b **A** is 1×3 and **C** is 3×2 $\therefore$ **AC** is 1×2

$$\mathbf{AC} = \begin{bmatrix} 1 & 3 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 3 \\ 1 & 4 \end{bmatrix} = [1 \times 1 + 3 \times 2 + 5 \times 1 \quad 1 \times 0 + 3 \times 3 + 5 \times 4]$$
$$= [12 \ 29]$$

EXERCISE 14E.2

1 Explain why **AB** cannot be found for $\mathbf{A} = \begin{bmatrix} 4 & 2 & 1 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$.

2 If **A** is $2 \times n$ and **B** is $m \times 3$:

a When can we find **AB**? **b** If **AB** can be found, what is its order?

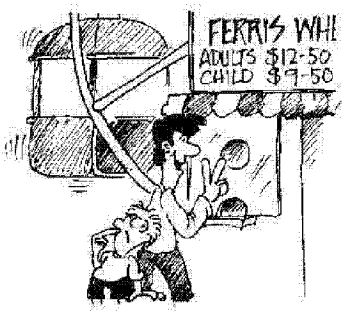
c Why can **BA** never be found?

3 **a** For $\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 5 & 6 \end{bmatrix}$, find **BA**.

b For $\mathbf{A} = \begin{bmatrix} 2 & 0 & 3 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$ find **i** **AB** **ii** **BA**.

4 Find: **a** $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ **b** $\begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$

5



At the Fair, tickets for the Ferris Wheel are \$12.50 per adult and \$9.50 per child. On the first day of the Fair, 2375 adults and 5156 children ride this wheel. On the second day the figures are 2502 adults and 3612 children.

a Write the costs matrix **C** as a 2×1 matrix and the numbers matrix **N** as a 2×2 matrix.

b Find **NC** and interpret the resulting matrix.

c Find the total income for the two days.

6 You and your friend each go to your local hardware stores A and B to price items you wish to purchase. You want to buy 1 hammer, 1 screwdriver and 2 cans of white paint and your friend wants 1 hammer, 2 screwdrivers and 3 cans of white paint. The prices of these goods are:

	Hammer	*Screwdriver*	*Can of paint*
Store A	\$7	\$3	\$19
Store B	\$6	\$2	\$22

a Write the requirements matrix **R** as a 3×2 matrix.

b Write the prices matrix **P** as a 2×3 matrix.

c Find **PR**.

d What are your costs at store A and your friend's costs at store B?

e Should you buy from store A or store B?

EXERCISE 14D

1 **a** $3A$ **b** O **c** $-C$ **d** O **e** $2A + 2B$
f $-A - B$ **g** $-2A + C$ **h** $4A - B$ **i** $3B$

2 **a** $X = A - B$ **b** $X = C - B$ **c** $X = 2C - 4B$
d $X = \frac{1}{2}A$ **e** $X = \frac{1}{3}B$ **f** $X = A - B$
g $X = 2C$ **h** $X = \frac{1}{2}B - A$ **i** $X = \frac{1}{4}(A - C)$

3 **a** $X = \begin{bmatrix} 3 & 6 \\ 9 & 18 \end{bmatrix}$ **b** $X = \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ \frac{3}{4} & \frac{5}{4} \end{bmatrix}$ **c** $X = \begin{bmatrix} -1 & -6 \\ 1 & -\frac{1}{2} \end{bmatrix}$

EXERCISE 14E.1

1 **a** $[11]$ **b** $[22]$ **c** $[16]$ **2** $\begin{bmatrix} w & x & y & z \end{bmatrix} \begin{bmatrix} \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \end{bmatrix}$

3 **a** $P = \begin{bmatrix} 27 & 35 & 39 \end{bmatrix}$ $Q = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$

b total cost $= \begin{bmatrix} 27 & 35 & 39 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix} = \$291$

4 **a** $P = \begin{bmatrix} 10 & 6 & 3 & 1 \end{bmatrix}$ $N = \begin{bmatrix} 3 \\ 2 \\ 4 \\ 2 \end{bmatrix}$

b total points $= \begin{bmatrix} 10 & 6 & 3 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 4 \\ 2 \end{bmatrix} = 56$ points

EXERCISE 14E.2

1 Number of cols. in A does not equal no. of rows in B.

2 **a** $m = n$ **b** 2×3 **c** B has 3 columns, A has 2 rows

3 **a** $\begin{bmatrix} 28 & 29 \end{bmatrix}$ **b** **i** $\begin{bmatrix} 8 \end{bmatrix}$ **ii** $\begin{bmatrix} 2 & 0 & 3 \\ 8 & 0 & 12 \\ 4 & 0 & 6 \end{bmatrix}$

4 **a** $\begin{bmatrix} 3 & 5 & 3 \end{bmatrix}$ **b** $\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

5 **a** $C = \begin{bmatrix} 12.5 \\ 9.5 \end{bmatrix}$ $N = \begin{bmatrix} 2375 & 5156 \\ 2502 & 3612 \end{bmatrix}$

b $\begin{bmatrix} 78\,669.5 \\ 65\,589 \end{bmatrix}$ income from adult rides and children's rides **c** \$144 258.50

6 **a** $R = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 2 & 3 \end{bmatrix}$ **b** $P = \begin{bmatrix} 7 & 3 & 19 \\ 6 & 2 & 22 \end{bmatrix}$ **c** $\begin{bmatrix} 48 & 70 \\ 52 & 76 \end{bmatrix}$

d My costs at store A are \$48, my friend's costs at store B are \$76. **e** store A

