[08-09-30-HH-11]

Example 3

If $\mathbf{A} = \begin{bmatrix} 3 & 4 & 8 \\ 2 & 1 & 0 \\ 1 & 4 & 7 \end{bmatrix}$

and $\mathbf{B} = \begin{bmatrix} 2 & 0 & 6 \\ 3 & 0 & 4 \\ 5 & 2 & 3 \end{bmatrix}$

find $\mathbf{A} - \mathbf{B}$.

$$\mathbf{A} - \mathbf{B} = \begin{bmatrix} 3 & 4 & 8 \\ 2 & 1 & 0 \\ 1 & 4 & 7 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 6 \\ 3 & 0 & 4 \\ 5 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3-2 & 4-0 & 8-6 \\ 2-3 & 1-0 & 0-4 \\ 1-5 & 4-2 & 7-3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 4 & 2 \\ -1 & 1 & -4 \\ -4 & 2 & 4 \end{bmatrix}$$

EXERCISE 14B

1 If $\mathbf{A} = \begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$ and $\mathbf{C} = \begin{bmatrix} -3 & 7 \\ -4 & -2 \end{bmatrix}$, find:

a $\mathbf{A} + \mathbf{B}$ **b** $\mathbf{A} + \mathbf{B} + \mathbf{C}$ **c** $\mathbf{B} + \mathbf{C}$ **d** $\mathbf{C} + \mathbf{B} - \mathbf{A}$

2 If $\mathbf{P} = \begin{bmatrix} 3 & 5 & -11 \\ 10 & 2 & 6 \\ -2 & -1 & 7 \end{bmatrix}$ and $\mathbf{Q} = \begin{bmatrix} 17 & -4 & 3 \\ -2 & 8 & -8 \\ 3 & -4 & 11 \end{bmatrix}$, find: **a** $\mathbf{P} + \mathbf{Q}$ **b** $\mathbf{P} - \mathbf{Q}$ **c** $\mathbf{Q} - \mathbf{P}$

3 A restaurant served 85 men, 92 women and 52 children on Friday night. On Saturday night they served 102 men, 137 women and 49 children.

a Express this information in *two* column matrices.

b Use the matrices to find the totals of men, women and children served over the Friday-Saturday period.

4 On Monday David bought shares in five companies and on Friday he sold them. The details are:

	Cost price/share	*Selling price/share*
A	\$1.72	\$1.79
B	\$27.85	\$28.75
C	\$0.92	\$1.33
D	\$2.53	\$2.25
E	\$3.56	\$3.51

a Write down David's

i cost price column matrix

ii selling price column matrix.

b What matrix operation is needed to find David's profit/loss matrix?

c Find David's profit/loss matrix.

5 In November, Lou E Gee sold 23 fridges, 17 stoves and 31 microwave ovens and his partner Rose A Lee sold 19 fridges, 29 stoves and 24 microwave ovens.

In December Lou's sales were: 18 fridges, 7 stoves and 36 microwaves while Rose's sales were: 25 fridges, 13 stoves and 19 microwaves.

a Write their sales for November as a 3×2 matrix.

b Write their sales for December as a 3×2 matrix.

c Write their total sales for November and December as a 3×2 matrix.

6 Find x and y if: **a** $\begin{bmatrix} x & x^2 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} y & 4 \\ 3 & y+1 \end{bmatrix}$ **b** $\begin{bmatrix} x & y \\ y & x \end{bmatrix} = \begin{bmatrix} -y & x \\ x & -y \end{bmatrix}$

7 **a** If $\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix}$ find $\mathbf{A}+\mathbf{B}$ and $\mathbf{B}+\mathbf{A}$.

b Explain why $\mathbf{A}+\mathbf{B} = \mathbf{B}+\mathbf{A}$ for all 2×2 matrices **A** and **B**.

8 **a** For $\mathbf{A} = \begin{bmatrix} -1 & 0 \\ 1 & 5 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 3 & 4 \\ -1 & -2 \end{bmatrix}$ and $\mathbf{C} = \begin{bmatrix} 4 & -1 \\ -1 & 3 \end{bmatrix}$ find $(\mathbf{A}+\mathbf{B})+\mathbf{C}$ and $\mathbf{A}+(\mathbf{B}+\mathbf{C})$.

b Prove that, if **A**, **B** and **C** are any 2×2 matrices then $(\mathbf{A}+\mathbf{B})+\mathbf{C} = \mathbf{A}+(\mathbf{B}+\mathbf{C})$.

(**Hint:** Let $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$ and $\mathbf{C} = \begin{bmatrix} w & x \\ y & z \end{bmatrix}$, say.)

C MULTIPLES OF MATRICES

In the pantry there are 6 cans of peaches, 4 cans of apricots and 8 cans of pears.

This information could be represented by the column vector $\mathbf{C} = \begin{bmatrix} 6 \\ 4 \\ 8 \end{bmatrix}$.

Doubling these cans in the pantry we would have $\begin{bmatrix} 12 \\ 8 \\ 16 \end{bmatrix}$ which is $\mathbf{C}+\mathbf{C}$.

Now if we let $\mathbf{C}+\mathbf{C}$ be $2\mathbf{C}$ we notice that to get $2\mathbf{C}$ from **C** we simply multiply all matrix elements by 2.

