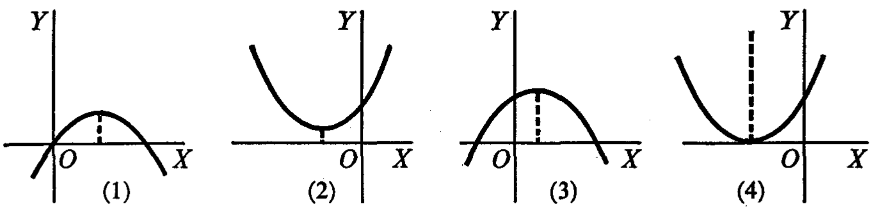
Miscellaneous Exercises [B]

[08-12-01-P2-10]

1. A quadratic function is such that $f(0) = f(4) = 4$. The maximum value of f is 8. Find the zeros of f and determine $f(x)$.
2. If x_1, x_2 are the roots of $4x^2 + 4px = 2p + 1$, show that
$$(x_1 - x_2)^2 = (p + 1)^2.$$
3. If x_1, x_2 are the roots of $ax^2 + bx + c = 0$, show that (a) $\frac{1}{x_1} + \frac{1}{x_2} = -\frac{b}{c}$, (b) $x_1{}^2 + x_2{}^2 = \frac{b^2 - 2ac}{a^2}$.
4. Find the value of k for which the parabola $y = x^2 - 5x + k$ has a chord 3 units long on the x-axis.

5. In the equation $ax^2 + bx + c = 0$, state the conditions satisfied by a, b, c if (a) the sum of the roots equals the product of the roots, (b) -1 is a root, (c) one root is the negative of the other, (d) one root is the reciprocal of the other, (e) the roots are equal.
6. Diagrams (1)–(4) are graphs of $y = ax^2 + bx + c$. In each case, state what may be deduced from the graph concerning the value of the discriminant $b^2 - 4ac$, and concerning the values of any of the individual coefficients a, b, c.



7. Evaluate $ax^2 - bx$ when $x = \frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}$.
8. Show that if the roots of $ax^2 + bx + c = 0$ differ by k, the discriminant has the value a^2k^2.
9. Show that the roots of $4x^2 - (4a + 8)x + a^2 + 4a = 0$ differ by 2.
10. If x_1 is a root of $ax^2 + bx + c = 0$, state the value of $ax_1{}^2 + bx_1$ and show that $(x - x_1)(ax + ax_1 + b)$ is identical with $ax^2 + bx + c$.
11. Show that for any real value of x the value of $\frac{x}{x^2 + 1}$ is not greater than $\frac{1}{2}$ or less than $-\frac{1}{2}$. Sketch the graph of the function defined by $y = \frac{x}{x^2 + 1}$.
12. (a) Find the value of k so that the roots of $(x - 2)(4 - x) = x + k$ shall be equal.
    (b) Using the value of k determined in (a), draw the graphs of $y = (x - 2)(4 - x)$ and $y = x + k$.
    (c) State the geometric relationship of the line to the parabola.
13. Find the values of a, b, c if the parabola $y = ax^2 + bx + c$ passes through the points (1, 1), (2, 4), (5, 1). Obtain the coordinates of the vertex of the parabola.
14. A quadratic function f is such that $f(1) = 10$, $f(2) = 25$, $f(3) = 35$. Determine $f(t)$.
15. (a) Find the equation of the parabola which has the point $(a, 0)$ as focus and the line $x = -a$ as directrix. (b) Show that if t is any real number, the point $(at^2, 2at)$ is on this parabola.

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1. $\{2 + 2\sqrt{2},\ 2 - 2\sqrt{2}\}$;

   $f(x) = 4 + 4x - x^2$

4. $k = 4$

5. a. $b + c = 0$

   b. $a - b + c = 0$

   c. $b = 0$

   d. $a - c = 0$

   e. $b^2 - 4ac = 0$

6. (1). $D>0,\ a<0,\ b>0,\ c=0$

   (2). $D<0,\ a>0,\ b>0,\ c>0$

   (3). $D>0,\ a<0,\ b>0,\ c>0$

   (4). $D=0,\ a>0,\ b>0,\ c>0$

7. $-c$

10. $ax_1^2 + bx_1 = -c$

12. a. $k = -1\frac{3}{4}$

    c. Tangent

13. $a = -1,\ b = 6,\ c = -4$;

    $(3,\ 5)$

14. $f(t) = -\frac{5}{2}(t^2 - 9t + 4)$

15. a. $y^2 = 4ax$