EXERCISE 14F

1 **a** $\begin{bmatrix} 16 & 18 & 15 \\ 13 & 21 & 16 \\ 10 & 22 & 24 \end{bmatrix}$ **b** $\begin{bmatrix} 10 & 6 & -7 \\ 9 & 3 & 0 \\ 4 & -4 & -10 \end{bmatrix}$

c $\begin{bmatrix} 22 & 0 & 132 & 176 & 198 \\ 44 & 154 & 88 & 110 & 0 \\ 176 & 44 & 88 & 88 & 132 \end{bmatrix}$ **d** $\begin{bmatrix} 115 \\ 136 \\ 46 \\ 106 \end{bmatrix}$

2 **a** $\begin{bmatrix} 3 & 3 & 2 \end{bmatrix}$ **b** $\begin{bmatrix} 125 & 150 & 140 \\ 44 & 40 & 40 \\ 75 & 80 & 65 \end{bmatrix}$ **c** $\begin{bmatrix} 657 & 730 & 670 \end{bmatrix}$

d $\begin{bmatrix} 369 & 420 & 385 \end{bmatrix}$ **e** $\begin{bmatrix} 657 & 730 & 670 \\ 369 & 420 & 385 \end{bmatrix}$

3 \$224 660

4 **a** $\begin{bmatrix} 125 & 195 & 225 \end{bmatrix} \times \begin{bmatrix} 15 & 12 & 13 & 11 & 14 & 16 & 8 \\ 4 & 3 & 6 & 2 & 0 & 4 & 7 \\ 3 & 1 & 4 & 4 & 3 & 2 & 0 \end{bmatrix}$

$- \begin{bmatrix} 85 & 120 & 130 \end{bmatrix} \times \begin{bmatrix} 15 & 12 & 13 & 11 & 14 & 16 & 8 \\ 4 & 3 & 6 & 2 & 0 & 4 & 7 \\ 3 & 1 & 4 & 4 & 3 & 2 & 0 \end{bmatrix}$

$= \$7125$

b $\begin{bmatrix} 125 & 195 & 225 \end{bmatrix} \times \begin{bmatrix} 15 & 12 & 13 & 11 & 14 & 16 & 8 \\ 4 & 3 & 6 & 2 & 0 & 4 & 7 \\ 3 & 1 & 4 & 4 & 3 & 2 & 0 \end{bmatrix}$

$- \begin{bmatrix} 85 & 120 & 130 \end{bmatrix} \times \begin{bmatrix} 20 & 20 & 20 & 20 & 20 & 20 & 20 \\ 15 & 15 & 15 & 15 & 15 & 15 & 15 \\ 5 & 5 & 5 & 5 & 5 & 5 & 5 \end{bmatrix}$

$= -\$9030$, i.e., a loss of \$9030

c $(\begin{bmatrix} 125 & 195 & 225 \end{bmatrix} - \begin{bmatrix} 85 & 120 & 130 \end{bmatrix}) \times \begin{bmatrix} 15 & 12 & 13 & 11 & 14 & 16 & 8 \\ 4 & 3 & 6 & 2 & 0 & 4 & 7 \\ 3 & 1 & 4 & 4 & 3 & 2 & 0 \end{bmatrix}$

EXERCISE 14G

1 $AB = \begin{bmatrix} -1 & 1 \\ -1 & 7 \end{bmatrix}$ $BA = \begin{bmatrix} 0 & 2 \\ 3 & 6 \end{bmatrix}$ $AB \neq BA$

2 $AO = OA = O$ **4** **b** $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

5 **a** $\begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$ **b** $\begin{bmatrix} 97 & -59 \\ 118 & 38 \end{bmatrix}$

6 **a** A^2 does not exist **b** when A is a square matrix

8 **a** $A^2 + A$ **b** $B^2 + 2B$ **c** $A^3 - 2A^2 + A$
d $A^3 + A^2 - 2A$ **e** $AC + AD + BC + BD$
f $A^2 + AB + BA + B^2$ **g** $A^2 - AB + BA - B^2$
h $A^2 + 2A + I$ **i** $9I - 6B + B^2$

9 **a** $A^3 = 3A - 2I$ $A^4 = 4A - 3I$
b $B^3 = 3B - 2I$ $B^4 = 6I - 5B$ $B^5 = 11B - 10I$
c $C^3 = 13C - 12I$ $C^5 = 121C - 120I$

10 **a** **i** $I + 2A$ **ii** $2I - 2A$ **iii** $10A + 6I$
b $A^2 + A + 2I$ **c** **i** $-3A$ **ii** $-2A$ **iii** A

11 **a** $AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ **b** $A^2 = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

c false as $A(A - I) = O$ does not imply that $A = O$ or $A - I = O$

d $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} a & b \\ \frac{a - a^2}{b} & 1 - a \end{bmatrix}, b \neq 0$

12 For example, $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, gives $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

13 **a** $a = 3, b = -4$ **b** $a = 1, b = 8$

14 $p = -2, q = 1$ **a** $A^3 = 5A - 2I$ **b** $A^4 = -12A + 5I$

EXERCISE 14H

1 **a** $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = 3I$, $\begin{bmatrix} 1 & -2 \\ -\frac{2}{3} & \frac{5}{3} \end{bmatrix}$

b $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} = 10I$, $\begin{bmatrix} 0.2 & 0.4 \\ -0.1 & 0.3 \end{bmatrix}$

c $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = 2I$, $\begin{bmatrix} -\frac{11}{2} & \frac{9}{2} & \frac{15}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 4 & -3 & -5 \end{bmatrix}$