Likewise, trebling the fruit cans in the pantry is $3\mathbf{C} = \begin{bmatrix} 3\times 6 \\ 3\times 4 \\ 3\times 8 \end{bmatrix} = \begin{bmatrix} 18 \\ 12 \\ 24 \end{bmatrix}$ and halving them is $\frac{1}{2}\mathbf{C} = \begin{bmatrix} \frac{1}{2}\times 6 \\ \frac{1}{2}\times 4 \\ \frac{1}{2}\times 8 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix}$

In general,

> if a scalar t is multiplied by a matrix **A** the result is matrix $t\mathbf{A}$ obtained by multiplying every element of **A** by t.

Example 4

If **A** is $\begin{bmatrix} 1 & 2 & 5 \\ 2 & 0 & 1 \end{bmatrix}$

find **a** $3\mathbf{A}$ **b** $\frac{1}{2}\mathbf{A}$

a $3\mathbf{A} = 3\begin{bmatrix} 1 & 2 & 5 \\ 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 15 \\ 6 & 0 & 3 \end{bmatrix}$

b $\frac{1}{2}\mathbf{A} = \frac{1}{2}\begin{bmatrix} 1 & 2 & 5 \\ 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 1 & 2\frac{1}{2} \\ 1 & 0 & \frac{1}{2} \end{bmatrix}$

EXERCISE 14C

1 If $\mathbf{B} = \begin{bmatrix} 6 & 12 \\ 24 & 6 \end{bmatrix}$ find: **a** $2\mathbf{B}$ **b** $\frac{1}{3}\mathbf{B}$ **c** $\frac{1}{12}\mathbf{B}$ **d** $-\frac{1}{2}\mathbf{B}$

2 If $\mathbf{A} = \begin{bmatrix} 2 & 3 & 5 \\ 1 & 6 & 4 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ find:

a $\mathbf{A} + \mathbf{B}$ **b** $\mathbf{A} - \mathbf{B}$ **c** $2\mathbf{A} + \mathbf{B}$ **d** $3\mathbf{A} - \mathbf{B}$

3 Isabelle sells clothing made by four different companies which we will call A, B, C and D. Her usual monthly order is:

$$\begin{array}{c c} & \begin{array}{cccc} \mathrm{A} & \mathrm{B} & \mathrm{C} & \mathrm{D} \end{array} \\ \begin{array}{c} \text{skirt} \\ \text{dress} \\ \text{evening} \\ \text{suit} \end{array} & \begin{bmatrix} 30 & 40 & 40 & 60 \\ 50 & 40 & 30 & 75 \\ 40 & 40 & 50 & 50 \\ 10 & 20 & 20 & 15 \end{bmatrix} \end{array}$$

Find her order, to the nearest whole number, if:

a she increases her total order by 15%
b she decreases her total order by 15%.

4 During weekdays a video store finds that its average hirings are: 75 movies (VHS), 27 movies (DVD) and 102 video/computer games. On the weekends the average figures are: 43 DVD movies, 136 VHS movies and 129 games.

a Represent the data using *two* column matrices.
b Find the sum of the matrices in **a**.
c What does the sum matrix of **b** represent?

$$\begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} \begin{array}{l} \leftarrow \text{VHS} \\ \leftarrow \text{DVD} \\ \leftarrow \text{games} \end{array}$$

5 A builder builds a block of 12 identical flats. Each flat is to contain 1 table, 4 chairs, 2 beds and 1 wardrobe.

If $\mathbf{F} = \begin{bmatrix} 1 \\ 4 \\ 2 \\ 1 \end{bmatrix}$ is the matrix representing the furniture in one flat, what, in terms of $\mathbf{F}$, is the matrix representing the furniture in **all** flats?

ZERO MATRIX (SOMETIMES CALLED NULL MATRIX)

For real numbers, it is true that $a + 0 = 0 + a = a$ for all values of a.

The question: "Is there a matrix $\mathbf{O}$ in which $\mathbf{A} + \mathbf{O} = \mathbf{O} + \mathbf{A} = \mathbf{A}$ for any matrix $\mathbf{A}$?"

Simple examples like: $\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ suggest that $\mathbf{O}$ consists of all zeros.

A **zero matrix** is a matrix in which all elements are zero.

For example, the 2×2 zero matrix is $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, the 2×3 zero matrix is $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

Zero matrices have the property that:

If $\mathbf{A}$ is a matrix of any order and $\mathbf{O}$ is the corresponding **zero matrix**, then $\mathbf{A} + \mathbf{O} = \mathbf{O} + \mathbf{A} = \mathbf{A}$.

Notice that the rules for addition (and subtraction) of matrices are identical to those of real numbers but we must be careful with scalar multiplication in matrix equations.

