.** Not linear

7. $x = 2t$
$y = t$

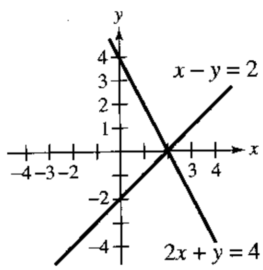
9. $x = 1 - s - t$
$y = s$
$z = t$

11. $x_1 = 5$
$x_2 = 3$

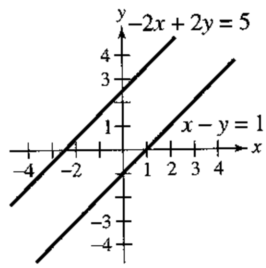
13. $x = \frac{3}{2}$
$y = \frac{3}{2}$
$z = 0$

15. $x_1 = -t$
$x_2 = 2t$
$x_3 = t$

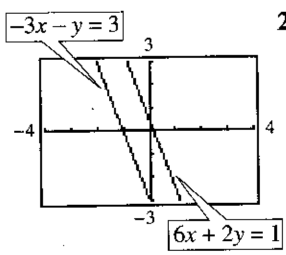
17. $x = 2$
$y = 0$



19. No solution

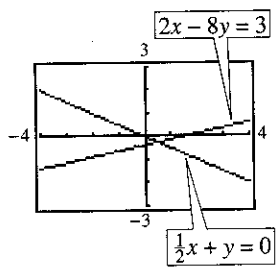


21.



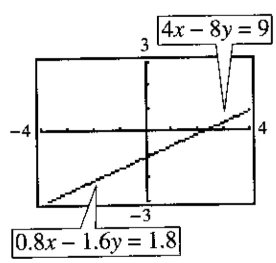
Inconsistent

23.



$x = \frac{1}{2}$
$y = -\frac{1}{4}$

25.



$x = \frac{9}{4} + 2t$
$y = t$

27. $x_1 = -1$
$x_2 = -1$

29. $u = 40$
$v = 40$

31. $x = -\frac{1}{3}$
$y = -\frac{2}{3}$

33. $x = 7$
$y = 1$

35. $x_1 = 8$
$x_2 = 7$

37. $x = 1$
$y = 2$
$z = 3$

39. No solution

41. $x_1 = \frac{5}{2} - \frac{1}{2}t$
$x_2 = 4t - 1$
$x_3 = t$

43. No solutio

45. $x = 1$
$y = 0$
$z = 3$
$w = 2$

47. $x_1 = 11.2415$
$x_2 = -60.9029$
$x_3 = 40.7674$
$x_4 = 27.4267$

49. $x_1 = 8.1124$
$x_2 = -4.5588$
$x_3 = -9.0448$

51. $x = 0$
$y = 0$
$z = 0$

53. $x = -\frac{3}{5}t$
$y = \frac{4}{5}t$
$z = t$

55. (a) True
(b) False
(c) False

57. $3x_1 - x_2 = 4$
$-3x_1 + x_2 = -4$
(The answer is not unique.)

59. $x = 3$
$y = -4$

61. $x = \cos\theta$
$y = \sin\theta$

63. $k = -2$

65. All $k \neq \pm 1$

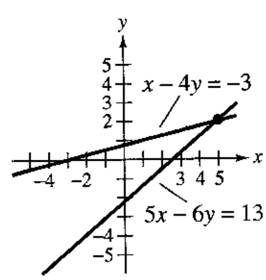
67. $k = \frac{8}{3}$

69. $k = 1, -2$

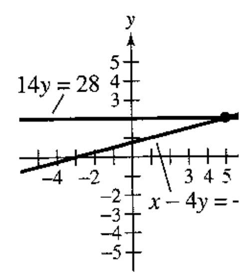
71. (a) Three lines intersecting at one point
(b) Three coincident lines
(c) Three lines having no common point

73. Answers will vary. (*Hint:* Choose three different val of x and solve the resulting system of linear equation the variables a, b, and c.)

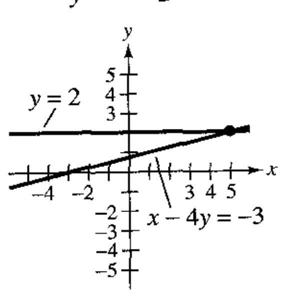
75. $x - 4y = -3$
$5x - 6y = 13$



$x - 4y = -3$
$14y = 28$



$x - 4y = -3$
$y = 2$



$x = 5$
$y = 2$