HH-REV 8A

REVIEW SET 8A

1 Find real numbers a and b such that:

a $a+ib=4$ b $(1-2i)(a+bi)=-5-10i$ c $(a+2i)(1+bi)=17-19i$

2 If $z=3+i$ and $w=-2-i$, find in simplest form:

a $2z-3w$ b $\frac{z^*}{w}$ c z^3

3 Prove the following: zw^*-z^*w is purely imaginary or zero for all complex numbers z and w.

4 Expand and simplify: a $(3x^3+2x-5)(4x-3)$ b $(2x^2-x+3)^2$

5 Carry out the following divisions: a $\frac{x^3}{x+2}$ b $\frac{x^3}{(x+2)(x+3)}$

6 State and prove the Remainder theorem.

7 $-2+bi$ is a solution to $z^2+az+[3+a]=0$. Find a and b given that they are real.

8 Find all zeros of $2z^4-5z^3+13z^2-4z-6$.

9 Factorise $z^4+2z^3-2z^2+8$ into linear factors.

10 Find a quartic polynomial with rational coefficients having $2-i\sqrt{3}$ and $\sqrt{2}+1$ as two of its zeros.

11 If $f(x)=x^3-3x^2-9x+b$ has $(x-k)^2$ as a factor, show that there are two possible values of k. For each of these two values of k, find the corresponding value for b and hence solve $f(x)=0$.

12 Find exact x-values when: a $x^2+2x\geqslant 5$ b $x<\frac{9}{x}$ c $\left|\frac{x}{8-x}\right|\leqslant 2$

13 Find k if the line with equation $y=2x+k$ does not meet the circle with equation $x^2+y^2+8x-4y+2=0$.

Hint: Solve simultaneously to get a quadratic and find k for which $\Delta<0$.

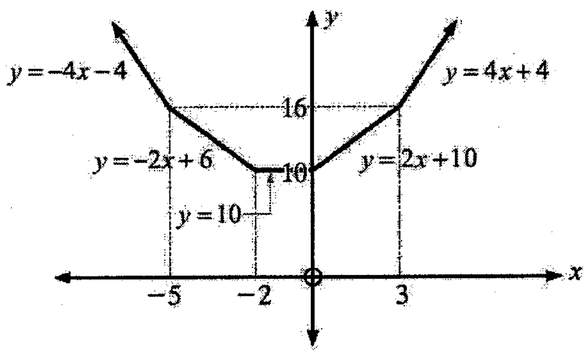
14 When $P(x)=x^n+3x^2+kx+6$ is divided by $x+1$ the remainder is 12. When $P(x)$ is divided by $x-1$ the remainder is 8. Find k and n given that $34<n<38$.

15 If α and β are two of the roots of $x^3-x+1=0$, show that $\alpha\beta$ is a root of $x^3+x^2-1=0$. [**Hint:** Let $x^3-x+1=(x-\alpha)(x-\beta)(x-\gamma)$.]

REVIEW SET 8B, 8C, 8D
Click on the icon to obtain printable review sets and answers

REVIEW SET 8B REVIEW SET 8C REVIEW SET 8D

4 **a**



b **ii** Anywhere between O and Q, minimum length of cable is 10 km.

iii At O, minimum length of cable is 17 km.

REVIEW SET 8A

1 **a** $a=4,\ b=0$ **b** $a=3,\ b=-4$
c $a=3,\ b=-7$ or $a=14,\ b=-\frac{3}{2}$

2 **a** $12+5i$ **b** $-1+i$ **c** $18+26i$

4 **a** $12x^4-9x^3+8x^2-26x+15$
b $4x^4-4x^3+13x^2-6x+9$

5 **a** $x^2-2x+4-\dfrac{8}{x+2}$ **b** $x-5+\dfrac{19x+30}{(x+2)(x+3)}$

6 "If a polynomial $P(x)$ is divided by $x-k$ until a constant remainder R is obtained then $R=P(k)$."

7 $a=7,\ b=0$ or $a=4,\ b=\pm\sqrt{3}$ **8** $1,-\frac{1}{2},1\pm i\sqrt{5}$

9 $(z+2)^2(z-1+i)(z-1-i)$

10 $P(z)=z^4-6z^3+14z^2-10z-7$

11 $k=3,\ b=27,\ x=3,-3;\ k=-1,\ b=-5,\ x=-1,5$

12 **a** $x\in\,]-\infty,-1-\sqrt{6}]$ or $x\in[-1+\sqrt{6},\infty[$
b $x\in\,]-\infty,-3\,[$ or $x\in\,]\,0,3\,[$
c $x\in\,]-\infty,\frac{16}{3}]$ or $x\in[16,\infty[$

13 $k\in\,]-\infty,\ 10-3\sqrt{10}\,[$ or $k\in\,]\,10+3\sqrt{10},\infty[$

14 $k=-2,\ n=36$

15 **Another hint:** Show that: $(\alpha\beta)^3+(\alpha\beta)^2-1=0$

EXERCISE 9A

1 24 **2** **a** 4 **b** 8 **c** 24 **3** 6 **4** 42 **5** 1680
6 **a** 125 **b** 60 **7** 17 576 000 **8** **a** 4 **b** 9 **c** 81

EXERCISE 9B

1 **a** 13 **b** 20 **c** 19 **d** 32

EXERCISE 9C

1 1, 1, 2, 6, 24, 120, 720, 5040, 40 320, 362 880, 3 628 800

2 **a** 6 **b** 30 **c** $\frac{1}{7}$ **d** $\frac{1}{30}$ **e** 100 **f** 21

3 **a** n **b** $(n+2)(n+1)$ **c** $(n+1)n$

4 **a** $\frac{7!}{4!}$ **b** $\frac{10!}{8!}$ **c** $\frac{11!}{6!}$ **d** $\frac{13!}{10!3!}$ **e** $\frac{3!}{6!}$ **f** $\frac{4!16!}{20!}$