EXERCISE 14D

1 Simplify:

a $\mathbf{A}+2\mathbf{A}$	**b** $3\mathbf{B}-3\mathbf{B}$	**c** $\mathbf{C}-2\mathbf{C}$
d $-\mathbf{B}+\mathbf{B}$	**e** $2(\mathbf{A}+\mathbf{B})$	**f** $-(\mathbf{A}+\mathbf{B})$
g $-(2\mathbf{A}-\mathbf{C})$	**h** $3\mathbf{A}-(\mathbf{B}-\mathbf{A})$	**i** $\mathbf{A}+2\mathbf{B}-(\mathbf{A}-\mathbf{B})$

2 Find $\mathbf{X}$ in terms of $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ if:

a $\mathbf{X}+\mathbf{B}=\mathbf{A}$	**b** $\mathbf{B}+\mathbf{X}=\mathbf{C}$	**c** $4\mathbf{B}+\mathbf{X}=2\mathbf{C}$
d $2\mathbf{X}=\mathbf{A}$	**e** $3\mathbf{X}=\mathbf{B}$	**f** $\mathbf{A}-\mathbf{X}=\mathbf{B}$
g $\frac{1}{2}\mathbf{X}=\mathbf{C}$	**h** $2(\mathbf{X}+\mathbf{A})=\mathbf{B}$	**i** $\mathbf{A}-4\mathbf{X}=\mathbf{C}$

3 **a** If $\mathbf{M}=\begin{bmatrix}1 & 2\\ 3 & 6\end{bmatrix}$, find $\mathbf{X}$ if $\frac{1}{3}\mathbf{X}=\mathbf{M}$.

b If $\mathbf{N}=\begin{bmatrix}2 & -1\\ 3 & 5\end{bmatrix}$, find $\mathbf{X}$ if $4\mathbf{X}=\mathbf{N}$.

c If $\mathbf{A}=\begin{bmatrix}1 & 0\\ -1 & 2\end{bmatrix}$, and $\mathbf{B}=\begin{bmatrix}1 & 4\\ -1 & 1\end{bmatrix}$, find $\mathbf{X}$ if $\mathbf{A}-2\mathbf{X}=3\mathbf{B}$.

E MATRIX MULTIPLICATION

Suppose you go to a shop and purchase 3 soft drink cans, 4 chocolate bars and 2 icecreams

and the prices are

soft drink cans	chocolate bars	ice creams
\$1.30	\$0.90	\$1.20

Each of these can be represented using matrices,

i.e., $\mathbf{A}=\begin{bmatrix}3\\4\\2\end{bmatrix}$ and $\mathbf{B}=\begin{bmatrix}1.30 & 0.90 & 1.20\end{bmatrix}$.

To work out the total cost, the following *product* could be found:

$$\begin{aligned}\mathbf{BA} &= \begin{bmatrix}1.30 & 0.90 & 1.20\end{bmatrix}\begin{bmatrix}3\\4\\2\end{bmatrix}\\ &= (1.30\times 3)+(0.9\times 4)+(1.20\times 2)\\ &= 3.90+3.60+2.40\\ &= 9.90\end{aligned}$$

Thus the total cost is \$9.90.

Notice that we write the **row matrix** first and the **column matrix** second

and that $\begin{bmatrix}a & b & c\end{bmatrix}\begin{bmatrix}p\\q\\r\end{bmatrix}=ap+bq+cr.$

EXERCISE 14E.1

1 Determine:

a $\begin{bmatrix} 3 & -1 \end{bmatrix}\begin{bmatrix} 5 \\ 4 \end{bmatrix}$ **b** $\begin{bmatrix} 1 & 3 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ 1 \\ 7 \end{bmatrix}$ **c** $\begin{bmatrix} 6 & -1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -1 \\ 4 \end{bmatrix}$

2 Show that the sum of w, x, y and z is given by $\begin{bmatrix} w & x & y & z \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$.
Represent the average of w, x, y and z in the same way.

3 Lucy buys 4 shirts, 3 skirts and 2 blouses costing \$27, \$35 and \$39 respectively.
- **a** Write down a quantities matrix **Q** and a price matrix **P**.
- **b** Show how to use **P** and **Q** to determine the total cost.

4 In the interschool public speaking competition a first place is awarded 10 points, second place 6 points, third place 3 points and fourth place 1 point. One school won 3 first places, 2 seconds, 4 thirds and 2 fourths.
- **a** Write down this information in terms of points matrix **P**, and numbers matrix **N**.
- **b** Show how to use **P** and **N** to find the total number of points awarded to the school.

Now consider more complicated matrix multiplication.

In **Example 1**, Lisa needed $\begin{array}{c}\begin{bmatrix} 2 & 3 & 1 \end{bmatrix} \\ \text{bread} \quad \text{milk} \quad \text{butter}\end{array}$ and at store A, the costs matrix was $\begin{bmatrix} 2.65 \\ 1.55 \\ 2.35 \end{bmatrix}$.

To find the *total cost* Lisa needs to multiply the number of items by their respective cost,

i.e., $2 \times \$2.65 + 3 \times \$1.55 + 1 \times \$2.35 = \$12.30$

As the quantities do not change, her total cost in Store B is

$$2 \times \$2.25 + 3 \times \$1.50 + 1 \times \$2.20 = \$11.20$$

To do this using matrices notice that:

$$\begin{bmatrix} 2 & 3 & 1 \end{bmatrix} \times \begin{bmatrix} 2.65 & 2.25 \\ 1.55 & 1.50 \\ 2.35 & 2.20 \end{bmatrix} = \begin{bmatrix} 12.30 & 11.20 \end{bmatrix}$$

orders: 1×3 $\quad 3 \times 2$ $\quad 1 \times 2$

the same

resultant matrix

Now suppose Lisa's friend Olu needs 1 bread, 2 milk and 2 butter.

