Function

■ Terminology and concepts

■ Function

A **function** is a rule or correspondence that assigns exactly one member of a set $\mathcal{B}$ to each member of a set $\mathcal{A}$. The set $\mathcal{A}$ is called the domain of the function. The set $\mathcal{B}$ is called the range of the function.

More formally,

A **function** is a set of ordered pairs such that each first component is paired with one and only one second component. That is, **F** is a function if $(x_1, y_1) \in \boldsymbol{F}$ and $(x_1, y_2) \in \boldsymbol{F}$ implies $y_1 = y_2$.

■ Domain, codomain, and range

We call the set of objects from which the first elements of the ordered pairs the **domain** the function.

The **codomain** set within which the second elements of the ordered pairs lie.

We call the set of second elements of the ordered pairs the **range** of a function is.

Understanding a function

■ Now

Domain

Range

Asymptotic Behavior

Zeros

Symmetry

Increasing - Decreasing

■ Soon

One-to-One

■ Later

Onto

Maximum - Minimum

■ The idea of a function

Think of $y = 3x + 2$ as a rule that pairs values of y with values of x. For example, this rule pairs $y = 5$ with $x = 1$. Additionally, the rule $y = 3x + 2$ pairs *exactly one* value of y with each value of x. A rule that pairs exactly one value of y with each value of x is called a **function**. Since the rule $y = 3x + 2$ pairs a unique value of y with each value of x, the rule $y = 3x + 2$ is a function and we say the rule gives *y as a function of x*.

It may at first seem that every rule must pair a unique y with every x. This is not the case. Consider the rule $y^2 = x$. When $x = 4$, y may equal *2* or -2. The rule pairs a value of x with two values of y. The rule $y^2 = x$ is not a function.

Note that while a function assigns a unique value of y to every value of x, it need not assign a unique value of x to every value of y. For example, $x = 2$ is a function that pairs all real x with *2*; that is $f(-1) = 2$, $f(10) = 2$, $f(\sqrt{3}) = 2$.

■ Notation

When we wish to draw attention to the fact that the equation $y = 3x + 2$ gives y as a function of x, we write $f(x) = 3x + 2$. Just as y is the dependant variable in $y = 3x + 2$, so also is $f(x)$ the dependant variable in $f(x) = 3x + 2$. The symbol $f(x)$ is pronounced "*f* of *x*" or "*f* at *x*". Thus, we say "*f* of *x* is $3x$ plus *2*". Do note that the symbol $f(x)$ does not denote the function, but the *value* of the function at x. When we wish to refer to the function, as opposed to the result of function operating of x, we simply write *f*. The symbol *f* denotes the function. The symbol $f(x)$ denotes the function evaluated at x.

This notation provides a convenient way to refer to the result of substituing *1* for x in $3x + 2$. We write $f(1) = 5$, because $5 = 3(1) + 2$. Accordingly, we write $f(4) = 14$, because $14 = 3(4) + 2$. The collection of symbols $f(4) = 14$ is pronounced "*f* of *4* equals *14*". That is, the function evaluated at x equal to 4 yields *14*.

■ More ideas associated with the idea of a function

As mentioned, a function is a rule that pairs values of x with unique values of y, or, as we say now, with $f(x)$. More precisely, we say that a function is a rule that pairs elements of one set with elements of another set. Equivalently, we say that a function is a correspondance between elements of one set with elements of another set such that no element of the first set corresponds to more than one element of the second set. We list the corresponding pairs by using the ordered pair notation. In the example $f(x) = 3x + 2$, *1* corresponds to *5* and *4* corresponds to *14*; thus, we write (1, 5), (4, 14); in general, $(x, f(x))$.