5 **a** $6\times4!$ **b** $10\times10!$ **c** $57\times6!$ **d** $131\times10!$
e $81\times7!$ **f** $62\times6!$ **g** $10\times11!$ **h** $32\times8!$

6 **a** 11! **b** 9! **c** 8! **d** 9 **e** 34 **f** $n+1$
g $(n-1)!$ **h** $(n+1)!$

EXERCISE 9D

1 **a** W, X, Y, Z
b WX, WY, WZ, XW, XY, XZ, YW, YX, YZ, ZW, ZX, ZY
c WXY, WXZ, WYX, WYZ, WZX, WZY, XWY, XWZ, XYW, XYZ, XZW, XZY, YWX, YWZ, YXW, YXZ, YZX, YZW, ZWX, ZWY, ZXW, ZXY, ZYW, ZYX

2 **a** AB, AC, AD, AE, BA, BC, BD, BE, CA, CB, CD, CE, DA, DB, DC, DE, EA, EB, EC, ED
b ABC, ABD, ABE, ACB, ACD, ACE, ADB, ADC, ADE, AEB, AEC, AED, BAC, BAD, BAE, BCA, BCD, BCE, BDA, BDC, BDE, BEA, BEC, BED, CAB, CAD, CAE, CBA, CBD, CBE, CDA, CDB, CDE, CEA, CEB, CED, DAB, DAC, DAE, DBA, DBC, DBE, DCA, DCB, DCE, DEA, DEB, DEC, EAB, EAC, EAD, EBA, EBC, EBD, ECA, ECB, ECD, EDA, EDB, EDC
2 at a time: 20 3 at a time: 60

3 **a** 120 **b** 336 **c** 5040 **4** **a** 12 **b** 24 **c** 36

5 720 **a** 24 **b** 24 **c** 48 **6** **a** 343 **b** 210 **c** 120

7 720, 72 **8** **a** 648 **b** 64 **c** 72 **d** 136

9 **a** 120 **b** 48 **c** 72 **10** **a** 3 628 800 **b** 241 920

EXERCISE 9E

1 **a** 8 **b** 28 **c** 56 **d** 28 **e** 1

3 ABC, ABD, ABE, ACD, ACE, ADE, BCD, BCE, BDE, CDE

4 $C_{11}^{17}=12\,376$ **5** $C_5^9=126,\ C_1^1\,C_4^8=70$

6 $C_3^{13}=286,\ C_1^1\,C_2^{12}=66$

7 $C_5^{12}=792$ **a** $C_2^2\,C_3^{10}=120$ **b** $C_1^2\,C_4^{10}=420$

8 $C_3^3\,C_0^1\,C_6^{11}=462$

9 **a** $C_1^1\,C_3^9=84$ **b** $C_0^2\,C_4^8=70$ **c** $C_0^2\,C_1^1\,C_3^7=35$

10 **a** $C_5^{16}=4368$ **b** $C_3^{10}\,C_2^6=1800$ **c** $C_5^{10}\,C_0^6=252$
d $C_3^{10}\,C_2^6+C_4^{10}\,C_1^6+C_5^{10}\,C_0^6=3312$
e $C_5^{16}-C_5^{10}\,C_0^6-C_0^{10}\,C_5^6=4110$

11 **a** $C_2^6\,C_1^3\,C_2^7=945$ **b** $C_2^6\,C_3^{10}=1800$
c $C_5^{16}-C_0^9\,C_5^7=4347$

12 $C_2^{20}-20=170$ **13** **a** **i** $C_2^{12}=66$ **ii** $C_1^{11}=11$
b **i** $C_3^{12}=220$ **ii** $C_2^{11}=55$

14 $C_4^9=126$

15 **a** the different committees of 4 to be selected from 5 men and 6 women in all possible ways **b** C_r^{m+n}

16 **a** $\dfrac{C_6^{12}}{2}=462$ **b** $\dfrac{C_4^{12}\,C_4^8\,C_4^4}{3!}=5775$

EXERCISE 9F

1 **a** x^3+3x^2+3x+1 **b** $x^3+6x^2+12x+8$
c $x^3-12x^2+48x-64$ **d** $8x^3+12x^2+6x+1$
e $8x^3-12x^2+6x-1$ **f** $27x^3-27x^2+9x-1$
g $8x^3+60x^2+150x+125$ **h** $8x^3+12x+\frac{6}{x}+\frac{1}{x^3}$

2 **a** $x^4+8x^3+24x^2+32x+16$
b $x^4-8x^3+24x^2-32x+16$
c $16x^4+96x^3+216x^2+216x+81$
d $81x^4-108x^3+54x^2-12x+1$
e $x^4+4x^2+6+\frac{4}{x^2}+\frac{1}{x^4}$
f $16x^4-32x^2+24-\frac{8}{x^2}+\frac{1}{x^4}$

3 **a** $x^5+10x^4+40x^3+80x^2+80x+32$
b $x^5-10x^4+40x^3-80x^2+80x-32$
c $32x^5+80x^4+80x^3+40x^2+10x+1$
d $32x^5-80x^3+80x-\frac{40}{x}+\frac{10}{x^3}-\frac{1}{x^5}$

4 **a** 1 6 15 20 15 6 1