The quantities matrix for both Lisa and Olu would be $\begin{array}{cl}\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} & \begin{array}{l}\leftarrow \text{Lisa} \\ \leftarrow \text{Olu}\end{array} \\ \text{bread} \quad \text{milk} \quad \text{butter} & \end{array}$

Lisa's *total cost* at Store A is \$12.30 and at store B is \$11.20

Olu's *total cost* at Store A is $1 \times \$2.65 + 2 \times \$1.55 + 2 \times \$2.35 = \$10.45$

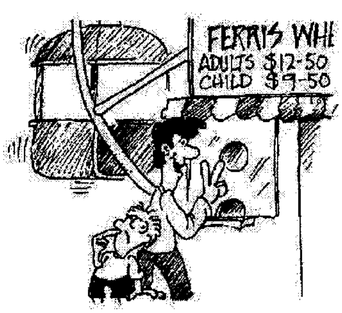
Store B is $1 \times \$2.25 + 2 \times \$1.50 + 2 \times \$2.20 = \$9.65$

b $\mathbf{A}$ is 1×3 and $\mathbf{C}$ is 3×2 $\quad \therefore$ $\mathbf{AC}$ is 1×2

$$\mathbf{AC} = \begin{bmatrix} 1 & 3 & 5 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & 3 \\ 1 & 4 \end{bmatrix} = [1 \times 1 + 3 \times 2 + 5 \times 1 \quad 1 \times 0 + 3 \times 3 + 5 \times 4]$$
$$= [12 \ 29]$$

EXERCISE 14E.2

1. Explain why $\mathbf{AB}$ cannot be found for $\mathbf{A} = \begin{bmatrix} 4 & 2 & 1 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$.
2. If $\mathbf{A}$ is $2 \times n$ and $\mathbf{B}$ is $m \times 3$:
   - **a** When can we find $\mathbf{AB}$?
   - **b** If $\mathbf{AB}$ can be found, what is its order?
   - **c** Why can $\mathbf{BA}$ never be found?
3. **a** For $\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 5 & 6 \end{bmatrix}$, find $\mathbf{BA}$.

   b For $\mathbf{A} = \begin{bmatrix} 2 & 0 & 3 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$ find **i** $\mathbf{AB}$ **ii** $\mathbf{BA}$.
4. Find: **a** $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ **b** $\begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$
5. 

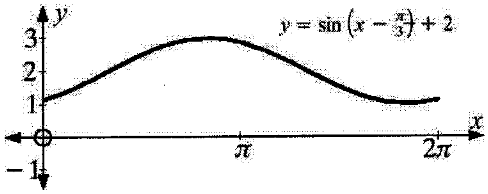
   At the Fair, tickets for the Ferris Wheel are $12.50 per adult and $9.50 per child. On the first day of the Fair, 2375 adults and 5156 children ride this wheel. On the second day the figures are 2502 adults and 3612 children.
   - **a** Write the costs matrix $\mathbf{C}$ as a 2×1 matrix and the numbers matrix $\mathbf{N}$ as a 2×2 matrix.
   - **b** Find $\mathbf{NC}$ and interpret the resulting matrix.
   - **c** Find the total income for the two days.
6. You and your friend each go to your local hardware stores A and B to price items you wish to purchase. You want to buy 1 hammer, 1 screwdriver and 2 cans of white paint and your friend wants 1 hammer, 2 screwdrivers and 3 cans of white paint. The prices of these goods are:

	Hammer	*Screwdriver*	*Can of paint*
Store A	$7	$3	$19
Store B	$6	$2	$22

   - **a** Write the requirements matrix $\mathbf{R}$ as a 3×2 matrix.
   - **b** Write the prices matrix $\mathbf{P}$ as a 2×3 matrix.
   - **c** Find $\mathbf{PR}$.
   - **d** What are your costs at store A and your friend's costs at store B?
   - **e** Should you buy from store A or store B?

3 **a** 6π **b** $\frac{\pi}{2}$

4



5 $T \doteqdot 7.05 \sin \frac{\pi}{6}(t - 4.5) + 24.75$

6 **a** $x \doteqdot 0.392, 2.75, 6.675$ **b** $x \doteqdot 7.235$

7 **a** $x \doteqdot 3.25, 4.69$ **b** $x \doteqdot 1.445, 5.89, 7.73$

8 **a** $x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$ **b** $x = \frac{-7\pi}{4}, \frac{-5\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$

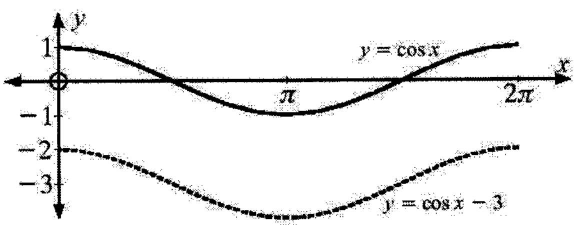
9 **a** $x = \frac{4\pi}{9}, \frac{5\pi}{9}, \frac{10\pi}{9}, \frac{11\pi}{9}, \frac{16\pi}{9}, \frac{17\pi}{9}$ **b** $x = \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}$

