

### ■ Statement verses rule

Consider two utterances.

- i) The author of the Odyssey was Homer.
- ii) Walk on the right side of the hallway.

The first utterance is a statement, the second articulates a rule. It makes sense to ask whether or not (i) is true, but it is nonsense to ask whether or not (ii) is true. A statement is either true or false. As for a rule, it is not that it is neither true or false, but rather that the categories *true* and *false* do not apply to it.

Now consider two more utterances.

- i') Seven equals two times three plus one.
- ii') Multiply two by three, then add one to the product.

Like (i), (i') is a statement. It makes sense to ask whether (i') is true or false. Like (ii), (ii') is a rule and it is nonsense to ask whether it is true or false.

It is easy to write (i') in mathematical symbols. We write

$$7 = 2 (3) + 1 \tag{1}$$

Here is how we write the rule expressed by (ii')

$$y = 2 (3) + 1 \tag{2}$$

In EQ2, no assertion is made that  $y$  equals  $2(3) + 1$ . Rather, the idea is that a computation is to be carried out. The letter  $y$  is merely standing for the result of the computation. It is like old arithmetic books in which " $2 + 5 = \blacksquare$ " indicated to the student that the sum of 2 and 5 was to be computed.

Typically, we are interested in rules that are more general than EQ2. Like the following

$$y = 2x + 1 \tag{3}$$

EQ3 says to multiply a number by 2, then add 1 to that product. Again, the letter  $y$  merely stands for the result of the computation. What number is  $y$ ? Well, that depends on what number is put into the rule; if we put 3 for  $x$ , then the rule produces a value of 7. Put in 4, the rule spits out 9. The rule gives  $y$  as a function of  $x$ .

There is a very nice notation that is typically used when we refer to a rule that gives us a number as a function of another number. Using this notation, we would write EQ3 this way

$$f(x) = 2x + 1 \tag{4}$$

Here the symbol  $f(x)$  stands for the result of applying the rule to a number  $x$ . We pronounce the symbol  $f(x)$  as "f of x". Using this notation,

$$f(-1) = -1, f(0) = 1, f(1) = 3, f(2) = 5, f\left(\frac{1}{2}\right) = 2, f(\pi) = 2\pi + 1.$$