10 **a** 5000 **b** 3000, 7000
c $0.5 < t < 2.5$ and $6.5 < t < 8$

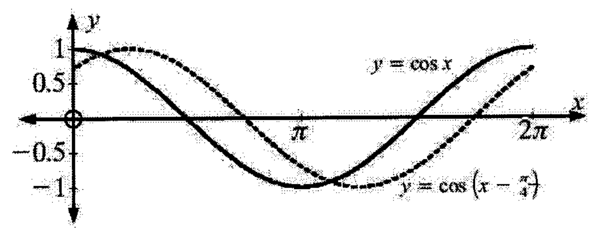
REVIEW SET 13B

1 **a** $x = \frac{3\pi}{2} + k2\pi$ **b** $x = \left.\begin{matrix}\frac{\pi}{6}\\ \frac{5\pi}{6}\end{matrix}\right\} + k\pi$

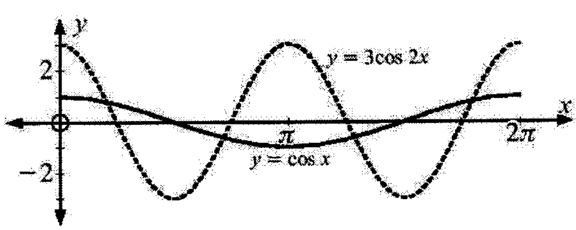
2 **a**



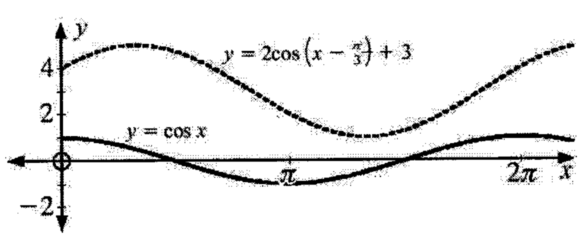
b



c



d



3 **a** 28 milligrams per m^3 **b** 8.00 am Monday

4 **a** $y = -4\cos 2x$ **b** $y = \cos \frac{\pi}{4}x + 2$

5 **a** $x \doteqdot 1.12, 5.17, 7.40$ **b** $x \doteqdot 0.184, 4.616$

6 **a** $x = \left.\begin{matrix}0.317\\ 1.254\end{matrix}\right\} + k\frac{\pi}{2}$ **b** $x \doteqdot 0.912, 2.23, 4.05$

7 **a** $x = \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{11\pi}{4}, \frac{13\pi}{4}$ **b** $x = -\pi, -\frac{\pi}{3}, \pi, \frac{5\pi}{3}$

8 **a** $x = 0, \frac{3\pi}{2}, 2\pi, \frac{7\pi}{2}, 4\pi$ **b** $x = \left.\begin{matrix}\frac{\pi}{6}\\ \frac{5\pi}{6}\end{matrix}\right\} + k\pi$

9 **a** $\cos\theta$ **b** $-\sin\theta$ **c** $2\cos\theta$ **d** $5\cos^2\theta$ **e** $-\cos\theta$

10 **a** $4\sin^2\alpha - 4\sin\alpha + 1$ **b** $1 - \sin 2\alpha$

EXERCISE 14A

1 **a** 1×4 **b** 2×1 **c** 2×2 **d** 3×3

2 **a** $\begin{bmatrix} 2 & 1 & 6 & 1 \end{bmatrix}$ **b** $\begin{bmatrix} 1.95 \\ 2.35 \\ 0.15 \\ 0.95 \end{bmatrix}$ **c** total cost of groceries

3 $\begin{bmatrix} 1000 & 1500 & 1250 \\ 1500 & 1000 & 1000 \\ 800 & 2300 & 1300 \\ 1200 & 1200 & 1200 \end{bmatrix}$ **4** $\begin{bmatrix} 40 & 50 & 55 & 40 \\ 25 & 65 & 44 & 30 \\ 35 & 40 & 40 & 35 \\ 35 & 40 & 35 & 50 \end{bmatrix}$

EXERCISE 14B

1 **a** $\begin{bmatrix} 9 & 1 \\ 3 & 3 \end{bmatrix}$ **b** $\begin{bmatrix} 6 & 8 \\ -1 & 1 \end{bmatrix}$ **c** $\begin{bmatrix} 3 & 4 \\ -6 & -1 \end{bmatrix}$ **d** $\begin{bmatrix} 0 & 0 \\ -11 & -3 \end{bmatrix}$

2 **a** $\begin{bmatrix} 20 & 1 & -8 \\ 8 & 10 & -2 \\ 1 & -5 & 18 \end{bmatrix}$ **b** $\begin{bmatrix} -14 & 9 & -14 \\ 12 & -6 & 14 \\ -5 & 3 & -4 \end{bmatrix}$ **c** $\begin{bmatrix} 14 & -9 & 14 \\ -12 & 6 & -14 \\ 5 & -3 & 4 \end{bmatrix}$

3 **a** Friday $\begin{bmatrix} 85 \\ 92 \\ 52 \end{bmatrix}$ Saturday $\begin{bmatrix} 102 \\ 137 \\ 49 \end{bmatrix}$ **b** $\begin{bmatrix} 187 \\ 229 \\ 101 \end{bmatrix}$

4 **a** **i** $\begin{bmatrix} 1.72 \\ 27.85 \\ 0.92 \\ 2.53 \\ 3.56 \end{bmatrix}$ **ii** $\begin{bmatrix} 1.79 \\ 28.75 \\ 1.33 \\ 2.25 \\ 3.51 \end{bmatrix}$ **b** subtract cost price from selling price **c** $\begin{bmatrix} 0.07 \\ 0.90 \\ 0.41 \\ -0.28 \\ -0.05 \end{bmatrix}$

5 **a** $\begin{array}{cc} \begin{matrix} \text{L} & \text{R} \end{matrix} & \\ \begin{bmatrix} 23 & 19 \\ 17 & 29 \\ 31 & 24 \end{bmatrix} & \begin{matrix} \text{fr} \\ \text{st} \\ \text{mi} \end{matrix} \end{array}$ **b** $\begin{array}{cc} \begin{matrix} \text{L} & \text{R} \end{matrix} & \\ \begin{bmatrix} 18 & 25 \\ 7 & 13 \\ 36 & 19 \end{bmatrix} & \begin{matrix} \text{fr} \\ \text{st} \\ \text{mi} \end{matrix} \end{array}$ **c** $\begin{array}{cc} \begin{matrix} \text{L} & \text{R} \end{matrix} & \\ \begin{bmatrix} 41 & 44 \\ 24 & 42 \\ 67 & 43 \end{bmatrix} & \begin{matrix} \text{fr} \\ \text{st} \\ \text{mi} \end{matrix} \end{array}$

6 **a** $x = -2,\ y = -2$ **b** $x = 0,\ y = 0$

7 **a** $\mathbf{A} + \mathbf{B} = \begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix}$ $\mathbf{B} + \mathbf{A} = \begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix}$

8 **a** $(\mathbf{A} + \mathbf{B}) + \mathbf{C} = \begin{bmatrix} 6 & 3 \\ -1 & 6 \end{bmatrix}$ $\mathbf{A} + (\mathbf{B} + \mathbf{C}) = \begin{bmatrix} 6 & 3 \\ -1 & 6 \end{bmatrix}$

EXERCISE 14C

1 **a** $\begin{bmatrix} 12 & 24 \\ 48 & 12 \end{bmatrix}$ **b** $\begin{bmatrix} 2 & 4 \\ 8 & 2 \end{bmatrix}$ **c** $\begin{bmatrix} \frac{1}{2} & 1 \\ 2 & \frac{1}{2} \end{bmatrix}$ **d** $\begin{bmatrix} -3 & -6 \\ -12 & -3 \end{bmatrix}$

2 **a** $\begin{bmatrix} 3 & 5 & 6 \\ 2 & 8 & 7 \end{bmatrix}$ **b** $\begin{bmatrix} 1 & 1 & 4 \\ 0 & 4 & 1 \end{bmatrix}$

c $\begin{bmatrix} 5 & 8 & 11 \\ 3 & 14 & 11 \end{bmatrix}$ **d** $\begin{bmatrix} 5 & 7 & 14 \\ 2 & 16 & 9 \end{bmatrix}$

3 **a** $\begin{array}{c} \begin{matrix} \text{A} & \text{B} & \text{C} & \text{D} \end{matrix} \\ \begin{bmatrix} 35 & 46 & 46 & 69 \\ 58 & 46 & 35 & 86 \\ 46 & 46 & 58 & 58 \\ 12 & 23 & 23 & 17 \end{bmatrix} \end{array}$ **b** $\begin{array}{c} \begin{matrix} \text{A} & \text{B} & \text{C} & \text{D} \end{matrix} \\ \begin{bmatrix} 26 & 34 & 34 & 51 \\ 43 & 34 & 26 & 64 \\ 34 & 34 & 43 & 43 \\ 9 & 17 & 17 & 13 \end{bmatrix} \end{array}$

4 **a** $\begin{array}{cc} \begin{bmatrix} 75 \\ 27 \\ 102 \end{bmatrix} & \begin{matrix} \leftarrow \text{VHS} \\ \leftarrow \text{DVD} \\ \leftarrow \text{gam.} \end{matrix} \end{array}$ $\begin{array}{cc} \begin{bmatrix} 136 \\ 43 \\ 129 \end{bmatrix} & \begin{matrix} \leftarrow \text{VHS} \\ \leftarrow \text{DVD} \\ \leftarrow \text{gam.} \end{matrix} \end{array}$ **b** $\begin{array}{cc} \begin{bmatrix} 211 \\ 70 \\ 231 \end{bmatrix} & \begin{matrix} \leftarrow \text{VHS} \\ \leftarrow \text{DVD} \\ \leftarrow \text{gam.} \end{matrix} \end{array}$

c total weekly average hirings **5** 12F

EXERCISE 14D

1 **a** $3\mathbf{A}$ **b** $\mathbf{O}$ **c** $-\mathbf{C}$ **d** $\mathbf{O}$ **e** $2\mathbf{A}+2\mathbf{B}$
f $-\mathbf{A}-\mathbf{B}$ **g** $-2\mathbf{A}+\mathbf{C}$ **h** $4\mathbf{A}-\mathbf{B}$ **i** $3\mathbf{B}$

2 **a** $\mathbf{X}=\mathbf{A}-\mathbf{B}$ **b** $\mathbf{X}=\mathbf{C}-\mathbf{B}$ **c** $\mathbf{X}=2\mathbf{C}-4\mathbf{B}$
d $\mathbf{X}=\frac{1}{2}\mathbf{A}$ **e** $\mathbf{X}=\frac{1}{3}\mathbf{B}$ **f** $\mathbf{X}=\mathbf{A}-\mathbf{B}$
g $\mathbf{X}=2\mathbf{C}$ **h** $\mathbf{X}=\frac{1}{2}\mathbf{B}-\mathbf{A}$ **i** $\mathbf{X}=\frac{1}{4}(\mathbf{A}-\mathbf{C})$

3 **a** $\mathbf{X}=\begin{bmatrix}3 & 6\\ 9 & 18\end{bmatrix}$ **b** $\mathbf{X}=\begin{bmatrix}\frac{1}{2} & -\frac{1}{4}\\ \frac{3}{4} & \frac{5}{4}\end{bmatrix}$ **c** $\mathbf{X}=\begin{bmatrix}-1 & -6\\ 1 & -\frac{1}{2}\end{bmatrix}$

EXERCISE 14E.1

1 **a** $[11]$ **b** $[22]$ **c** $[16]$ **2** $\begin{bmatrix}w & x & y & z\end{bmatrix}\begin{bmatrix}\frac{1}{4}\\ \frac{1}{4}\\ \frac{1}{4}\\ \frac{1}{4}\end{bmatrix}$

3 **a** $\mathbf{P}=\begin{bmatrix}27 & 35 & 39\end{bmatrix}$ $\mathbf{Q}=\begin{bmatrix}4\\ 3\\ 2\end{bmatrix}$

b total cost $=\begin{bmatrix}27 & 35 & 39\end{bmatrix}\begin{bmatrix}4\\ 3\\ 2\end{bmatrix}=\$291$

4 **a** $\mathbf{P}=\begin{bmatrix}10 & 6 & 3 & 1\end{bmatrix}$ $\mathbf{N}=\begin{bmatrix}3\\ 2\\ 4\\ 2\end{bmatrix}$

b total points $=\begin{bmatrix}10 & 6 & 3 & 1\end{bmatrix}\begin{bmatrix}3\\ 2\\ 4\\ 2\end{bmatrix}=56$ points

EXERCISE 14E.2

1 Number of cols. in A does not equal no. of rows in B.

2 **a** $m=n$ **b** 2×3 **c** B has 3 columns, A has 2 rows

3 **a** $\begin{bmatrix}28 & 29\end{bmatrix}$ **b** **i** $\begin{bmatrix}8\end{bmatrix}$ **ii** $\begin{bmatrix}2 & 0 & 3\\ 8 & 0 & 12\\ 4 & 0 & 6\end{bmatrix}$

4 **a** $\begin{bmatrix}3 & 5 & 3\end{bmatrix}$ **b** $\begin{bmatrix}-2\\ 1\\ 1\end{bmatrix}$

5 **a** $\mathbf{C}=\begin{bmatrix}12.5\\ 9.5\end{bmatrix}$ $\mathbf{N}=\begin{bmatrix}2375 & 5156\\ 2502 & 3612\end{bmatrix}$

b $\begin{bmatrix}78\,669.5\\ 65\,589\end{bmatrix}$ income from adult rides and children's rides **c** $144 258.50

6 **a** $\mathbf{R}=\begin{bmatrix}1 & 1\\ 1 & 2\\ 2 & 3\end{bmatrix}$ **b** $\mathbf{P}=\begin{bmatrix}7 & 3 & 19\\ 6 & 2 & 22\end{bmatrix}$ **c** $\begin{bmatrix}48 & 70\\ 52 & 76\end{bmatrix}$

d My costs at store A are \$48, my friend's costs at store B are \$76. **e** store A

EXERCISE 14F

1 **a** $\begin{bmatrix}16 & 18 & 15\\ 13 & 21 & 16\\ 10 & 22 & 24\end{bmatrix}$ **b** $\begin{bmatrix}10 & 6 & -7\\ 9 & 3 & 0\\ 4 & -4 & -10\end{bmatrix}$

c $\begin{bmatrix}22 & 0 & 132 & 176 & 198\\ 44 & 154 & 88 & 110 & 0\\ 176 & 44 & 88 & 88 & 132\end{bmatrix}$ **d** $\begin{bmatrix}115\\ 136\\ 46\\ 106\end{bmatrix}$

2 **a** $\begin{bmatrix}3 & 3 & 2\end{bmatrix}$ **b** $\begin{bmatrix}125 & 150 & 140\\ 44 & 40 & 40\\ 75 & 80 & 65\end{bmatrix}$ **c** $\begin{bmatrix}657 & 730 & 670\end{bmatrix}$

d $\begin{bmatrix}369 & 420 & 385\end{bmatrix}$ **e** $\begin{bmatrix}657 & 730 & 670\\ 369 & 420 & 385\end{bmatrix}$

3 $224 660

4 **a** $\begin{bmatrix}125 & 195 & 225\end{bmatrix}\times\begin{bmatrix}15 & 12 & 13 & 11 & 14 & 16 & 8\\ 4 & 3 & 6 & 2 & 0 & 4 & 7\\ 3 & 1 & 4 & 4 & 3 & 2 & 0\end{bmatrix}$

$-\begin{bmatrix}85 & 120 & 130\end{bmatrix}\times\begin{bmatrix}15 & 12 & 13 & 11 & 14 & 16 & 8\\ 4 & 3 & 6 & 2 & 0 & 4 & 7\\ 3 & 1 & 4 & 4 & 3 & 2 & 0\end{bmatrix}$

$=\$7125$

b $\begin{bmatrix}125 & 195 & 225\end{bmatrix}\times\begin{bmatrix}15 & 12 & 13 & 11 & 14 & 16 & 8\\ 4 & 3 & 6 & 2 & 0 & 4 & 7\\ 3 & 1 & 4 & 4 & 3 & 2 & 0\end{bmatrix}$

$-\begin{bmatrix}85 & 120 & 130\end{bmatrix}\times\begin{bmatrix}20 & 20 & 20 & 20 & 20 & 20 & 20\\ 15 & 15 & 15 & 15 & 15 & 15 & 15\\ 5 & 5 & 5 & 5 & 5 & 5 & 5\end{bmatrix}$

$=-\$9030$, i.e., a loss of \$9030

c $(\begin{bmatrix}125 & 195 & 225\end{bmatrix}-\begin{bmatrix}85 & 120 & 130\end{bmatrix})$
$\times\begin{bmatrix}15 & 12 & 13 & 11 & 14 & 16 & 8\\ 4 & 3 & 6 & 2 & 0 & 4 & 7\\ 3 & 1 & 4 & 4 & 3 & 2 & 0\end{bmatrix}$

EXERCISE 14G

1 $\mathbf{AB}=\begin{bmatrix}-1 & 1\\ -1 & 7\end{bmatrix}$ $\mathbf{BA}=\begin{bmatrix}0 & 2\\ 3 & 6\end{bmatrix}$ $\mathbf{AB}\neq\mathbf{BA}$

2 $\mathbf{AO}=\mathbf{OA}=\mathbf{O}$ **4** **b** $\mathbf{I}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

5 **a** $\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$ **b** $\begin{bmatrix}97 & -59\\ 118 & 38\end{bmatrix}$

6 **a** $\mathbf{A}^2$ does not exist **b** when $\mathbf{A}$ is a square matrix

8 **a** $\mathbf{A}^2+\mathbf{A}$ **b** $\mathbf{B}^2+2\mathbf{B}$ **c** $\mathbf{A}^3-2\mathbf{A}^2+\mathbf{A}$
d $\mathbf{A}^3+\mathbf{A}^2-2\mathbf{A}$ **e** $\mathbf{AC}+\mathbf{AD}+\mathbf{BC}+\mathbf{BD}$
f $\mathbf{A}^2+\mathbf{AB}+\mathbf{BA}+\mathbf{B}^2$ **g** $\mathbf{A}^2-\mathbf{AB}+\mathbf{BA}-\mathbf{B}^2$
h $\mathbf{A}^2+2\mathbf{A}+\mathbf{I}$ **i** $9\mathbf{I}-6\mathbf{B}+\mathbf{B}^2$

9 **a** $\mathbf{A}^3=3\mathbf{A}-2\mathbf{I}$ $\mathbf{A}^4=4\mathbf{A}-3\mathbf{I}$
b $\mathbf{B}^3=3\mathbf{B}-2\mathbf{I}$ $\mathbf{B}^4=6\mathbf{I}-5\mathbf{B}$ $\mathbf{B}^5=11\mathbf{B}-10\mathbf{I}$
c $\mathbf{C}^3=13\mathbf{C}-12\mathbf{I}$ $\mathbf{C}^5=121\mathbf{C}-120\mathbf{I}$

10 **a** **i** $\mathbf{I}+2\mathbf{A}$ **ii** $2\mathbf{I}-2\mathbf{A}$ **iii** $10\mathbf{A}+6\mathbf{I}$
b $\mathbf{A}^2+\mathbf{A}+2\mathbf{I}$ **c** **i** $-3\mathbf{A}$ **ii** $-2\mathbf{A}$ **iii** $\mathbf{A}$

11 **a** $\mathbf{AB}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ **b** $\mathbf{A}^2=\begin{bmatrix}\frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2}\end{bmatrix}$

c false as $\mathbf{A}(\mathbf{A}-\mathbf{I})=\mathbf{O}$ does not imply that $\mathbf{A}=\mathbf{O}$ or $\mathbf{A}-\mathbf{I}=\mathbf{O}$

d $\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$, $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, $\begin{bmatrix}a & b\\ \frac{a-a^2}{b} & 1-a\end{bmatrix}$, $b\neq 0$

12 For example, $\mathbf{A}=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$, gives $\mathbf{A}^2=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

13 **a** $a=3, b=-4$ **b** $a=1, b=8$

14 $p=-2, q=1$ **a** $\mathbf{A}^3=5\mathbf{A}-2\mathbf{I}$ **b** $\mathbf{A}^4=-12\mathbf{A}+5\mathbf{I}$

EXERCISE 14H

1 **a** $\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}=3\mathbf{I}$, $\begin{bmatrix}1 & -2\\ -\frac{2}{3} & \frac{5}{3}\end{bmatrix}$

b $\begin{bmatrix}10 & 0\\ 0 & 10\end{bmatrix}=10\mathbf{I}$, $\begin{bmatrix}0.2 & 0.4\\ -0.1 & 0.3\end{bmatrix}$

c $\begin{bmatrix}2 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 2\end{bmatrix}=2\mathbf{I}$, $\begin{bmatrix}-\frac{11}{2} & \frac{9}{2} & \frac{15}{2}\\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2}\\ 4 & -3 & -5\end{bmatrix}$